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Models of Soil Moisture Dynamics in Ecohydrology: A Comparative Study

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[1] An accurate description of plant ecology requires an understanding of the interplay between precipitation, infiltration, and evapotranspiration. A simple model for soil moisture dynamics, which does not resolve spatial variations in saturation, facilitates analytical expressions of soil and plant behavior as functions of climate, soil, and vegetation characteristics. Proper application of such a model requires knowledge of the conditions under which the underlying simplifications are appropriate. To address this issue, we compare predictions of evapotranspiration and root zone saturation over a growing season from a simple bucket-filling model to those from a more complex, vertically resolved model. Dimensionless groups of key parameters measure the quality of the match between the models. For a climate, soil, and woody plant characteristic of an African savanna the predictions of the two models are quite similar if the plant can extract water from locally wet regions to make up for roots in dry portions of the soil column; if not, the match is poor. *INDEX TERMS:* 1818 Hydrology: Evapotranspiration; 1851 Hydrology: Plant ecology; 1866 Hydrology: Soil moisture; 1875 Hydrology: Unsaturated zone; *KEYWORDS:* ecohydrology, soil moisture, transpiration, modeling, scale

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1. Introduction

[2] The emerging field of ecohydrology encompasses work that attempts to identify linkages between climate, soil moisture dynamics, and vegetation in natural environments [*Rodriguez-Iturbe*, 2000]. While many factors, such as fire, grazing, and nutrient availability, impact the growth and persistence of plants, much of the initial focus has been on those ecosystems that are water controlled [e.g., *Laio et al.*, 2001a; *Ridolfi et al.*, 2000; *Rodriguez-Iturbe et al.*, 1999]. The stochastic nature of rainfall coupled with the nonlinear processes of infiltration, evaporation, transpiration, and drainage lead to rich and complex variability in soil moisture. The dynamics of soil moisture, in turn, have a significant impact on vegetation stress and the suitability of various plant species to particular climate and soil conditions.

[3] Analytical expressions of the time-varying nature of soil moisture provide insight into interactions among climate, soil, and vegetation. Owing to the complexity of the dynamics, however, obtaining such solutions requires simplification of the processes affecting soil moisture. A common simplification is to ignore the spatial variability of soil moisture within the root zone of a plant. This prohibits representation of wetting-front propagation and necessitates the description of losses, such as evaporation and transpiration, as functions of average saturation over the root zone. *Rodriguez-Iturbe et al.* [1999] developed a model based on this simplification and determined an analytical solution for the steady state probability density function (pdf) of soil moisture. *D'Odorico et al.* [2000] used this model to investigate the impacts of year-to-year climate fluctuations on soil moisture. In a similar effort, *Ridolfi et al.* [2000] explored the effects of climate variability on vegetation water stress and found that slight changes in climate from year to year can lead to significant year-to-year variations in plant stress. A recent series of papers [*Rodriguez-Iturbe et al.*, 2001; *Laio et al.*, 2001b; *Porporato et al.*, 2001; *Laio et al.*, 2001a] covers the development of a model of soil moisture dynamics similar to that of *Rodriguez-Iturbe et al.* [1999], the subsequent analytical expressions for the steady state distributions of soil moisture and vegetation stress (both static and dynamic), and application of the model to an African savanna, Texas scrubland, and Colorado steppe.

[4] While these works provide insight into relationships between climate, soil, and vegetation, proper interpretation of their results requires knowledge of the applicability and limitations of the simplifications employed in the representation of the soil moisture dynamics. Undoubtedly, these simplifications are acceptable under some climate and soil conditions. For example, *Salvucci* [2001] found that the representation of the losses as a function of average saturation used in the above model compares well to an estimate of the loss function based on data from Illinois. For other situations, however, the simplifications may not be appropriate. Identification of the circumstances when the

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simple model is and is not appropriate will enhance its utility. To address this issue, we compare a model of soil moisture dynamics, based on the works above, to a vertically resolved model that accounts for the spatial variability of soil moisture within the root zone. We identify the key differences between the models and develop dimensionless quantities to characterize these differences. We compare the predictions of soil and vegetation behavior from the two models for the case of a woody species from an African savanna and discuss how differences in the model formulations lead to differences in the model predictions.

2. Methodology

[5] We compare two models of soil moisture dynamics with respect to their ability to represent the health of vegetation in water-controlled ecosystems. The first is a zero-dimensional bucket-filling model based on the work of *Laio et al.* [2001b]. The second is a one-dimensional, vertically resolved model that uses Richards' equation to represent water movement in the unsaturated zone. Throughout this paper, we refer to the first as the bucket model, and the second as the Richards model. With both models, we simulate soil moisture dynamics and plant behavior over a growing season. The climate, vegetation, and soil conditions are characteristic of an African savanna with the woody species, *Burkea africana*, as described by *Scholes and Walker* [1993]. For these conditions, we compare predictions of the daily variations of average root zone saturation and plant transpiration, and the relationship between the two. We identify conditions under which the two models give similar results as well as those conditions for which results from the simpler bucket model may not be appropriate.

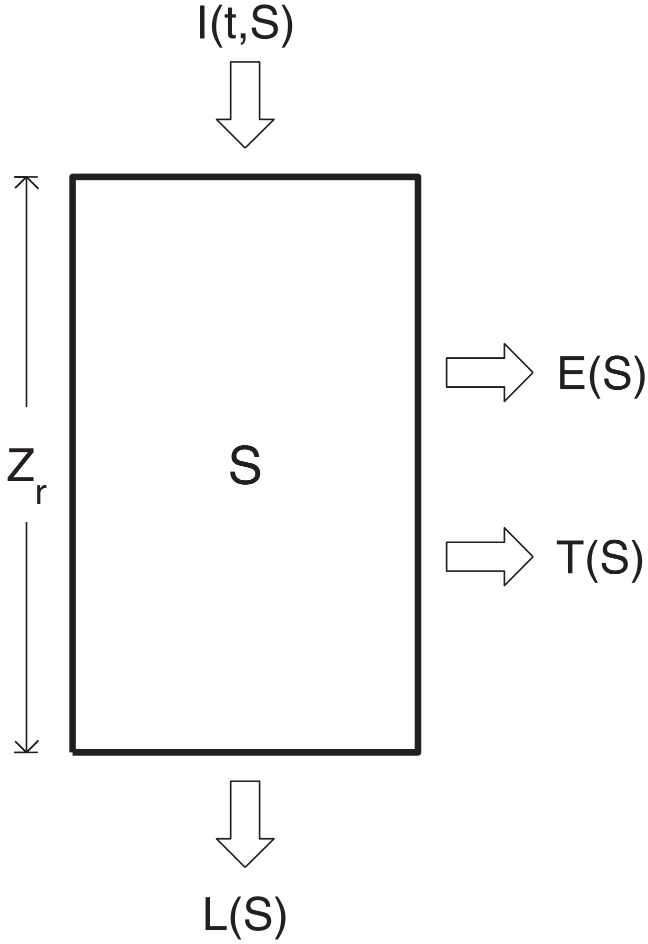

Figure 1. Schematic representation of the vertically averaged bucket model.

2.1. Bucket Model

[6] The bucket model represents soil moisture dynamics with a volume-balance equation applied over the root zone of a plant [*Rodriguez-Iturbe et al.*, 1999]:

$$nZ_r \frac{dS}{dt} = I(S,t) - L(S) - T(S) - E(S) \tag{1}$$

where S is the average saturation over the root zone, n is the porosity, Z_r is the depth of the root zone, $I(S, t)$ is the infiltration rate to the root zone, $L(S)$ is the rate of leakage from the root zone, and $T(S)$ and $E(S)$ are the transpiration and evaporation rates, respectively. In this simple representation, propagation of wetting fronts is ignored, and a single value of saturation represents the soil moisture of the entire root zone. Leakage and evapotranspiration losses are functions of the root zone saturation only. Figure 1 presents a schematic of the bucket model, and complete descriptions are given by *Rodriguez-Iturbe et al* [1999] and *Laio et al.* [2001b].

2.1.1. Infiltration and drainage

[7] Because the duration of a rainfall event is shorter than the daily time scale at which this bucket model applies, storm events are modeled as shots of water concentrated in time, arriving as a Poisson process with rate λ. Since the precipitation for an individual storm is not distributed in time, the infiltration over a growing season can be represented by a sum of dirac delta functions

$$I(S,t) = \sum_{i=1}^{M} I'\left(S\left(t_i^-\right), t_i\right) \cdot \delta(t - t_i) \tag{2}$$

where M is the number of storms that produce infiltration over the growing season, $S(t_i^-)$ is the saturation an instant before time t_i, and $I'(S(t_i^-), t_i)$ is the depth of the infiltration for storm i occurring at time t_i. This quantity is given by the minimum of the depth of precipitation minus interception and the unsaturated volume of the root zone:

$$I'\left(S\left(t_i^-\right), t_i\right) = \min\left[P(t_i), nZ_r\left(1 - S\left(t_i^-\right)\right)\right] \tag{3}$$

where $P(t_i)$ is the depth of precipitation minus interception. As specified in (3), either all of the water enters the root zone, or it fills the root zone to full saturation; there is no Hortonian (infiltration excess) mechanism for runoff generation. The depth of precipitation is modeled as an exponentially distributed random variable with mean, α, and the depth of interception, Δ, is taken to be a fixed quantity. This implies that the depths, $P(t_i)$, are exponentially distributed with mean, α, and the rate of arrival of

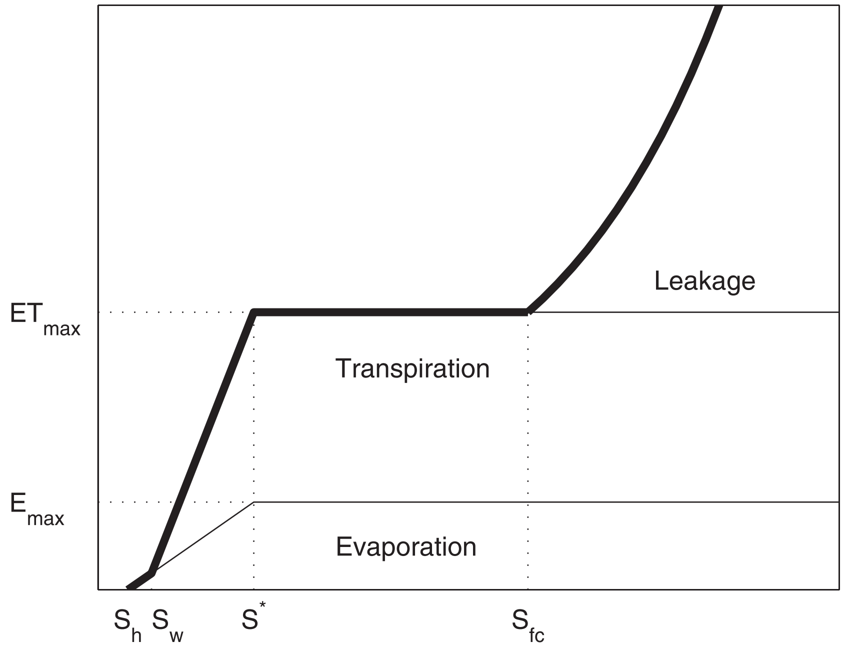

Figure 2. Losses from the bucket model due to evaporation, transpiration, and leakage as a function of average saturation over the root zone.

storms that generate infiltration is $\lambda' = \lambda e^{-\Delta/\alpha}$ [*Rodriguez-Iturbe et al.*, 1999].

[8] As in the work of *Laio et al.* [2001b], leakage out of the root zone is set equal to the unsaturated hydraulic conductivity, described with an exponential form:

$$L(S) = K_{sat} \frac{e^{\beta(S-S_{fc})} - 1}{e^{\beta(1-S_{fc})} - 1} \tag{4}$$

where K_{sat} is the saturated hydraulic conductivity, and β is a parameter of the soil. The field capacity, S_{fc}, is the saturation at which the rate of gravity drainage becomes negligible relative to evapotranspiration [*Laio et al.*, 2001b].

2.1.2. Plant uptake and evaporation

[9] Extraction of water from the root zone by the plant is governed by two factors: the atmospheric demand and the supply of water in the soil. If the supply is sufficient, uptake will equal the demand. If the soil is too dry, however, the plant will be unable to extract enough water to meet the demand, and the uptake will be reduced. In the bucket model, this relationship is represented by an extraction function that depends on the average root zone saturation. When the saturation is greater than a threshold, S^*, the uptake is equal to the demand. The uptake decreases linearly between S^* and S_w, the saturation at which the uptake is zero and the plant wilts. This uptake function is described mathematically by

$$T(S) = \left\{ \begin{array}{cc} 0 & S \le S_w \\ \frac{S-S_w}{S^*-S_w} \cdot T_{\max} & S_w < S < S^* \\ T_{\max} & S \ge S^* \end{array} \right\} \tag{5}$$

where $T_{\max}$ represents the maximum transpiration rate as dictated by the atmospheric demand. Since the interpretation of the model predictions are intended for the timescale of a day, $T_{\max}$ represents the maximum depth of water transpired over a day. This can be significantly less than the instantaneous maximum rate of transpiration.

[10] Evaporation from the root zone is represented in a manner analogous to the plant uptake:

$$E(S) = \left\{ \begin{array}{cc} 0 & S \le S_h \\ \frac{S-S_h}{S^*-S_h} \cdot E_{\max} & S_h < S < S^* \\ E_{\max} & S \ge S^* \end{array} \right\} \tag{6}$$

where S_h is the hygroscopic saturation, at which evaporation ceases, and $E_{\max}$ is the maximum daily evaporation rate. The average saturation at which E reaches its maximum, S^*, is the same as that for transpiration (see equation 5). The combination of maximum daily evaporation and transpiration is denoted by $ET_{\max}$. These formulations for evapotranspiration are based on the work of *Laio et al.* [2001b], though those authors did not separate evaporation from transpiration. Figure 2 presents a graphical representation of the combination of losses from the root zone (leakage, evaporation, and plant uptake) as a function of average root zone saturation.

2.1.3. Simulation and solution

[11] The simplicity of the loss function enables the temporal integration of the losses between storm events to be evaluated analytically. Therefore simulation of the soil moisture dynamics entails the generation of a sequence of storm arrivals and depths, and evaluation of the loss integral for any time between storms.

2.2. Richards Model

[12] In the Richards model, the soil column is resolved in the vertical dimension, and the soil moisture dynamics are described by Richards' equation:

$$\frac{\partial(ns)}{\partial t} - \frac{\partial}{\partial z}K\frac{\partial h}{\partial z} + \frac{\partial K}{\partial z} = -e' - \sigma' \tag{7}$$

where s is the local saturation, K is the unsaturated hydraulic conductivity (with units of length per time), h is the pressure head in the water (with units of length), and z is positive downward. The functions e' and σ' are the local rates of evaporation and plant uptake in units of depth per depth per time. Evaporation takes place over a depth, Z_e, and uptake of water by the plant occurs over the depth of the root zone, Z_r. For a soil column that is discretized uniformly in space, the volume balance for layer i becomes

$$\Delta z\frac{\partial(ns)_i}{\partial t} - K_{i+\frac{1}{2}}\left(\frac{h_{i+1}-h_i}{\Delta z} - 1\right) - K_{i-\frac{1}{2}}\left(\frac{h_i - h_{i-1}}{\Delta z} - 1\right) = -e_i - \sigma_i \tag{8}$$

where Δz is the spatial discretization, and e_i and σ_i are the rates of evaporation and plant uptake from layer i in units of depth per time. Figure 3 gives a schematic representation of the Richards model.

[13] We use modified versions of the relative permeability and retention curves presented by *Laio et al.* [2001b]. The retention curve relating the pressure head and saturation is given by

$$h(s) = h_e \cdot \left(\frac{s - s_h}{1 - s_h}\right)^{-b} \tag{9}$$

where h_e is the entry pressure head, s_h is the hygroscopic saturation, and the exponent, b, describes the shape of the curve. The unsaturated hydraulic conductivity is given by

$$K(s) = K_{sat} \cdot \left(\frac{s - s_h}{1 - s_h}\right)^{2b+3} \tag{10}$$

where K_{sat} is the hydraulic conductivity when the soil is fully saturated.

2.2.1. Infiltration and drainage

[14] In the Richards model, precipitation is introduced as a boundary condition at the surface of the soil column. Each rainfall event is characterized by an intensity and duration. The duration for a given storm is taken from a uniform distribution, and the intensity is calculated so that the depth of water is equal to the depth of precipitation minus interception specified in the bucket model. This rainfall rate is set as a flux boundary condition on the top of the soil column, provided that the intensity is less than the potential infiltration rate for the soil (the infiltration rate if the water pressure at the surface were atmospheric). If the rainfall rate exceeds the potential infiltration rate, then the boundary flux is set equal to the potential rate, and the remaining precipitation runs off. This formulation allows both Hortonian (infiltration excess) and Dunne (saturation excess) mechanisms of runoff generation.

[15] Leakage of soil water to depths greater than the root zone is governed by Richards' equation. The bottom boundary of the soil column is made deep enough so as to have minimal impact on the soil moisture dynamics within the root zone.

2.2.2. Plant uptake and evaporation

[16] The vertical resolution of the Richards model enables a more sophisticated uptake function that depends not only on the total water within the root zone but also its spatial distribution. We represent the uptake of water by the plant with a model based on the work of *Gardner* [1960], *Cowan* [1965], *Federer* [1979, 1982], and *Lhomme* [1998]. These works assert that plant uptake is proportional to a difference in water potential in the soil and in the plant. This type of plant uptake model is sometimes referred to as a Type I model [e.g., *Cardon and Letey*, 1992; *Shani and Dudley*, 1996]. The uptake is limited by two resistances: one associated with water movement through the root tissue and one associated with movement of the water through the soil to the roots. The local uptake function is described mathematically by

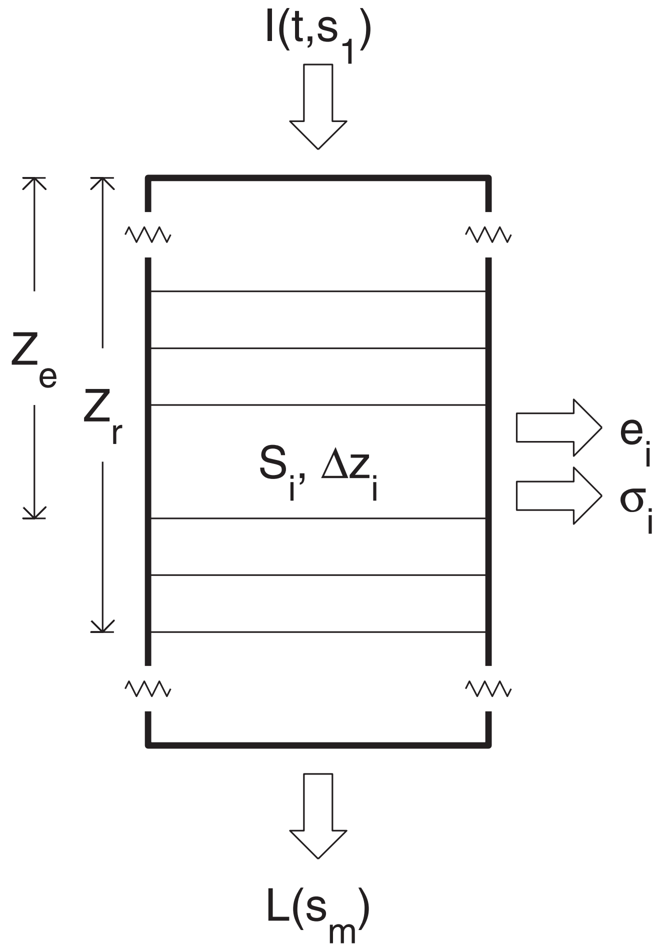

Figure 3. Schematic representation of the one-dimensional Richards model.

$$\sigma_i = \Delta z_i \cdot \frac{(h_i - H_p)}{r_{s,i} + r_{r,i}} \tag{11}$$

with the restriction that σ_i cannot be negative. In (11), $r_{s,\,i}$ and $r_{r,\,i}$ are the local soil and root resistances, respectively, and Δz_i is the thickness of the soil layer. The driving force for the uptake is a difference between the water potential in the soil, h_i, and the water potential in the plant, H_p. In our representation, variations of H_p within the plant are ignored.

[17] As in the work of *Lhomme* [1998], the local soil resistance is inversely proportional to the unsaturated

hydraulic conductivity and the local root density (length of roots per volume of soil):

$$r_{s,i} = \frac{C_s}{K(s_i) \cdot RW_i} \quad (12)$$

where C_s is a dimensionless constant that accounts for root diameter, geometry, and arrangement. The local root resistance is inversely proportional to the root density:

$$r_{r,i} = \frac{C_r}{RW_i} \quad (13)$$

where C_r is a constant parameter of the plant with units of time per length. Rewriting the local root density as the product of the average root density over the entire root zone times a local, relative, root density gives the following expression for the local uptake:

$$\sigma_i = \Delta z_i \cdot rw_i \cdot \frac{(h_i - H_p)}{\frac{C_s}{K(s_i)RW_0} + \frac{C_r}{RW_0}} \quad (14)$$

where RW_0 is the average root density over the entire root zone, and rw_i is the relative root density as a function of depth. The local uptake function, (14), contains three unknowns: H_p, C_s, and C_r. We choose values for these unknowns to effect a match between the plant uptake from the bucket and Richards models.

[18] The value of H_p, the plant potential, is based on two constraints. The first is that H_p must be greater than or equal to the plant wilting potential, h_w, the pressure head corresponding to S_w from the bucket model. This ensures that the plant cannot extract water from regions in which the water potential is less than h_w. Additionally, the plant cannot transpire more over a day than the atmospheric demand, $T_{\max}$. Thus either the plant potential is such that the plant extracts enough water from the entire root zone to meet demand, or the plant extracts as much as it can without its potential dropping below h_w.

[19] The bucket model incorporates a reduction in the plant uptake when the root zone saturation drops below S^*. The uptake in the Richards model, however, depends not only on the total soil moisture but also its spatial distribution. At one extreme, an average saturation of S^* can be achieved by a uniform saturation of s^* over the entire root zone. In this case, the total transpiration over the root zone is given by

$$T = \sum_{z_i \le Z_r} \sigma_i = Z_r \cdot \frac{(h^* - H_p)}{\frac{C_s}{K(s^*)RW_0} + \frac{C_r}{RW_0}} \quad (15)$$

To match the bucket model, we stipulate that for this saturation profile the plant uptake over the entire root zone is equal to the transpiration demand when the plant potential is at its lowest value, h_w:

$$T_{\max} = Z_r \cdot \frac{(h^* - h_w)}{\frac{C_s}{K(s^*)RW_0} + \frac{C_r}{RW_0}} \quad (16)$$

If the potential in the soil were to drop below h^*, i.e., if the average saturation were to drop below S^*, then the extraction would not meet the demand because the plant potential can not go below h_w. This constraint corresponds to the break point in the extraction curve used in the bucket model (see equation 5).

[20] If the soil moisture is distributed nonuniformly over the root zone, the plant may compensate for some of its roots being dry by extracting additional water from roots in a wet region. Ability to do so is a function of the magnitude of the root resistance, r_r, since the soil resistance will be negligible at high saturations. If the root resistance is small, the plant can extract water at high rates from wet regions to compensate for portions of the root zone that are dry. This behavior can be expressed by a factor, f, defined as the minimum fraction of the roots that must be at full saturation ($h = 0$) in order for the plant to withdraw enough water to meet the transpiration demand if extraction from elsewhere in the soil column is zero:

$$T_{\max} = f \cdot Z_r \cdot \frac{-h_w}{\frac{C_s}{K_{sat}RW_0} + \frac{C_r}{RW_0}} \quad (17)$$

If f is close to one, the plant has little ability to compensate; if f is close to zero, the plant can readily compensate for dry regions within the root zone. Specification of f enables the simultaneous solution of (16) and (17) for C_r and C_s.

[21] Unlike the plant uptake, evaporation from the upper soil layers is presumed to be a function of the local saturation only. The extraction from a layer is described by

$$e_i = \Delta z_i \cdot ew_i \cdot \frac{E(s_i)}{\sum_{i=1}^{m_e} \Delta z_i \cdot ew_i} \quad (18)$$

where $E(s_i)$ is the function given by (6) with the local saturation as its argument, m_e is the number of soil layers over which evaporation is nonzero, and ew_i is a depth-dependent weight. If the soil layers from the ground surface to Z_e were uniformly saturated, the total evaporation as a function of saturation would be equal to that for the bucket model.

[22] Since the models presented here are intended to be interpreted at the daily timescale, we include only a rudimentary diurnal variation of σ and e. The extraction function is set to zero for twelve hours each day, and for the remaining twelve hours the values of $T_{\max}$ and $E_{\max}$ used in (14)–(18) are set to twice those used in the bucket model. Also, it is worth noting that we have simplified the comparison by fixing the root density; neither the bucket nor the Richards model accounts for the added complexity associated with root growth.

2.2.3. Simulation and solution

[23] Because of the highly nonlinear nature of the governing equations for the Richards model, analytical solution is infeasible. To simulate soil moisture dynamics and plant uptake, we use a one-step, fully implicit, temporal scheme with finite difference approximations in space. We use the mass-conserving modified Picard iterations, described by *Celia et al.* [1990], for solution of the nonlinear equations.

2.3. Key Differences Between the Models

[24] The two model formulations above have some key differences that affect the similarity of their representations of soil moisture dynamics and plant uptake. These differ-

ences are interrelated but can be put into three categories: (1) infiltration dynamics, (2) runoff generation, and (3) extraction functions for evaporation and plant uptake.

[25] The first difference is related to the temporal and spatial resolution of the two models. The bucket model is a point model, and therefore cannot represent the migration of a wetting front through the soil column. Consequently, only an average saturation over the root zone can be resolved, giving no information about the spatial distribution of that water within the soil. The Richards model represents the vertical movement explicitly and resolves the spatial distribution of soil moisture.

[26] Even if one were to use the Richards model with a single layer only, however, it would not be equivalent to the bucket model. In the bucket model, the intensity of rain events is ignored, and all water reaching the ground surface is presumed to enter the soil column until the root zone is fully saturated. In the Richards model, the precipitation rate must be specified, and runoff can occur via either the Dunne or Hortonian mechanism.

[27] The last difference pertains to how the water losses due to evaporation and plant uptake are implemented. In the bucket model, these are simple functions of the average saturation over the entire root zone. In the Richards model, evapotranspiration is a function of the local saturation, the distribution of soil moisture over the entire root zone, and the weighting functions, $rw(z)$ and $ew(z)$. The driving force for the plant uptake is a potential difference, and the function is constrained to match the important endpoints of the bucket model.

2.4. Characterization With Nondimensional Groups

[28] The impact of the differences described in the previous section can be characterized by nondimensional groups of parameters. The simplified representation of infiltration dynamics in the bucket model ignores both the temporal and spatial distributions of infiltration. The impact of the temporal simplification can be quantified with a temporal infiltration index:

$$I_{I,t} = \frac{\min(\alpha/t_P, K_{sat})}{ET_{\max}} \tag{19}$$

where t_P is the characteristic duration of a rain event. $I_{I,\,t}$ is the ratio of the characteristic infiltration rate to the maximum rate of evapotranspiration. If $I_{I,\,t}$ is much greater than one, then infiltration occurs much faster than evapotranspiration, and representation of the process as instantaneous will not impact the results significantly.

[29] Similarly, the impact of the spatial distribution of infiltration can be characterized by a spatial infiltration index:

$$I_{I,x} = \frac{\overline{Z}_i}{Z_r} \tag{20}$$

where

$$\overline{Z}_i = \frac{\alpha}{n(s_{fc} - s_h)} \tag{21}$$

$\overline{Z}_i$ is a measure of the depth of infiltration from a characteristic storm. Thus $I_{I,\,x}$ is the ratio of the infiltration depth to the total depth of the bucket model. A small value of this ratio indicates that the spatial distribution of soil moisture after a rain event can be far from uniform, and this may lead to differences in the predictions of the two models. If both $I_{I,\,t}$ and $I_{I,\,x}$ are large, then differences in the model results due to their different representations of infiltration dynamics are likely to be small.

[30] Under some climate and soil conditions, the omission of Hortonian overland flow from the bucket model may lead to differences in results when compared to the Richards model. The impact of this omission can be characterized by a runoff index:

$$R_I = \frac{K_{sat}}{\alpha/t_P} \tag{22}$$

If R_I is large, Hortonian runoff is unlikely to occur, and differences between the models due to runoff generation will be insignificant.

[31] The impact of the different representations of evaporation and transpiration between the bucket and Richards models will depend on whether the climate is characteristically wet or dry. To quantify this, *Milly* [2001] used the index of dryness, the ratio of the maximum daily evapotranspiration rate to the average rainfall rate:

$$D_I = \frac{ET_{\max}}{\lambda' \cdot \alpha} \tag{23}$$

where λ' is the rate of storm arrivals. An index of dryness greater than one indicates that evapotranspiration will be limited by the amount of water supplied through precipitation, not the atmospheric demand. In this case, leakage is likely to be small. If runoff is also negligible, then at steady state, or over a long enough time, the average evapotranspiration rate will equal the rainfall rate. Therefore predictions of average evapotranspiration from all models that are mass-conserving and have a plateau of $ET_{\max}$ will be the same. Differences may arise in the rate of approach to steady state, the partitioning between evaporation and transpiration, the time history of the average root zone saturation, and the timing and intensity of evapotranspiration.

[32] One way to quantify the differences in timing and intensity of evaporation and transpiration is to compare their characteristic rates immediately following a precipitation event. The ratios of these rates will be denoted by Γ:

$$\Gamma_E = \frac{E_b^*}{E_R^*} \tag{24a}$$

$$\Gamma_T = \frac{T_b^*}{T_R^*} \tag{24b}$$

where E_b^* and E_R^* are the evaporation rates just after a rain event for the bucket and Richards models, respectively, and T_b^* and T_R^* are the transpiration rates.

[33] In a dry climate with infrequent rain events, between storms the soil moisture will decay to a value close to the wilting point saturation, below which the loss rate is very small. Consequently, for the bucket model applied to water-controlled ecosystems, the evapotranspiration rate, $E_b^* + T_b^*$, can be approximated as the rate at an average saturation

Table 1. Values of Climate Parameters for an African Savanna

Parameter	Symbol	Value	Units
Storm arrival rate	λ	1/6	days^{-1}
Mean rainfall depth	α	1.5	cm
Minimum storm duration in Richards model		0.05	days
Maximum storm duration in Richards model		0.15	days

equal to the saturation at the wilting point plus the change in saturation due to a characteristic rain event:

$$E_b^* + T_b^* = E(S_w + \Delta S) + T(S_w + \Delta S) \quad (25)$$

where $\Delta S = \alpha/nZ_r$.

[34] For the Richards model, the evapotranspiration rate immediately following a storm can be characterized by the evaporation and transpiration rates calculated from (18) and (14), presuming that the soil column is saturated to field capacity to a depth, $\overline{Z}_i$, and that there is no extraction from elsewhere in the soil column. This rate can be approximated by

$$E_R^* + T_R^* = f_E \cdot E(s_{fc}) + T_{\max} \cdot \min\left(1, \frac{f_T}{f}\right) \quad (26a)$$

$$f_E = \frac{\int_0^{\overline{Z}_i} ew(z)dz}{Z_e} \quad (26b)$$

$$f_T = \frac{\int_0^{\overline{Z}_i} rw(z)dz}{Z_r} \quad (26c)$$

In (26), f is the minimum fraction of roots under fully saturated conditions needed to achieve a total uptake of $T_{\max}$; f_T and f_E are the fraction of the roots and the fraction of the zone of evaporation, respectively, that are wetted by a characteristic precipitation event.

[35] If Γ_E and Γ_T are close to one, the losses from the bucket and Richards models are similar. Values of Γ larger than one indicate that the bucket model predicts a higher loss rate than the Richards model, and vice versa for a value less than one. Note that if $I_{I,\,x}$ is greater than one, a typical precipitation event will cover the entire soil column with a saturation greater than S_{fc}. Since S_{fc} is generally larger than S^*, this implies that, following a storm, the Richards and bucket model will both predict an evapotranspiration rate equal to $ET_{\max}$. Therefore if $I_{I,\,x}$ is greater than one, there is no need to compute Γ_E and Γ_T.

2.5. Parameters Used in the Comparison

[36] The nondimensional groups defined in the previous section identify a number of ways in which the bucket and Richards models can differ. Rather than attempt to account for all of them, we limit our comparison of the two models to a specific plant, soil, and climate condition for which extensive data exist. We investigate the ability of the two models to represent the soil moisture dynamics and plant uptake for *Burkea africana*, a woody species from an African savanna. The data describing the vegetation, soil, and climate are taken from *Scholes and Walker* [1993].

[37] Table 1 presents the parameter values used to describe the climate, including the rate of storm arrivals and mean storm depth. These values are the same as those used by *Laio et al.* [2001a]. Table 2 gives the parameter values that describe the soil. These values represent a combination of two soil types presented by *Scholes and Walker* [1993]. Because of the slight modification of the functional forms for the unsaturated conductivity and reten-

Table 2. Parameter Values Describing the Soil Characteristics

Parameter	Symbol	Value	Units
Porosity	n	0.42	–
Saturated conductivity	K_{sat}	109.8	cm/day
Hygroscopic saturation	s_h, S_h	0.02	
Field capacity	s_{fc}, S_{fc}	0.29	
Entry pressure head	h_e	3.0	cm
Retention curve parameter	b	2.25	
Drainage curve parameter	β	9.0	

Table 3. Values of Vegetation Parameters for *Burkea africana*, a Woody Species From an African Savanna

Parameter	Symbol	Value	Units
Depth of interception	Δ	0.2	cm
Maximum daily evaporation rate	$E_{\max}$	0.15	cm/day
Maximum daily transpiration rate	$T_{\max}$	0.325	cm/day
Potential at the point of stomatal closure	h^*	730	cm
Saturation at the point of stomatal closure	S^*	0.105	–
Minimum plant potential	h_w	31,600	cm
Saturation at the wilting point	S_w	0.036	
Evaporation depth	Z_e	30	cm
Root zone depth	Z_r	100	cm
Parameter 1 for *ew* distribution	A	0.9	
Parameter 2 for *ew* distribution	B	5.0	
Parameter 1 for *rw* distribution	A	2.0	
Parameter 2 for *rw* distribution	B	2.0	
Mean root density	RW_0	0.02	cm/cm^3
Fraction of roots needed to supply $T_{\max}$	f	0.1, 0.8	

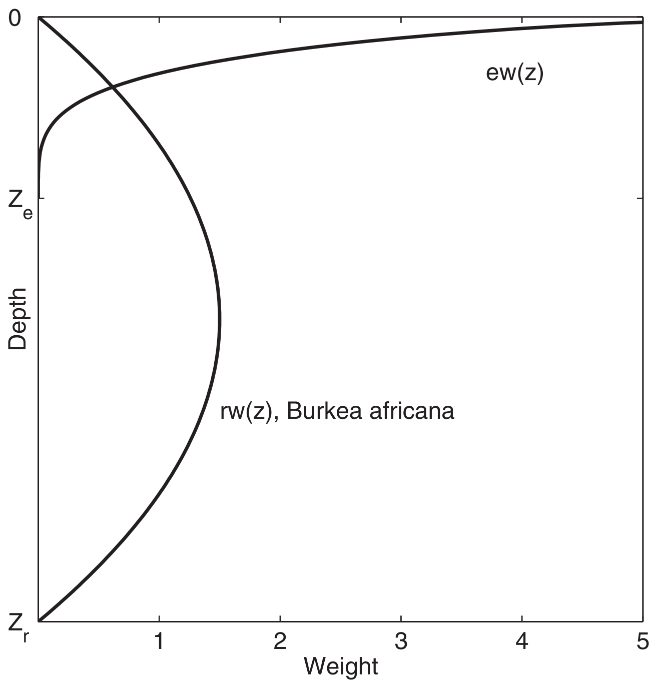

Figure 4. Weights for evaporation and transpiration as functions of depth.

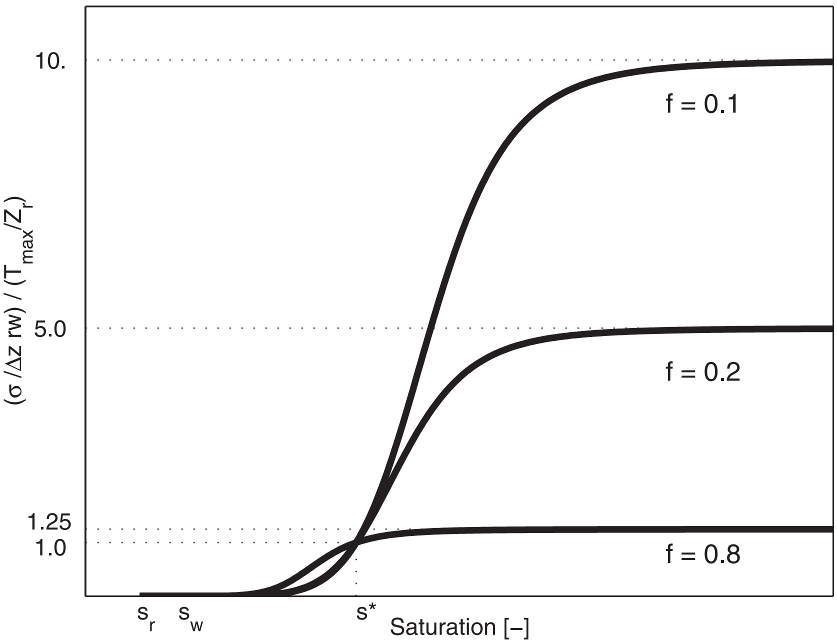

Figure 5. Normalized transpiration per unit of roots versus saturation for the Richards model when $H_p = h_w$.

tion curves, the parameter values differ slightly from those used by *Laio et al.* [2001a].

[38] Table 3 presents the values used to describe the plant, *Burkea africana*. We use beta distributions to describe the weighting functions, $ew(z)$ and $rw(z)$:

$$rw(z), ew(z) = \frac{\Gamma(A+B)}{\Gamma(A)\cdot\Gamma(B)} \cdot z^{(A-1)} \cdot (1-z)^{(B-1)} \tag{27}$$

where A and B are nonnegative shape parameters. Data on the root density distribution for *Burkea africana* are from *Scholes and Walker* [1993]. Figure 4 presents the functions $rw(z)$ and $ew(z)$ for the simulations we conducted.

[39] Because we do not know f for Burkea africana, we consider two values in our comparison: 0.1 and 0.8. To demonstrate how this parameter affects transpiration, Figure 5 presents the local uptake per unit of roots ($\sigma_i/\Delta z_i \cdot rw_i \cdot RW_0$) as a function of the local saturation, under the condition that $H_p = h_w$, for three values of f. The ordinate is normalized by $T_{max}/Z_r \cdot RW_0$, the average uptake per unit of roots when $T = T_{max}$. This figure shows that, locally, the plant can withdraw up to ten times the amount needed per unit of roots to meet the demand, T_{max}, when f is 0.1. As f moves closer to one, however, the plant loses its ability to withdraw water at high rates to compensate for the spatial variability of soil moisture.

[40] As another interpretation of the plant uptake, Figure 6 displays the rate of uptake over the entire root zone, normalized by T_{max}, as a function of the depth, Z_i, to which the soil column is saturated to field capacity. The remaining portion of the soil column is taken to be at the wilting saturation and therefore not contributing to the plant uptake. When $f = 0.1$, only the top twenty percent of the soil column needs to be wetted in order for the plant to uptake T_{max} (the fraction is not 10% due to the nonuniformity of the root distribution). When $f = 0.8$, more than seventy percent of the soil column needs to be saturated. Under the presumed soil moisture profile, the average saturation over the root zone is a function of Z_i; also plotted in Figure 6 is the corresponding uptake predicted by the bucket model. When $Z_i = 0.27 \cdot Z_r$, the average root zone saturation equals S^*, and the transpiration for the bucket model equals T_{max}.

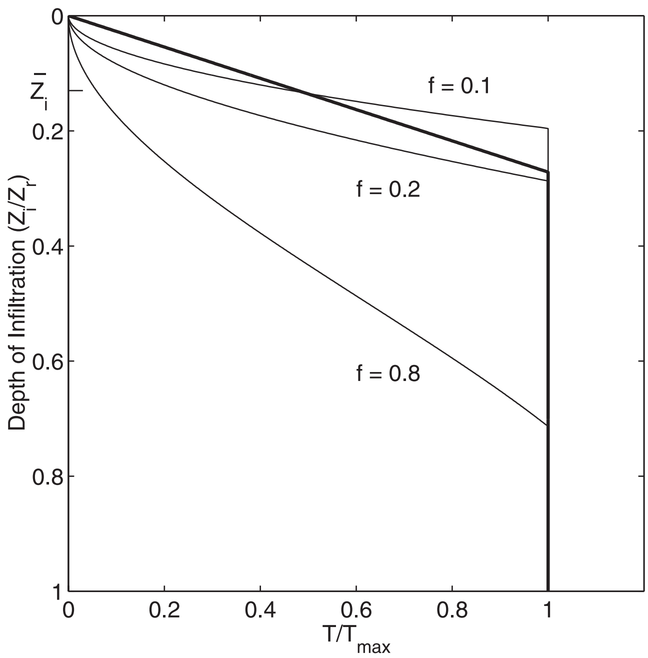

Figure 6. Transpiration rate as a function of depth of infiltration, Z_i, presuming a uniform saturation of s_{fc} from the ground surface to Z_i, and a uniform saturation of s_w below Z_i. The thick line represents the bucket model; the thin solid lines represent the Richards model for various values of f.

Table 4. Values of Dimensionless Numbers Quantifying the Comparison of the Bucket and Richards Models of Soil Moisture Dynamics for *Burkea africana* in an African Savanna

Nondimensional Group	Symbol	Value
Temporal infiltration index	$I_{I,t} = \min(K_{sat}, \alpha/t_P)/ET_{\max}$	32
Spatial infiltration index	$I_{I,x} = \overline{Z}_i/Z_r$	0.13
Runoff index	$R_I = K_{sat}/(\alpha/t_p)$	7.3
Index of dryness	$D_I = ET_{\max}/(\lambda'\alpha)$	2.1
Ratio of poststorm evaporation	$\Gamma_E = E_b^*/E_R^*$	0.64
Ratio of poststorm transpiration, $f = 0.8$	$\Gamma_T = T_b^*/T_R^*$	8.4
Ratio of poststorm transpiration, $f = 0.1$	Γ_T	1.0

[41] We simulated three realizations of climate for a two-hundred day growing season. The three realizations comprise a wet, average, and dry season, all generated from the same stochastic process. The differences among the realizations are due to variations in storm depth; all three have similar storm arrival rates. In none of the three cases did the rainfall rate exceed the saturated hydraulic conductivity; therefore Hortonian runoff does not occur. For all simulations, the initial saturation is uniform over the root zone with a value of 0.1. The vertical resolution of the Richards model is one centimeter, and the saturation at twice Z_r is held fixed at s_{fc}.

[42] Table 4 presents the values of the nondimensional groups, discussed above, for the particular climate, soil, and vegetation conditions under consideration. Inspection of these indicates some anticipated behavior. First, the large value of the runoff index indicates that differences in runoff generation mechanisms are not important for this comparison. Second, the temporal infiltration index is much greater than one, indicating that representation of rain events as instantaneous is reasonable. The spatial infiltration index, however, is much less than one, which implies that the distribution of soil moisture in the Richards model is likely to be far from uniform. This may affect the match between the models, since the uptake in the Richards model depends on the local saturation. The value of Γ_E less than one indicates that evaporation immediately following a storm will be higher for the Richards model than the bucket model. The value of Γ_T close to one for $f = 0.1$ indicates that the bucket model may be a good approximation to this Richards model; the value of Γ_T larger than one for the Richards model with $f = 0.8$ indicates that the bucket model predicts higher rates of uptake immediately following a rain event. The difference in Γ_E and Γ_T indicates that the paritioning of infiltration between evaporation and transpiration may be different for the bucket and Richards model.

3. Results

[43] Table 5 presents a summary of the results from the simulations conducted with the bucket and Richards models. The results include the values of λ' and α estimated from the climate realizations, the depth of total infiltration over the growing season, the cumulative transpiration and evaporation over the growing season (as depths and as percentages of total infiltration), and the temporally averaged root zone saturation. For the simulations with the Richards model, capillary rise into the root zone contributed 1.8 cm of moisture over the growing season; this accounts for the difference in infiltration values between the bucket and Richards model. Though leakage is negligible, in each simulation the sum of evaporation and transpiration does not equal the infiltration because of changes in storage in the root zone over the growing season.

[44] For all climate realizations, the bucket model predicts the highest values of transpiration over the growing season. The bucket model also predicts the lowest values of average saturation over the root zone. Across the three seasons (dry, average, and wet), the fraction of infiltration that ends up as transpiration is nearly constant for the bucket model. For the Richards model, however, the fraction increases with increasing wetness of the climate. For example, the transpiration goes from 54% to 57% to 66% of infiltration for the case with $f = 0.1$. Correspondingly, the fraction of infiltration that goes as evaporation decreases from the dry to the wet climate. For the dry climate, losses are split evenly between evaporation and transpiration, while for the wet climate the ratio is nearly one to two. The match between the bucket and Richards model improves as the climate gets wetter and as the value of f gets smaller, i.e., as the plant's ability to compensate for spatial variability in the soil moisture distribution improves.

[45] Average values over the entire growing season do not give the complete picture, however. Figure 7 shows a time history of the vertically integrated root zone saturation over the growing season under average climate conditions, as predicted by the bucket model and Richards model with $f = 0.8$ and 0.1. This figure is in general agreement with the results mentioned above: the trace for the bucket model is lower than the two others, and it is closer to the Richards

Table 5. Cumulative Infiltration, Transpiration, and Average Saturation Over a Growing Season of 200 Days as Predicted by the Bucket and Richards Models

Season	$\widehat{\lambda}'$, days^{-1}	$\widehat{\alpha}$, cm	Model	f	Infiltration, cm	Transpiration, cm (%)	Evaporation, cm (%)	Saturation
Dry	0.14	1.2	bucket	–	35.2	23.5 (67%)	14.4 (41%)	7%
Dry	0.14	1.2	Richards	0.8	37.0	17.9 (48%)	19.4 (52%)	10%
Dry	0.14	1.2	Richards	0.1	37.0	20.1 (54%)	17.0 (46%)	9%
Average	0.15	1.5	bucket	–	45.4	30.8 (68%)	17.2 (38%)	8%
Average	0.15	1.5	Richards	0.8	47.3	25.0 (53%)	22.6 (48%)	12%
Average	0.15	1.5	Richards	0.1	47.3	27.1 (57%)	20.3 (43%)	10%
Wet	0.14	2.2	bucket	–	58.2	39.6 (68%)	20.4 (35%)	12%
Wet	0.14	2.2	Richards	0.8	60.1	36.5 (61%)	20.4 (34%)	17%
Wet	0.14	2.2	Richards	0.1	60.2	40.0 (66%)	19.2 (32%)	14%

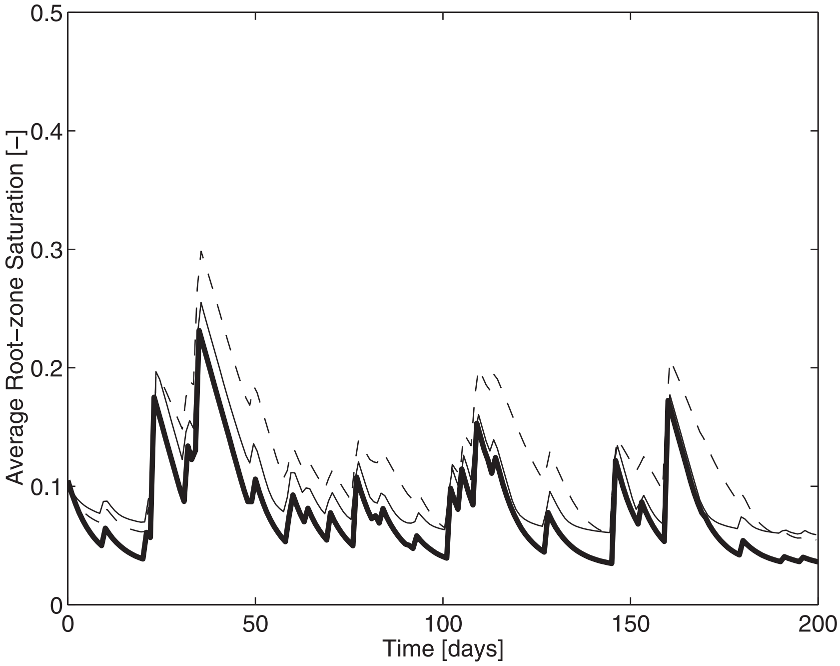

Figure 7. Traces of root zone saturation over the growing season for the average climate realization. The thick line is the result from the bucket model; the thin solid line is the result from the Richards model with $f = 0.1$; the dashed line is the result from the Richards model with $f = 0.8$.

model with $f = 0.1$ than the case with $f = 0.8$. The lowest saturations for the Richards model simulations level out near seven percent, while those for the bucket model drop down to four percent. Except for the behavior at very low saturations, the results from the bucket model and the Richards model with $f = 0.1$ are in close agreement. The predictions of the Richards model with $f = 0.8$ follow the same trends, but the details of the trace are different.

[46] Figure 8 presents a time-history of the daily transpiration rate for the same three cases. This figure shows some dramatic differences among the models. While the bucket model and the Richards model with $f = 0.1$ predict daily

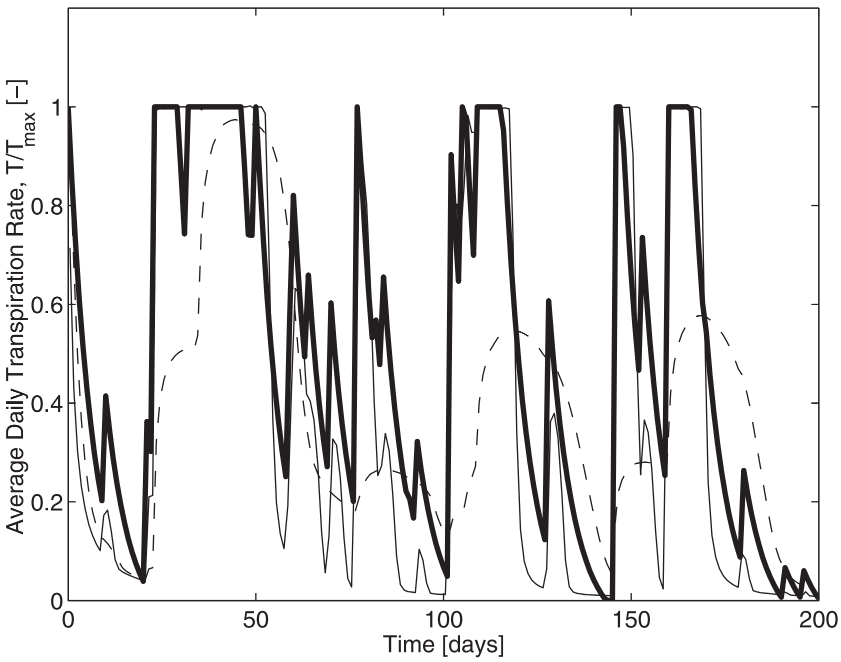

Figure 8. Traces of daily transpiration over the growing season for the average climate realization. The thick line is the result from the bucket model; the thin solid line is the result from the Richards model with $f = 0.1$; the dashed line is the result from the Richards model with $f = 0.8$.

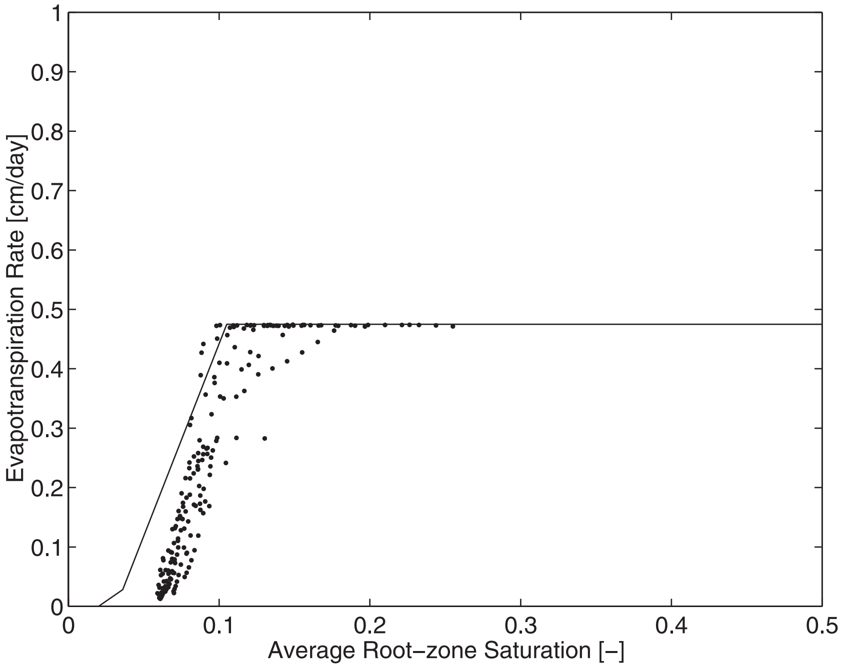

Figure 9. Relationship between total evapotranspiration and average root zone saturation for the average climate realization. Points represent upscaled results from the Richards model with $f = 0.1$, and the solid line represents the function used in the bucket model.

transpiration rates that reach T_{max}, the transpiration rate for the Richards model with $f = 0.8$ never does. The frequency of the transpiration fluctuations for the Richards model with $f = 0.8$ is also much lower than the two other cases. In general, the transpiration predictions from the bucket model and the Richards model with $f = 0.1$ are in good agreement. After a significant rain event, however, the decline of the transpiration rate is much faster for the Richards model (see examples near days 120 and 170). This model shows a rapid decrease in total transpiration from T_{max} to nearly zero.

[47] In addition to the temporal evolution of transpiration, we also investigated the dependence of total evapotranspiration on the average root zone saturation. Figure 9 presents this relationship for the Richards model with $f = 0.1$ for the

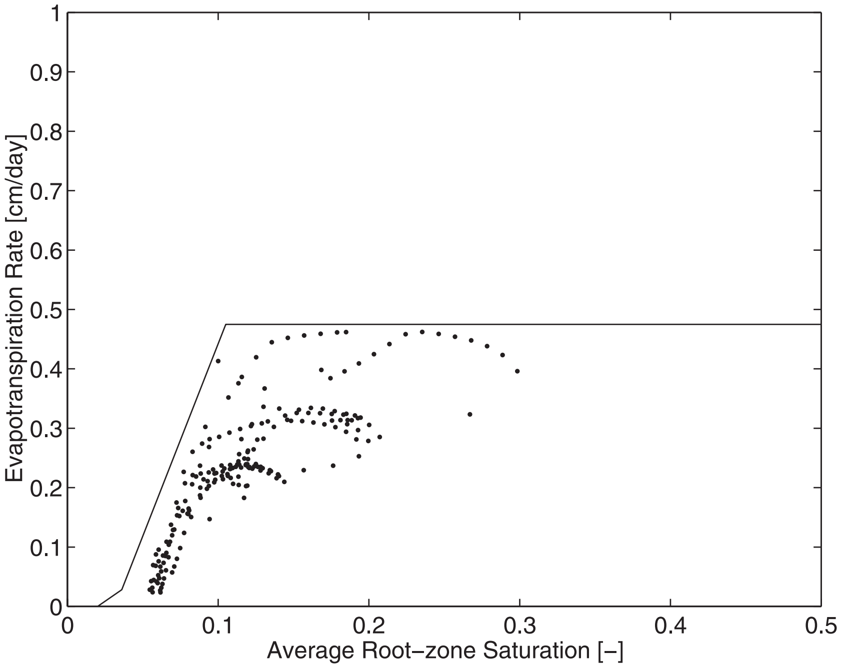

Figure 10. Relationship between total evapotranspiration and average root zone saturation for the average climate realization. Points represent upscaled results from the Richards model with $f = 0.8$, and the solid line represents the function used in the bucket model.

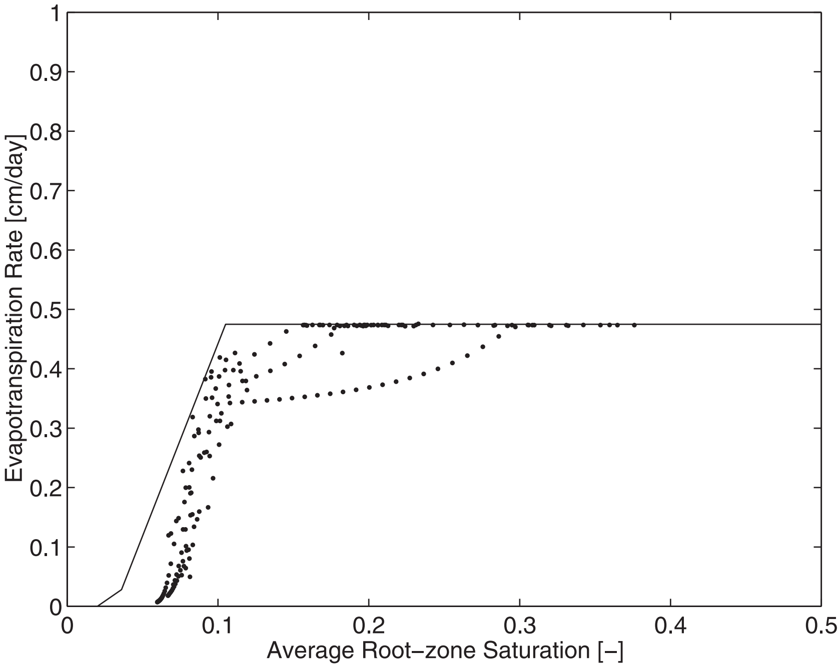

Figure 11. Relationship between total evapotranspiration and average root zone saturation for the wet climate realization. Points represent upscaled results from the Richards model with $f = 0.1$, and the solid line represents the function used in the bucket model.

average climate realization. Each discrete point corresponds to the total evapotranspiration and average saturation over one day. Plotted on the same figure is a solid line depicting the corresponding relationship for the bucket model. This figure shows that the relationship between total evapotranspiration and average saturation for the Richards model is not single-valued. For an average root zone saturation of 0.1, the evapotranspiration rate ranges from around 0.25 cm/day up to $ET_{\max}$ = 0.475 cm/day. Figure 9 also shows that the evapotranspiration rate for the bucket model deviates significantly from the values for the Richards model at low saturations. Despite these differences, however, the upscaled evapotranspiration functions for the two models are not too different.

[48] Figure 10 presents the relationship between saturation and evapotranspiration for the bucket model and the Richards model with $f = 0.8$. In this case, evapotranspiration as a function of average saturation exhibits even greater variation. In fact, one can identify traces along which the average saturation is decreasing but the total evapotranspiration is increasing. This results from propagation of wetting fronts through the soil column and the plateau of the extraction function—that is, the same amount of water spread over twice the depth can produce higher rates of evapotranspiration. As shown in this figure, the relationships between evapotranspiration and root zone saturation for the bucket and Richards model with $f = 0.8$ are quite different.

[49] Generally, the plots for the wet and dry climate realizations are similar to those already presented. The relationship between evapotranspiration rate and average saturation for the Richards model with $f = 0.1$ coupled with the wet climate shows some interesting behavior, however. Figure 11 presents these results. The figure looks similar to that for the average climate conditions, except for a few traces along which the evapotranspiration drops below $ET_{\max}$ even for saturations well above S^*. This phenomenon results from the limited spatial extent of evaporation. The evapotranspiration rate after a large storm remains at $ET_{\max}$ for a number of days during which the upper soil layers quickly dry out due to the intensity of the evaporative uptake. At this point, the evaporation is reduced even though the average saturation of the soil column is still fairly high. The figure shows traces of the evapotranspiration rate approaching the value of $ET_{\max} - E_{\max} = T_{\max}$.

4. Discussion

[50] The two models of soil moisture dynamics presented here are underlain by differences in the infiltration dynamics, runoff generation, and uptake for evapotranspiration. Because of the climate and soil characteristics, differences in runoff generation are negligible for this comparison, as indicated by a large value of R_I. The temporal infiltration index is also much greater than one, so the representation of rainfall as instantaneous shots of water in the bucket model is reasonable. Therefore the differences in the results can be attributed to the spatial variability in soil moisture and its effect on local losses due to evaporation and transpiration. The biggest differences in the predictions from the bucket and Richards models are in the relationship between evapotranspiration and average root zone saturation, the timing and intensity of transpiration, and the partitioning of uptake between evaporation and transpiration. Whether or not these differences are significant will depend on the objective of the modeling study. For example, if one is interested in the time history of soil moisture, the relationship between evapotranspiration and saturation is important, but the

partitioning of the losses between evaporation and transpiration is less significant. Conversely, if one is interested in vegetation health, transpiration may be the most significant quantity, and the time history of soil moisture may not be important. Therefore the appropriate use of the bucket or Richards model will depend on the goals of the study.

[51] For the behavior of the bucket and Richards models to match, the upscaled relationships between evapotranspiration and average saturation must be similar. For all cases presented here, the evapotranspiration at low average saturations is higher for the bucket model than for the Richards model. Part of this difference is due to the difference in the steepness of the supply curve and part to the limited depth over which evaporation takes place. For a uniform saturation profile, Figure 5 gives the total, normalized, plant uptake when the saturation is less than s^*. As the saturation goes from s^* to s_w, the transpiration rate depicted by this curve decays more quickly than that for the bucket model. This difference in the steepness of the upscaled transpiration curves between the bucket and Richards models is consistent with the different rates of decline in the transpiration rate after a storm, depicted in Figure 8. For the Richards model, the steepness of the curve is directly related to the unsaturated hydraulic conductivity. The shape of the relative permeability curve influences both the rate of change of the transpiration rate as a function of soil moisture and also the saturation below which uptake effectively ceases. Even though transpiration does not go to zero until the wilting saturation, the steepness of the supply curve causes rates of uptake at saturations below five percent to be negligible. Since relative permeability is a difficult quantity to measure at low saturations, whether the losses from the Richards model at low saturations are a better representation than those from the bucket model is not clear. To obtain a better match between the two models, one could reassign S_w from $s(h_w)$, as determined from the retention curve, to a value closer to seven percent, at which point the unsaturated hydraulic conductivity is a factor of fifty less than the value at S^*. Alternately, one could use a different representation of the relative permeability curve for the plant uptake function, or a different uptake function altogether, that would bring the supply function for the Richards model closer to that for the bucket model. Of course, the appropriate representation depends on available data and their reliability.

[52] In addition to the differences arising from the formulation of the uptake function, the spatial variability of evaporation leads to differences in the relationship between total evapotranspiration and average saturation for the two models. Because evaporation occurs over just a fraction of the root zone, the average saturation over the entire root zone could be much higher than hygroscopic even when evaporation is nearly zero.

[53] Figures 9, 10 and 11 also indicate that the relationship between transpiration and average saturation is not unique. Figure 4 of *Federer* [1982] shows similar scatter in the relationship between transpiration and average saturation. Because of the variation in root density and the nonlinearity of the retention and relative permeability curves with respect to saturation, the transpiration is strongly dependent on the distribution of soil moisture. The total uptake from a soil column with an average saturation of fifteen percent can be very different if the water is distributed evenly over the soil column versus if the top fifteen percent of an otherwise dry root zone is fully saturated. As the resistance to flow in the root tissue decreases, the plant can compensate for the spatial variability of soil moisture, and the nonuniqueness of transpiration as a function of saturation is less severe for smaller values of f. For *Burkea africana* with $f = 0.1$, representation of the relationship between transpiration and average saturation with a single-valued function would be reasonable. For the case with $f = 0.8$, however, such a representation is not appropriate.

[54] Because of the shallow depth over which evaporation occurs, the spatial variability in soil moisture over this depth is likely to be regulated purely by the process of drying. Thus there is likely to be a one-to-one relationship between the distribution of soil moisture and the average saturation. Consequently, one would anticipate that representing evaporation as a function of an average saturation would be appropriate, provided that the average saturation were calculated over the depth of evaporation only. Representation of evaporation as a function of the average saturation over the entire root zone is not appropriate when only a fraction of that depth is affected by the process of evaporation.

[55] Figure 8 shows that the timing and intensity of transpiration for the Richards model is a strong function of the parameter f. When the root resistance is small, i.e., when f is small, the plant can extract water from wet regions at high rates to compensate for roots in dry parts of the soil column. In such a case, the transpiration rate fluctuates rapidly. When f is close to one, however, the changes in transpiration rate over time are more subdued. For the cases under consideration here, predictions of the transpiration rate from the bucket model more closely match those from the Richards model when $f = 0.1$ than when $f = 0.8$. This is consistent with the values of Γ_T for these two cases: 1.0 and 8.4, respectively. This consistency is depicted graphically in Figure 6, which shows that, when $Z_i = \overline{Z}_i$, the transpiration rates for the bucket model and the Richards model with $f = 0.1$ are close, but the rate for the bucket model is far greater than the rate for the Richards model with $f = 0.8$.

[56] Though f is not known for *Burkea africana*, a sense for the appropriateness of the values can be obtained by comparing the associated values of C_r to measured values for other plant species. The values of C_r for the Richards model are 7.8×10^4 days/cm when $f = 0.8$ and 9.7×10^3 days/cm when $f = 0.1$. *Steudle et al.* [1987] subjected many samples of young maize roots (with a diameter of approximately one millimeter) to an applied pressure gradient and measured the resulting root conductivities. When converted to the same units as C_r, the values range from 1.3×10^4 to 3.1×10^5 days/cm. The values used in the Richards model for *Burkea africana* are comparable, but given the range of measured values and the difference in species, a definitive value of f for *Burkea africana* can not be obtained.

[57] A final difference between the bucket and Richards model is in the partitioning of evapotranspiration between evaporation and transpiration. If the environment were not water limited, i.e., if the index of dryness, D_I, were much less than one, indicating deep and frequent storms, the ratio of transpiration to evaporation would be $T_{\max}/E_{\max}$, or 2.167 in this case. The approach to this asymptote for the water-controlled ecosystem under consideration varies between

the bucket and the Richards model. Since the soil column dries substantially between storms, both models predict that shallower storms will lead to a greater fraction of the uptake going as evaporation. In the bucket model, this is due to the shape of the loss function at low saturations, see Figure 2. For the Richards model, the weighting functions depicted in Figure 4 indicate that evaporation is a large component of local uptake at shallow depths; for large storms, however, more water infiltrates to depths at which the plant can use it, as indicated in Table 5. Following a characteristic storm, Γ_E indicates that the evaporative losses for the Richards model are about fifty percent larger than those for the bucket model. Over a season, the Richards model predicts that the split between transpiration and evaporation is approximately 55% and 45% of total evapotranspiration, respectively, for the average climate realization. This is in contrast to the bucket model, for which the ratio is closer to two to one. *Scholes and Walker* [1993] report that evapotranspiration comprises 45% transpiration and 55% evaporation. Direct comparison to the results of this paper is not appropriate, since the Scholes and Walker study includes more than one plant species, but the data indicate that the bucket model may require a better method of separating evaporation and transpiration.

5. Conclusions

[58] In this comparison, we characterize the predictions of soil moisture dynamics and plant uptake from the bucket and Richards models by three primary differences: the relationship between uptake and average root zone saturation, the timing and intensity of transpiration, and the partitioning of evapotranspiration into evaporation and transpiration. The dimensionless runoff index, R_I, and the temporal infiltration index, $I_{I,\,t}$, indicate that differences in model results are not attributable to differences in runoff generation or the timing of infiltration. The value of the spatial infiltration index is much less than one, however, and this indicates that the spatial distribution of soil moisture as predicted by the Richards model is typically far from uniform. Thus a dependence of evaporation and transpiration on the spatial distribution of soil moisture can lead to differences in the model predictions. If the plant has the ability to compensate for spatial variation in saturation, i.e., if f is small, the differences in the models are smaller, as in the case with $f = 0.1$. If f is large, however, the differences between the models are substantial.

[59] The dimensionless numbers, Γ_T and Γ_E, depend on the root distribution, the typical depth of infiltration, the parameter f, and the loss function used in the bucket model, and they quantify the match of transpiration and evaporation between the bucket and Richards models. When these numbers are close to one, the models predictions are similar; values greater than one indicate that the bucket model predicts higher losses, and vice versa for values less than one. In addition to the differences arising from the spatial variation in soil moisture, the formulations for the loss functions lead to other differences between the models at low saturations. The the linear decay of the bucket-model loss function with respect to saturation differs from the steep decline of the supply function in the Richards model. This leads to minimum and average saturations for the bucket model that are lower than those for the Richards models.

[60] For $f = 0.1$, the transpiration and saturation histories predicted by the bucket and Richards models match pretty well. The Richards model shows a more rapid decrease in the transpiration rate following a storm event, and the bucket model predicts lower minimum saturations, but both of these discrepancies are attibutable to the particular shape of the supply curve at low saturations. In this case, the soil moisture dynamics are adequately described by the bucket model, and a more complex representation is not necessary. When $f = 0.8$, however, use of the bucket model to predict soil moisture and plant behavior is not appropriate. This comparison of the bucket and Richards models applies to a specific set of climate, soil, and vegetation conditions. The dimensionless quantities identified herein, however, can be used to quantify the similarity of the two models under different conditions.

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