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Maintaining Stability with Distributed Generation in a Restructured Industry

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Maintaining Stability with Distributed Generation in a Restructured Industry

Judith Cardell Marija Ilic

Abstract

A set of reduced order, linearized, dynamic models for distributed generators is developed along with a framework for modeling the generators in a power distribution system. Analysis of this distributed system structure raises two issues. The first is that the simulations demonstrate, unexpectedly, that a small load disturbance is capable of causing frequency instability in the primary dynamics of the distributed generators. Eigenanalysis of the instability suggests that it is a system phenomenon. The second issue is that the system matrix is found to not *have a block diagonal dominant structure raising questions over the possible implementation of decentralized control strategies. A method to regain system stability along with an example of implementing this method are presented, along with the generator models.*

1. Introduction

The potential for an increased number of distributed generators in the existing power system raises a number of engineering questions concerning stability and control of the power system, both locally in the distribution system and at the high voltage and central control facilities. Significant research has been done in this area through investigations into the microgrid concept [13]. The standard stability issues are those of frequency and voltage, with an additional control interest focusing on the technical capability for decentralized control of generation, to parallel the growing decentralized ownership.

The research discussed in this paper focuses on developing dynamic, state space models of distributed generators and the distribution system. The paper describes the model development first, followed by a stability analysis based on eigenanalysis and sensitivity factors, and finally discusses an approach for allowing decentralized control within a distributed utility.

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2. The Model 2.1. Time Scale Definitions

The goals of the modeling are first to simulate the dynamic interactions of distributed generators in response to a system disturbance and second to analyze the effectiveness of different control strategies in maintaining system stability and allowing decentralization. The different dynamic phenomena and corresponding control responses can be distinguished by the time scale at which they occur. Primary dynamics from 5 seconds to 1 minute, and tertiary dynamics of several minutes to several hours. In practice, secondary level controls are designed assuming that the primary dynamics have settled, and tertiary controls assume that secondary dynamics have settled. Modeling and analysis of dynamic phenomena must mirror these assumptions. The results in this paper focus solely on primary dynamics.

2.2. Modeling Goals and Assumptions

The modeling effort is based on building decoupled, linearized state space models¹ for each type of distributed generator, and coupling² them through a distribution system model. State space models have been developed for steam turbines, hydroelectric turbines, combustion turbines, combined cycle plants, wind turbines and inverters (to be used with fuel cells and photovoltaics). Numerous dynamic models exist for each of these technologies, however the majority are very complex, involving a large number of state variables. In developing the models for this project, the objective is to represent each generator with a small number of state variables (three to four) so that interconnected system models, which each include a

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¹ Decoupled here refers to the assumption that for small disturbances frequency and voltage dynamics are essentially independent, and are related to real power and reactive power respectively.

² Coupling' here refers to the physical connection of the generators with each other by means of the distribution system.

number of the distributed generators, will not be overly complex. A second objective is to develop each set of local state equations such that they incorporate PG as the system coupling variable. The traditional system coupling variable is rotor angle, δ.

2.3. Individual Component Models

The models which include a synchronous generator all use a form of the swing equation as the generator state equation

$$
J\ddot{\mathbf{d}} + D\dot{\mathbf{d}} = P_m - P_e
$$

where $Pe \equiv P_G$, the electrical power output. Use of this equation facilitates the inclusion of the system coupling variable, P_G in each set of local state equations. This generator equation differs for different technologies, since the mechanical power from the turbine, P_m , has a different representation for each turbine type. A wind turbine – induction generator model is presented at the end of this section.

2.3.1. Steam-Turbine-Generator. The simplest model of this form is for the steam turbine where P_m is equivalent to P_t , the local state variable for the turbine. The other state variables are $\omega_{\rm G}$ for the generator (where $w_G \equiv \dot{d}$) and *a* for the governor. The full set of steam turbine-generator equations is

$$
M\dot{\mathbf{w}}_G = (e_T - D)\mathbf{w}_G + P_t - P_G
$$

\n
$$
T_u \dot{P}_t = -P_t + k_t a
$$

\n
$$
T_g \dot{a} = -\mathbf{w}_G - ra + \mathbf{w}^{ref}
$$

In these equations M is the inertia constant, e_T is a coefficient representing the turbine self-regulation, defined as $\partial P_t / \partial W_G$, D is the damping coefficient, T_u is the time constant representing the delay between the control valves and the turbine nozzles, k_t is a proportionality factor representing the control valve position variation relative to the turbine output variation, T_g is the time constant of the valveservomotor-turbine gate system, and r is the permanent speed droop of the turbine. These parameters are defined in references [1, 7, 8]. ω^{ref} is the reference frequency set by the secondary controls, and so is assumed constant in the primary dynamics time scale. P_G is defined as an input to this system of equations.

2.3.2. Hydro-Turbine-Generator. A slightly more complex set of equations than that for the steam turbine is that for a hydro turbine-generator. This model follows the model for a low-head hydro facility developed in [1], with additional information for parameter values from [6, 16]. The state variables for this technology are $\omega_{\rm G}$ for the generator equation, q for penstock flow, v for governor droop and *a* for gate position.

$$
M\dot{w}_G = -(e_H + D)w_G + k_q q - k_w a - P_G
$$

\n
$$
\dot{q} = w_G / T_f - q/T_q + a/T_w
$$

\n
$$
T_e \dot{v} = -v + r'a
$$

\n
$$
T_S \dot{a} = -w_G + v - (r_h + r')a + w^{ref}
$$

M and D are the inertia and damping constants as above. e_H , k_q and k_w are all ratios of constants from a standard hydro-turbine diagram referred to as the universal water turbine stead-state performance diagram (see for example Figure 8 in [1]), T_f , T_q , and T_w are also all ratios of constants from the same diagram, multiplied by T_c , the time constant of the penstock, T_e is the time constant of the valve-turbine gate system, T_s is the time constant of the servomotor gates, r_h is the permanent speed droop, and r' is the transient speed droop. These coefficients are contained in references [1, 7, 8].

2.3.3. Combustion-Turbine-Generator. The set of equations used for a combustion turbine are presented below. The equations represent the generator $(\omega_{\rm G})$, fuel controller (V_{CE}), and fuel flow (both W_F and $W_Fdot)$

$$
M\dot{\mathbf{v}}_G = -D\mathbf{v}_G + cW_F - P_G
$$

\n
$$
b\dot{V}_{CE} = -K_D \mathbf{v}_G - V_{CE}
$$

\n
$$
\dot{W}_F = W_F dot
$$

\n
$$
a\dot{W}_F dot = aV_{CE} - dW_F - bW_F dot
$$

These equations are derived from the equations and models found in [6, 15]. M and D are the inertia and damping coefficients respectively. a, b and c are transfer function coefficients for the fuel system, and K_D is the governor gain. β and δ are algebraic functions of the parameters in the references, defined as $\mathbf{b} \equiv b + c \mathbf{t}_F$ and $\mathbf{d} \equiv c + aK_F$, where τ_F is the

fuel system time constant, and K_F is the fuel system feedback gain.

2.3.4. Combined Cycle Plant. The combined cycle combustion turbine, CCCT, model has equations for both a combustion turbine and steam turbine driving the synchronous generator. The generator output (swing equation) has mechanical power from both the steam and combustion turbines as input. The model develop for the CCCT uses the equations for the fuel controller (V_{CE}), and the fuel flow (both W_F and WFdot) from the CT model. The fifth equation represents the thermodynamic coupling between the turbines, using the air flow, W_{air} as the coupling variable. The sixth and seventh equations are for the steam turbine, where P_{ST} represents the mechanical power output from the steam turbine.

The new parameters in this set of equations are T_v , the vane control time constant, d, the ratio of the fuel flow to rotor speed, T_M and T_B are time constants for a simplified steam turbine modeled in Figure 8 of [17], m and n represent the enthalpy in the mass flow of the air and fuel respectively, p is a function of the turbine exhaust temperature (see function f_1 in [15]), and the function f_2 , also defined in [15], represents the turbine torque. This model was derived from the models in [4, 9, 15, 17].

$$
W\dot{w}_G = -Dw_G + (f_2 + P_{ST}) - P_G
$$

\n
$$
b\dot{V}CE = -K_D V_G - V_{CE} + K_D w^{\text{ref}}
$$

\n
$$
\dot{w}_F = W_F \text{dot}
$$

\n
$$
a\dot{w}_F \text{dot} = aV_{CE} - gW_F - bW_F \text{dot}
$$

\n
$$
T_v\dot{w}_{\text{air}} = d\mathbf{w}_G + V_{CE} - W_{\text{air}}
$$

\n
$$
\dot{P}_{ST} = P_{ST} \text{dot}
$$

\n
$$
(T_M T_B) \dot{P}_{ST} \text{dot} = -p\mathbf{w}_G + nW_F + mW_{air} - P_{ST}
$$

\n
$$
-(T_M + T_B)P_{ST} \text{dot}
$$

2.3.5. Wind Turbine – Induction Generator. The model for the wind turbine system is based substantially on the work in [3], which specifically developed a model to be used for dynamic studies of dispersed wind turbine applications. The model below differs from that model in that it has a single torque input, T_w (defined as the wind torque), rather than both T_w and Tturbine. Turbine torque is expressed in terms of the turbine inertia and wind torque.

The wind turbine system is modeled as two rotating masses – the turbine and generator rotors – coupled by a tortional spring. The three equations represent the induction generator, ω_{G} , the tortional spring, d, and the wind turbine, ω _T. Note that the wind turbine system has no generator control, as in the other models, as is appropriate for a non-dispatchable technology.

 M_G , M_T , D_G and D_T are the generator and turbine inertias and damping coefficients. T_w is the wind torque, and is an input to the system of equations, as is PG, and K is the spring constant of the tortional spring used to model the drive train coupling between the two rotors. References [12, 16] were also used for developing this model.

$$
\dot{\mathbf{w}}_G = \frac{-(D_G - D_T)}{M_G} \mathbf{w}_G + \frac{(D_G - D_T)}{M_G} \mathbf{w}_T
$$
\n
$$
+ \frac{1}{M_G} T_W - \frac{1}{M_G} P_G
$$
\n
$$
\dot{\mathbf{d}} = -\mathbf{w}_G + \mathbf{w}_T
$$
\n
$$
\dot{\mathbf{w}}_T = \frac{D_T}{M_T} \mathbf{w}_G - \frac{K}{M_T} \mathbf{w}_T - \frac{D_T}{M_T} \mathbf{w}_T + \frac{1}{M_T} T_W
$$

Steam Turbine Parameters						
M	1.26	k_t	0.95			
D	\overline{c}	T_{g}	0.25			
e t	0.15	r	0.05			
T_u	0.2					
Hydro Turbine Parameters						
M	1.5	T_q	0.72			
D	\overline{c}	T_w	0.76			
e_h	-0.22	T_e				
k_q	2.78	r'	0.4			
k_w	1.52	T_s	0.1			
T_f	-3.6	r_h	0.05			
		Combustion Turbine Parameters				
M	11.5	a	0.45			
D		a	1			
\mathbf{C}	\overline{c} $\mathbf{1}$					
		t_F	0.4 0			
K D b	25 0.05	K F				
Wind Turbine Parameters						
M G	5	D T	1			
M T	11	K	400			

Table 1: Parameter values for generator models

2.4. Generator Model Parameter Values

The specific values for the parameters in the generator models, which are used in the system simulations in this paper are presented in Table 1. Complete development of these values is found in [2].

2.5. The Extended State Space

To build the complete system model, the individual generator models are coupled to each other via the distribution system. To achieve this coupling each set of equations representing a local generator state space is extended to include the system coupling variable, selected to be power output, or P_{Gi} . This choice of coupling variable, rather than the traditional choice of rotor angle d, follows directly from the process of linearizing the full system model. Through use of the Jacobian matrix it facilitates retaining in the extended state space those aspects of the system topology which directly impact the dynamic behavior (line strength, interconnections and electrical distances) [10].

The following equation for P_G is obtained, as defined fully in [2, 10]

$$
\dot{P}_G = K_P \mathbf{w}_G + D_P \dot{P}_L \tag{1}
$$

where \dot{P}_L , representing a load disturbance, is an input variable to the system, and the matrices K_P and D_P are derived from the Jacobian matrix. Equation (1) is added to each set of local state space equations to form what is referred to as the extended state space.

2.6. The Full System Model

The state equations for the individual generators can be written in matrix form and represented as

$$
\dot{x}_{LC} = A_{LC} x_{LC} + C_M P_G + Bu \tag{2}
$$

where x_{LC} is the local state vector, u is the input ω^{ref} , and the bold variables represent the matrices with the elements of $C_M = 1/M$ and A_{LC} defined as the local system matrix. The equations for each generator can be written in this form.

Incorporating P_G from equation (1) for the extended state space, the full system model can be written as

$$
\dot{x}_{ext} = A_{ext} x_{ext} + D_p \dot{P}_L \tag{3}
$$

where x_{ext} is the extended state space, state vector, and A is partitioned into block diagonal partitions composed of A_{LC} , C_M , $K_P E$ and 0. Numerous dynamic models exist for each of the various distributed technologies, however the majority are very complex, involving a large number of state variables. In developing the models for this project [2], the objective was to represent each generator with a small number of state variables so that full system models which include a number of the distributed generators would not be overly complex. A second objective was to ensure that the local state equations incorporated P_G making them mutually compatible in the extended state space, with the use of P_G as the system coupling variable.

2.7. Model Specification

For the results presented here, all distributed generators are located in the distribution system. Everything behind the local substation is grouped together and modeled as an infinite bus, filling the role of the slack bus for the system. Within the distribution system model every bus has either a load or a generator, or possibly both.

The system model is defined by specifying the distribution system topology, the location and size of loads and the location, size and type of the generators. The model inputs (forming the input vector to the system of state equations \dot{P}_L), specify the location and timing of system disturbance (a small increase of decrease in demand). The model simulates the dynamics due both to disturbances and to specified control actions, (for the results presented in this paper only primary controls are active).

The output from the simulation is the dynamic behavior of all the state variables, with frequency and real power output typically being of greater interest than the others.

3. Stability Analysis 3.1. Sample Systems

The distribution system modeled in the following examples is shown in Figure 1, with the line parameters defined in [5]. The first example discussed has a 1 p.u. steam turbine at bus 11, and a 1 p.u. combustion turbine at bus 23 (as well as a slack bus at the substation). The load disturbance at bus 21 is a 0.1 p.u. increase in demand at time equals 2 seconds.

Figure 2 shows the frequency deviation in the primary dynamics from the equilibrium point for this system. The third line in the figure represents the slack bus. The rotor frequencies for both distributed generators are seen to oscillate around the nominal 60Hz frequency, and settle to a slightly slower value. The behavior demonstrated by the system in Figure 2 is the expected behavior.

Figure 1: 30 Bus Distribution System [5]

Figure 2: Frequency Deviation from Equilibrium for Steam and Combustion Turbines

If the same distribution system is modeled with a single hydroelectric generator at bus 11 the frequency becomes unstable. With a combustion turbine added to the system at bus 23, the instability caused by the hydroelectric plant creates instability at the combustion turbine bus as well. See Figure 3. Note that the instability remains local to the distribution system; the slack bus frequency is unaffected, as a result of the large inertia used to represent the system behind the substation.

3.2. Eigenvalues and Participation Factors

Eigenanalysis of the system matrices, A_{LCi} and A, was used to begin identifying the cause of the instability. The eigenvalues for the individual generators and for the three sample systems introduced above are listed in Tables 2 and 3 respectively (the eigenvalues of A_{LC} for each generator and of A for each system). The tables clearly show that each generator is individually stable, while the systems that include a hydro plant are unstable. (Note that the zero eigenvalue for each system is inherent to the structure of power system, and does not represent a stability problem [10].)

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Figure 3: Frequency Deviation from Equilibrium for Hydroelectric and Combustion Turbines

The cause of the unstable eigenvalues is investigated next. If these unstable eigenvalues could be uniquely associated with one or more of the state variables, then the identified state variable could be directly controlled to regain system stability. Participation factors, developed fully in reference [14], were used for this part of the analysis. A participation factor p_{ii} is defined as

$$
p_{ij} = w_{ij} v_{ij}
$$

where w_{ij} is the i^{th} entry in the j^{th} left eigenvector, and v_{ij} is analogous for the right eigenvector. The p_{ij} provide a measure of the contribution of the ith state variable to the jth eigenvalue. Participation factors were calculated for the unstable modes for the systems discussed in this paper, as well as others with the generators or load disturbances located at different buses. This analysis identified different state variables as causing the instability in the system for each different system configuration. These results show that the instability is not caused by a single state variable, but is more appropriately identified as truly a system phenomenon.

Steam Turbine	Combustion Turbine	Hydro Turbine
$-0.50 + j1.63$	$-20.24 + j4.95$	$-0.03 + i1.48$
$-0.50 + j1.63$	$-20.24 - j4.95$	$-0.03 - i1.48$
-5.66	$-0.12 + j4.83$	-7.17
	$-0.12 + j4.83$	-0.36

Table 2: Eigenvalues of Individual Generator Models

4 CT System	4 CT con't	Hydro & CT
$-21.23 + j4.94$	$-0.46 + j2.95$	$-20.30 + j2.41$
$-21.23 - j4.94$	$-0.46 - j2.95$	$-20.30 - i2.41$
$-21.20 + i4.92$	-5.00	-6.62
$-21.20 - 14.92$	-0.67	$0.07 + i4.33$
$-20.31 + i2.41$	$-0.07 + i0.22$	$0.07 - i4.33$
$-20.31 - i2.41$	$-0.07 - i0.22$	$-0.47 + i2.68$
$-20.30 + i2.40$	-1.19	$-0.47 - i2.68$
$-20.30 - i2.40$	-0.19	-5.00
$0.18 + i5.72$	-1.61	-1.26
$0.18 - i5.72$	0.00	-0.9
$-0.06 + j4.97$		-0.17
$-0.06 - j4.97$		$-0.06 + i0.05$
$-0.25 + i3.77$		$-0.06 - j0.05$
$-0.25 - i3.77$		0.00
		$-20.30 + i2.42$

Table 3: Eigenvalues of 30 Bus System Examples

3.3. System Characteristics

Recognizing the instability as a characteristic of the system raises the question of: What are the significant differences, as related to stability, between the two systems, *i.e.,* between the high voltage network with large generators and a radial distribution system with smaller distributed generators? When modeling the hight voltage transmission system it is usually assumed that the local dynamics in x_{LC} are slow relative to the network dynamics, P_G . The implication of this assumption is that any change in x_{LC} is instantaneously transmitted through the system (via the K_PE term in the full system matrix A), so that the network itself has no affect on the local generators dynamics. The radial distribution system with relatively high impedance represents a basic change to the interconnecting network and its subsequent influence on local generator dynamics. A second distinction is that the generators on the high voltage grid are very large with correspondingly large inertias, in comparison to the distributed generators as modeled for this paper.

The two major differences identified, from the reference point of the distribution system are i) Machine inertias are relatively small, making the elements in matrix C_M relatively large, and ii) the impedance, R and L, of distribution lines is relatively large, affecting elements of the Jacobian derived matrix K_P.

These two properties result in strong coupling between the local state space x_{LC} and the system coupling variable P_G , as can be seen by referring back to an expanded matrix form of equation (2). The network term, represented K_P reflects a larger

coupling parameter between local generator frequencies and PG dynamics. The smaller inertias compound the effect on the local frequency by being too small to damp out the oscillations rapidly. These observation of large line impedance and small inertias are not surprising. What is unexpected is that they are significant enough to potentially affect stability within the distribution system.

3.4. Stabilizing the System

The stability problem suggests that new efforts may be required in designing local controls to ensure that stability will be maintained in a distribution system which has numerous distributed generators. Using the local state space, equation (2), bounds can be defined for the system parameters to ensure stability. Since each generator is individually stable, we investigate the assumption that if the local system matrix is allowed to dominate the local dynamics in the interconnected system, then the system as a whole will remain stable. Requiring the local dynamics to dominate in equation (6) results in the inequality

$$
|A_{LC}x_{LC}| \ge |C_M P_G| \tag{4}
$$

Focusing first in the right hand side of this inequality leads to setting bounds for C_M (= 1/M), or specifically to specifying a minimum inertia or size of plant installed (both machine size and rotation frequency determine machine inertia). Increasing the value of M for the hydro plant does stability the system in the models.

Alternatively, the range of allowed P_G values P_G^{min} $\epsilon = P_G \epsilon = P_G^{\text{max}}$, could be redefined such that P_G^{max} would be restricted to lower values. In actual operations this would mean a generator might not be able to respond to an increased demand for power, even if it were independently economically beneficial to do so.

Stability can also be addressed by focusing on the left hand side of the inequality (equation (4)). A general method for specifying local parameter value ranges for A_{LC} is to calculate eigenvalue sensitivity to the parameters, for the unstable system eigenvalues. This calculation is similar to that for the participation factors discussed earlier. The sensitivity matrix, S_i, for the ith eigenvalue is defined to be

$$
S_i = \left[\partial \mathbf{I}_i / \partial a_{jk}\right] = w_i v_i'
$$

where w_i and v_i are the left and right eigenvectors respectively for the ith eigenvalue (v_i ['] is a row vector), and the a_{ik} are the elements of the A_{LC} matrix. (Note that the diagonal elements of this matrix are identical to the participation factors.)

This matrix was calculated for the two unstable eigenvalues for each of the systems with a hydroelectric generator. The results show that for the local parameters, sensitivity is greatest to the parameters in the equation for the gate position. The time constant T_s is a factor in each of these parameters suggesting that T_s would be a good value to adjust.

Figure 4: Gate Opening *Ts* **Increased**

Figure 4 shows the system of Figure 3, with the time constant for the gate opening of the hydro plant *increased* so that it can not react as quickly to a disturbance, ,preventing it from resonating with the oscillations. Note that although this second solution does solve the stability problem, it also serves to challenge one of the anticipated benefits of distributed generation, specifically that the fast response capabilities of small generators would be *beneficial* in responding quickly to changes in demand and so help minimize any disturbance.

3.5. Options for Decentralized Control

As discussed in the introduction, the changing utility industry structure will most likely encourage increased penetration of distributed generators, and therefore increasingly emphasize decentralized versus centralized control of both individual generators and system level services. In linear system theory the ability to have decentralized control is represented by having a diagonal system matrix. If the system matrix is diagonalizable then the system can be represented by independent single order subsystems. If the system

matrix is not diagonalizable, but is instead diagonally dominant, the subsystems are not fully independent but are still identifiable as subsystems with weak interconnections. When the subsystems inherently include more than a single state variable, decentralization is represented by a system matrix with a block diagonal dominant (BDD) structure.

The definition of strict block diagonal dominance is

$$
\left\| A_{ii}^{-1} \right\|_{\infty} < \frac{1}{\sum_{i \neq j} \left\| A_{ij} \right\|_{\infty}} \tag{5}
$$

where the A_{ii} are the square diagonal blocks and the A_{ii} are the off-diagonal blocks across the same rows [11]. To obtain a block structure for the system matrix of equation (3) the state vector is reorganized as $[x_{\text{LC}}]$ P_{G1} x_{LC2} P_{G2} ...]^T. This ordering groups all state variables associated with a single generator together, and eliminates the lower right-hand block of zeros. Each diagonal block in the system matrix now represents a single generator's extended state space and has the following structure

$$
A_i^{ext} = \left[\frac{A_{LCi} \mid C_{Mi}}{K_p E \mid 0} \right]
$$

Each off-diagonal block in the full system matrix *A* contains a single network coupling tern *KPE* representing the coupling of generator *i* to the other generators on the system. Other elements in the offdiagonal blocks are 0.

The system matrix can be partitioned into the block structure as outlined above, with each generator representing a multivariable subsystem interconnected to the other generators via the K ^{*P* E} terms. As discovered in the section on stability though, these interconnections are not weak in the mathematical sense. Applications of the definition in equation (5) demonstrates that the system matrix is not block diagonally dominant. To facilitate decentralized control on the system this matrix must be made BDD – part of the continuing work in this research.

4. Summary

This paper has presented dynamic models and described the modeling approach used to simulate the decoupled frequency dynamics for a distribution system with small, distributed generators. One objective of the modeling was to represent each generator with a small number of state variables (3 to

4), and incorporate power output, P_G , as the system coupling variable (rather than the traditional variable, d, rotor angle). Unexpectedly, instability at the primary dynamics level was found, and was shown to be a system level phenomenon rather than one caused by a single state variable. Identification of the significant system level characteristics suggested various methods for stabilizing the system, requiring that close attention be paid to generator selection (size or inertia), operating parameters (specifically P_G^{max}) and local control design.

In addition to the instability at the system level, it was found that initially the system matrix is not block diagonally dominant. This suggest that parameters of the system matrix must be restricted to certain values or ranges in order to regain a block diagonally dominant structure and facilitate decentralized control.

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