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We reconsider the old problem of the dynamics of spontaneous symmetry breaking (SSB) using 3D lattice simulations. We develop a theory of tachyonic preheating, which occurs due to the spinodal instability of the scalar field. Tachyonic preheating is so efficient that SSB typically completes within a single oscillation as the field rolls towards the minimum of its effective potential. We show that, contrary to previous expectations, preheating in hybrid inflation is typically tachyonic. Our results may also be relevant for the theory of the formation of topological defects and of disoriented chiral condensates in heavy ion collisions.

Spontaneous symmetry breaking (SSB) is a basic feature of all realistic theories of elementary particles. In the simplest models, SSB appears because of the presence of tachyonic mass terms such as $-m^2\phi^2/2$ in the effective potential. This results in tachyonic instability: long wavelength quantum fluctuations ϕ_k of the field ϕ with momenta $k < m$ grow exponentially, $\phi_k \sim \exp(t\sqrt{m^2 - k^2})$, which leads to SSB.

This process may occur gradually, as in the theory of second order phase transitions, when the parameter m^2 slowly changes from positive to negative and the degree of symmetry breaking gradually increases in time [1]. Sometimes the symmetry breaking occurs discontinuously, due to a first order phase transition [1]. But there is also another possibility, which we will study in this paper: The tachyonic mass term may appear suddenly, on a time scale that is much shorter than the time required for symmetry breaking to occur. This may happen, for example, when the hot plasma created by heavy ion collisions in a "little big bang" suddenly cools down [2]. A more important application from the point of view of cosmology is the process of preheating in the hybrid inflation scenario [3,4], where inflation ends in a "waterfall" regime triggered by tachyonic instability.

During the last few years we have learned that particle production by an oscillating scalar field may occur within a dozen oscillations due to the nonperturbative process called preheating [5]. Usually preheating is associated with broad parametric resonance in the presence of a coherently oscillating inflaton field [5], but other mechanisms are also possible; see, e.g., [6].

In this paper we will concentrate on what we call *tachyonic preheating,* which occurs due to tachyonic (spinodal) instabilities in the field responsible for the symmetry breaking. The process of symmetry breaking has been studied before by advanced methods of perturbation theory; see, e.g., [7] and references therein. However,

SSB is a strongly nonlinear and nonperturbative effect. It usually leads to the production of particles with large occupation numbers inversely proportional to the coupling constants. As a result the perturbative description, including the Hartree and $1/N$ approximations, has limited applicability. It does not properly describe rescattering of created particles and other important features such as production of topological defects.

For further theoretical understanding of the issue one should go beyond perturbation theory. Fortunately, during the last few years new methods of lattice simulations have been developed. They are based on the observation that quantum states of Bose fields with large occupation numbers can be interpreted as classical waves and their dynamics can be analyzed by solving relativistic wave equations on a lattice in $(1 + 1)$ dimensions [8] and in $(3 + 1)$ dimensions [9]. In our paper we will for the first time apply the methods of Ref. [9] to the investigation of SSB and tachyonic preheating in $(3 + 1)$ dimensions.

We will show that tachyonic preheating can be extremely efficient. In many models it leads to the transfer of the initial potential energy density $V(0)$ into the energy of scalar particles within a single oscillation. Contrary to standard expectations, the first stage of preheating in hybrid inflation is typically tachyonic. Thus the stage of oscillations of a homogeneous component of the scalar fields driving hybrid inflation either does not exist at all or ends after a single oscillation. A detailed description of our results will be given elsewhere [10].

The simplest model of SSB is based on the theory with the effective potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 \equiv \frac{m^4}{4\lambda} - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4,$$

(1)

where $\lambda \ll 1$. $V(\phi)$ has a minimum at $\phi = \pm v$ and a maximum at $\phi = 0$ with curvature $V'' = -m^2$.

The development of tachyonic instability depends on the initial conditions. We will assume that initially the symmetry is restored so that the field ϕ does not have any homogeneous component, i.e., $\langle \phi \rangle = 0$. But then $\langle \phi \rangle$ remains zero at all later stages, and for the investigation of SSB one needs to find the spatial distribution of the field $\phi(x, t)$. To avoid this complication, many authors assume that there is a small but finite initial homogeneous background field $\phi(t)$, and even smaller quantum fluctuations $\delta \phi(x, t)$ that grow on top of it. This approximation does not describe the creation of topological defects, which, as we will see, is not a small nonperturbative effect but an important ingredient of the process.

For definiteness, we suppose that in the symmetric phase $\phi = 0$ there are the usual quantum fluctuations of a massless field with the mode functions $(1/\sqrt{2k})e^{-ikt+i\vec{k}\vec{x}}$. Then at $t = 0$ we "turn on" the term $-m^2\phi^2/2$ corresponding to the negative mass squared $-m^2$. The modes with $k = |\vec{k}| < m$ grow exponentially. Their dispersion can be estimated as

$$\langle \delta \phi^2 \rangle = \int_0^m \frac{dk^2}{8\pi^2} e^{2t\sqrt{m^2-k^2}} = \frac{e^{2mt}(2mt - 1) + 1}{16\pi^2 t^2}. \tag{2}$$

To get a qualitative understanding of the process of SSB, instead of many growing waves with momenta $k < m$ consider first a single sinusoidal wave $\delta \phi = \Delta(t)\cos kx$ with $k \sim m$ and with initial amplitude $\sim \frac{m}{2\pi}$ in one-dimensional space. The amplitude of this wave grows exponentially until it becomes $\mathcal{O}(v) \sim m/\sqrt{\lambda}$. This leads to the division of the universe into domains of size $\mathcal{O}(m^{-1})$ in which the field changes from $\mathcal{O}(v)$ to $\mathcal{O}(-v)$. The gradient energy density of domain walls separating areas with positive and negative ϕ will be $\sim k^2 \delta \phi^2 = O(m^4/\lambda)$. This energy is of the same order as the total initial potential energy $V(0) = m^4/4\lambda$. This is one of the reasons why any approximation based on perturbation theory and ignoring topological defect production cannot give a correct description of the process of symmetry breaking.

Thus a substantial part of the false vacuum energy $V(0)$ is transferred to the gradient energy of the field ϕ when it rolls down to the minimum of $V(\phi)$. Because the initial state contains many quantum fluctuations with different phases growing at a different rate, the resulting field distribution is very complicated, so it cannot give all of its gradient energy back and return to its initial state $\phi = 0$. That is why SSB and the main stage of preheating in this model may occur within a single oscillation of the field ϕ.

The tachyonic growth of all fluctuations with $k < m$ continues until $\sqrt{\langle \delta \phi^2 \rangle}$ reaches the value $\sim v/2$, since at $\phi \sim v/\sqrt{3}$ the curvature of the effective potential vanishes and instead of tachyonic growth one has the usual oscillations of all the modes. Equation (2) shows that this happens within the time $t_* \sim \frac{1}{2m} \ln \frac{C}{\lambda}$, where $C \sim 10^2$. The exponential growth of fluctuations up to that moment

can be interpreted as the growth of the occupation number of particles with $k \ll m$. These occupation numbers at the time t_* grow up to $n_k \sim \exp(2mt_*) \sim \mathcal{O}(10^2)\lambda^{-1} \gg 1$. For small λ the fluctuations with $k \ll m$ have very large occupation numbers, and therefore they can be interpreted as classical waves of the field ϕ.

When the field rolls down to the minimum of its effective potential at $\phi \sim v$, its fluctuations scatter off each other as classical waves. It is difficult to study this process analytically, but one can do it using lattice simulations [9]. We performed our simulations on lattices with either 128^3 and 256^3 grid points. A detailed description of our calculations will be given in [10]; here we will present only our main results.

Figure 1 illustrates the dynamics of symmetry breaking in the model (1) with a two-component scalar field $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$. It shows the probability distribution $P(\phi_i, t)$, which is the fraction of the volume containing the field ϕ at a time t if at $t = 0$ one begins with a probability distribution concentrated near $\phi = 0$, with the quantum mechanical dispersion (2). After a single oscillation this probability distribution stabilizes at $|\phi| \sim v$, which corresponds to SSB. The standard approximation representing the scalar field as a homogeneous background field with small fluctuations does not work at any stage of the process.

A detailed investigation of the spatial distribution of the field ϕ shows [10] that after the first oscillation the scalar field can be represented as a collection of classical waves oscillating near $|\phi| \sim v$ with an amplitude smaller than $v/2$. Thus SSB indeed occurs within a single oscillation of the field distribution. A small but nonvanishing height of the histogram in Fig. 1 at $\phi = 0$ is due to the presence
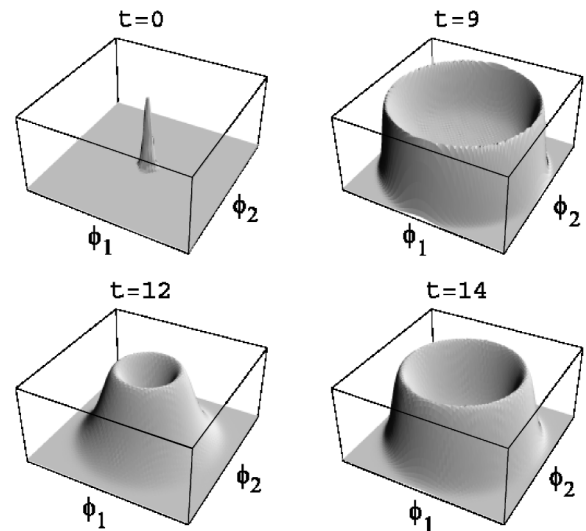

FIG. 1. The process of symmetry breaking in the model (1) for a complex field $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$. The field distribution falls down to the minimum of the effective potential at $|\phi| = v$ and experiences only small oscillations with rapidly decreasing amplitude $|\Delta\phi| \ll v$. Time is given in units m^{-1}.

of strings that have $\phi = 0$ in their cores. The process of thermalization takes much longer than spontaneous symmetry breaking [11].

Similar results can be obtained not only in this basic model of spontaneous symmetry breaking, but in other models as well. In particular, computer simulations show that SSB in the model

$$V = -\frac{\lambda}{3} v\phi^3 + \frac{\lambda}{4} \phi^4 + \frac{\lambda}{12} v^4 \qquad (3)$$

also completes within one oscillation.

The tachyonic mass $m^2(\phi) = -2\lambda v\phi + 3\lambda\phi^2$ in this model vanishes at $\phi = 0$. However, Eq. (2) implies that scalar field fluctuations with momentum $\sim k$ have initial amplitude $\langle \delta\phi^2 \rangle \sim \frac{k^2}{8\pi^2}$. They enter a self-sustained tachyonic regime if $k^2 < |m_{\text{eff}}^2| = 2\lambda v\sqrt{\langle\delta\phi^2\rangle} \sim \frac{\lambda v k}{2\pi}$, i.e., if $k < \frac{\lambda v}{2\pi}$. The average initial amplitude of the growing tachyonic fluctuations with momenta smaller than $\frac{\lambda v}{2\pi}$ is $\delta\phi_{\text{rms}} \sim \frac{\lambda v}{4\pi^2}$. These fluctuations grow until the amplitude of $\delta\phi$ becomes comparable to $2v/3$, and the effective tachyonic mass vanishes. At that moment the field can be represented as a collection of waves with dispersion $\sqrt{\langle\delta\phi^2\rangle} \sim v$, corresponding to coherent states of scalar particles with occupation numbers $n_k \sim (\frac{4\pi^2}{\lambda})^2 \gg 1$.

Because of the nonlinear dependence of the tachyonic mass on ϕ, a description of this process is more involved than in the theory (1). Even though the typical amplitude of the fluctuations is given by $\delta\phi_{\text{rms}}$, the speed of the growth of the fluctuations increases considerably if the initial amplitude is somewhat bigger than $\delta\phi_{\text{rms}}$. Thus even though the fluctuations with amplitude greater than $\delta\phi_{\text{rms}}$ are exponentially suppressed, they grow faster and may therefore have greater impact on the process than the fluctuations with amplitude $\delta\phi_{\text{rms}}$. Low probability fluctuations with $\delta\phi \gg \delta\phi_{\text{rms}}$ correspond to peaks of the initial Gaussian distribution of fluctuations of the field ϕ. Such peaks tend to be spherically symmetric [12]. As a result, the whole process looks not like a uniform growth of all modes, but more like bubble production (even though there are no instantons in this model). The results of our lattice simulations for this model are shown in Fig. 2. The bubbles (high peaks of the field distribution) grow, change shape, and interact with each other, rapidly dissipating the vacuum energy $V(0)$.

These results have important implications for the theory of reheating in the hybrid inflation scenario. The basic form of the effective potential in this scenario is [3]

$$V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2 + \frac{1}{2}m^2\phi^2. \qquad (4)$$

The point where $\phi = \phi_c = M/g$ and $\sigma = 0$ is a bifurcation point. Here $M \equiv \sqrt{\lambda} v$. The global minimum is located at $\phi = 0$ and $|\sigma| = v$. However, for $\phi > \phi_c$ the squares of the effective masses of both fields $m_\sigma^2 = g^2\phi^2 - \lambda v^2 + 3\lambda\sigma^2$ and $m_\phi^2 = m^2 + g^2\sigma^2$ are posi-
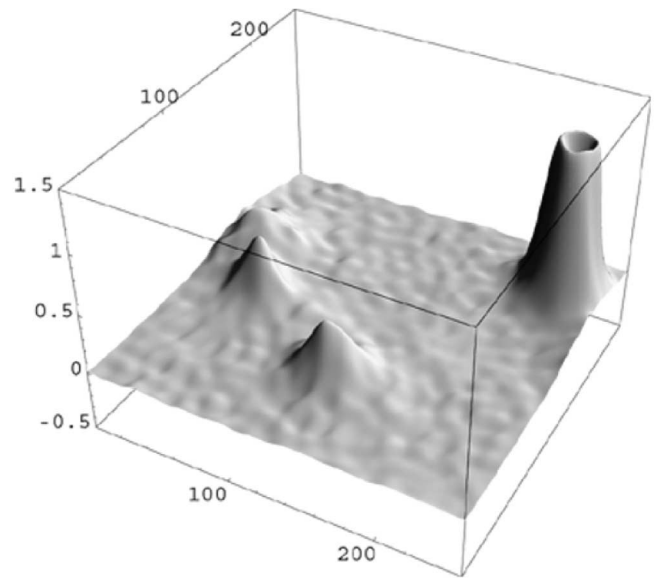

FIG. 2. Growth of fluctuations of the field ϕ in the cubic model (3) on a two-dimensional slice of our 3D lattice.

tive and the potential has a valley at $\sigma = 0$. Inflation in this model occurs while the ϕ field rolls slowly in this valley towards the bifurcation point. When ϕ reaches ϕ_c, inflation ends and the fields rapidly roll towards the global minimum at $\phi = 0$, $|\sigma| = v$. If σ is a real one-component scalar, this may lead to the formation of domain walls. To avoid this problem, we assume that σ is a complex field. In this case symmetry breaking after inflation produces cosmic strings instead of domain walls [3].

In realistic versions of this model the mass m and the velocity $\dot\phi$ of the field ϕ after inflation are extremely small. The fields fall down along a certain trajectory $\phi(t)$, $\sigma(t)$ in such a way that initially this trajectory is absolutely flat, then it rapidly falls down, and then it becomes flat again near the minimum of $V(\phi, \sigma)$. This implies that the curvature of the effective potential along this curve is initially negative. Therefore the fields should experience tachyonic instability along the way.

The decay of the homogeneous inflaton field and preheating in hybrid inflation were considered in [4,13]. Both papers were focused on the possibility of parametric resonance. However, in [4] it was also pointed out that for $g^2 \gg \lambda$ the field σ falls down only when the field ϕ reaches some point $\phi \ll \phi_c$. As a result, the motion of the field σ occurs just like the motion of the field ϕ in the theory (1). In this case one has a tachyonic instability and the fields relax near the minimum of $V(\phi, \sigma)$ within a single oscillation [4]. For all other relations between g^2 and λ the fields follow more complicated trajectories. One could expect that the fields would in general experience many oscillations [4,13].

We performed an investigation of preheating in hybrid inflation in the model (4) with two scalar fields (one real and one complex) and in SUSY-motivated F-term and
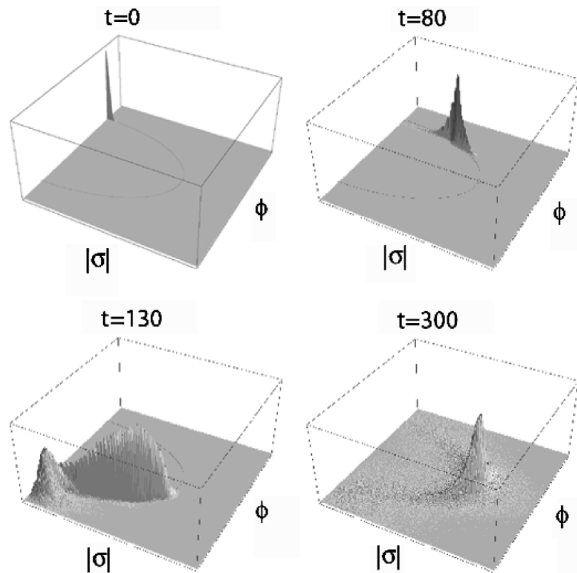
FIG. 3. The process of symmetry breaking in the hybrid inflation model (4) for $g^2 \ll \lambda$. The field distribution moves along the ellipse $g^2\phi^2 + \lambda\sigma^2 = g^2\phi_c^2$ from the bifurcation point $\phi = \phi_c = \sqrt{\lambda}\,v/g$, $\sigma = 0$ towards the point $\phi = -\phi_c$, $\sigma = 0$, bounces back, and finally relaxes at $\phi = 0$, $\sigma = v$.

D-term inflation models with three complex fields. We used methods similar to those that we applied in the previous section to the investigation of SSB, including 3D lattice simulations. We found that efficient tachyonic preheating is a generic feature of the hybrid inflation scenario, not only for $g^2 \gg \lambda$, but for $g^2 \sim \lambda$ and $g^2 \ll \lambda$ as well. For example, Fig. 3 shows the process of spontaneous symmetry breaking in the theory (4) for $g^2 = 10^{-4}$, $\lambda = 10^{-2}$, $M = 10^{15}$ GeV. The fields relax near the minimum of the effective potential within a single oscillation.

The theory of preheating in D-term inflation [14] is very similar to the theory discussed above. In the case $g^2 = 2\lambda$ the effective potential (4) has the same features as the effective potential of SUSY-inspired F-term inflation [15]. In this scenario the fields ϕ and σ fall down along a simple linear trajectory [13], so that instead of following each of these fields one may consider a linear combination of them and find the effective potential in this direction. This effective potential has exactly the same shape as our cubic potential (3). Thus all our results for tachyonic preheating in the theory (3) should be valid, with minor modifications, for D-term and F-term inflation. We confirmed these conclusions with lattice simulations of the F-term and D-term models.

Preheating in the noninflaton sector and the subsequent development of equilibrium in hybrid models were considered in [11]. Light bosonic fields interacting with scalars from the inflaton sector are dragged into the process of preheating. Excitations of these fields rapidly acquire large occupation numbers and further evolve into equilibrium together with the inflaton scalars.

From a more general point of view, the most important application of our results is to the general theory of spontaneous symmetry breaking. This theory constitutes the basis of all models of weak, strong, and electromagnetic interactions. The new methods developed during the last few years in application to the theory of reheating after inflation have been applied in this paper to the theory of spontaneous symmetry breaking. These methods have for the first time allowed us not only to calculate correlation functions and spectra of produced particles, but to actually *see* the process of spontaneous symmetry breaking and to reveal some of its rather unexpected features. We will return to the discussion of this issue in an upcoming publication [10].

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