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SPECTROSCOPIC LIMITS ON HIGH-REDSHIFT Lya EMISSION¹

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ABSTRACT

We have conducted a deep long-slit spectroscopic search for high-redshift (2.7 < z < 4.7) Ly α emission. Four pairs of deep, high-resolution ($R \approx 2000$), long-slit CCD frames were taken at the Multiple Mirror Telescope. The "blank sky" in each pair of frames was searched for faint emission features unresolved spatially and spectrally. No emission features were found down to a limiting line surface brightness (1σ) of $1-4 \times 10^{-18}$ ergs s⁻¹ cm⁻² arcsec⁻². The sensitivity of the search was calibrated using simulations with synthetic features added to the data frames. The data set upper limits on the mean space density $\langle n \rangle$ and line flux of randomly distributed Ly α -emitting clouds; for example, at $z \approx 4.5$ we have a 95% confidence limit of $\langle n \rangle < 1$ Mpc⁻³ h_{100}^3 at a total line flux level of $\sim 3 \times 10^{-17}$ ergs s⁻¹ cm⁻². These limits approach expected emission levels for fairly conservative published primeval galaxy models. In addition, Ly α emission was searched for, but not detected, from a known Ly α -limit absorption cloud toward QSO 0731+653. Assuming that the gas in this cloud is spatially resolved (size $\gtrsim 4h^{-1}$ kpc) we set an upper limit on the diffuse ionizing flux density at z = 2.912 of $J_{\nu} \lesssim 2 \times 10^{-19}$ ergs s⁻¹ cm⁻² sr⁻¹ Hz⁻¹.

Subject headings: cosmic background radiation - cosmology - galaxies: formation

I. INTRODUCTION

A natural way to probe the distribution of gas and ionizing radiation at high redshift is to look for $Ly\alpha$ line radiation. Such emission is expected from gas clouds ionized in the vicinity of QSOs, from collapsing primeval galaxies undergoing initial star formation, and from the hydrogen clouds thought to be responsible for the $Ly\alpha$ forest seen in QSO spectra, which are ionized by the ambient intergalactic ultraviolet radiation (Hogan and Weymann 1987). In this paper we report the initial results of a search for "blank sky" high-redshift line radiation and use them to place constraints on the abundance of starforming galaxies and the intensity of intergalactic radiation at redshifts $z \simeq 3$.

Low ($\simeq 90$ Å)-resolution interference-filter imaging of QSO environments or high-redshift galaxies has been very successful at discovering Lya emission (Djorgovski et al. 1987; Hu and Cowie 1987; McCarthy et al. 1987; Lilly 1988; Spinrad 1989; Chambers and Miley 1990; Elston 1988). By contrast, searches for Lya emission in blank sky using both narrow-band imaging and spectroscopic techniques have been interpreted as producing uniformly null results (see Koo 1986 for a review). Koo and Kron (1980) used slitless spectroscopy to survey 4000 arcmin², but reached only R = 20.6, considerably below the predictions of most primeval galaxy models. Pritchet and Hartwick (1987), using a 3.6 m telescope and a CCD with ~ 100 Å filters to search 7.26 arcmim² of sky, found no confirmed sources at $z \sim 5$ down to $R \sim 27$. Cowie (1988) conducted a deep long-slit Lya search of 2000 arcsec² at 11 Å resolution and a narrowband (60 Å) imaging search of 10,000 arsec² of blank sky; visual inspection revealed no obvious primeval galaxy candi-dates to a flux level of a few times 10^{-17} ergs s⁻¹ cm⁻². Our

¹ This research is based on data obtained at the Multiple Mirror Telescope, a joint facility of the University of Arizona and the Smithsonian Institution.

strategy has been to use higher spectral resolution in order to maximize the contrast of line emission over the sky background, whose noise is the limiting factor in all such searches. This leads to a better absolute flux-density sensitivity than is possible at lower spectral resolution, at the expense of lowering the volume of space searched.

Given a two-dimensional detector (in our case, an 800×800 CCD), high resolution may be obtained in two ways: narrowband imaging, especially Fabry-Perot imaging, and long-slit two-dimensional spectroscopy (Hogan and Rees 1979). In either case, the search maps a three-dimensional spatial volume onto the chip and can be configured to use efficiently the entire dynamic range of the chip using optimal sampling in spatial and spectral dimensions. In a sky-limited search, the absolute flux limit is determined by the spectral resolution (which should optimally match the line width), so both the volume searched and the limiting flux density are similar in the two cases. The first technique has the advantages of giving two-dimensional sky coverage and sampling a spatial volume that resembles a cube, with sides of comoving scale of order a few Mpc; since this is a scale where the galaxy-galaxy correlation is appreciable today, such a geometry is thus ideal for probing environments of known objects. The long-slit technique, on the other hand, samples a volume resembling a thin yard-stick seen end-on, covering a relatively small area of sky but reaching deep in redshift space; the dimensions of the search volumes described here are of the order of 0.02 Mpc \times 3 Mpc \times 150 Mpc in comoving units. The larger surface area of the search volume and large redshift coverage are thus well suited for detecting diffuse and uncorrelated emission, for example from absorption clouds spread over a wide range of redshift. The long-slit technique also has the advantage of sampling regions of space not physically associated with QSOs or other objects in the field.

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In this paper we report the results of a deep long-slit spectroscopic search for high-redshift $Ly\alpha$ emission conducted at the Multiple Mirror Telescope. The images were subjected to rigorous statistical tests to verify the sensitivity reached. The data have been analyzed with regard to the question of blank-sky emission; in this paper QSO light is used only to locate $Ly\alpha$ absorption systems along the line of sight, and no analysis is attempted of possible gas in environs of the QSOs themselves. The observations are discussed in § II, the statistical analysis in § III and IV.

II. OBSERVATIONS

The observations were made at the MMT using the CCD spectrograph (Schmidt, Weymann, and Folz 1989) on the nights of 1987 November 23 and 1988 October 16 and 17 (see Table 1). Two or four integrations, ranging from 3600 to 5000 s, were made of each of three fields centered on bright highredshift (3.0 < z < 3.5) QSOs selected from the Hewitt and Burbidge catalog (1987). The wavelength region observed was chosen in each case to cover the QSO's Lya emission line, except for the highest redshift pair, which covered redshifted C IV 1549 Å emission from Q0301-005; these wavelengths were chosen to search for Lya emission from redshifts both lower and higher than that of the QSO itself. Total coverage was ~ 600 Å per frame at 2.4 Å resolution, and the unvignetted spectrograph slit dimensions were $1.5 \times 160^{"}$. The CCD was binned in the spatial directions by a factor of 2 to reduce readout noise, yielding a spatial scale of 0.6 pixel⁻¹; the spectral scale was ≈ 0.8 Å pixel⁻¹. Between the two observations of each field, the telescope was moved from one image to the next. (This technique allows cosmic rays and cosmetic defects on the CCD [e.g., 'hot pixels"] to be rejected as candidate emission features.) The seeing on all three nights was better than 2", and sometimes as good as 1".

Data reduction was carried out using standard IRAF routines. The raw images were bias subtracted and flat-fielded to correct for variations in pixel sensitivity (except in the case of the Q0731 + 653 field, for which it was determined that the flat fields would have increased the noise fluctuations in the frame). The long-wavelength data from the 1988 October run showed significant spectral curvature, due to spectrograph optics; these frames were transformed to rectilinear coordinates, after first removing obvious cosmic-ray hits. which otherwise are spread out by the transformation. Finally, all the data were sky-subtracted, using as high signal-to-noise templates the averaged areas of "blank sky" bordering the QSO spectra, and flux-calibrated using standard stars. This method of sky subtraction was preferable to fitting a polynomial to the background-sky spectrum, since the high spectral resolution of the data resolved narrow night-sky emission lines that would have been inaccurately fitted by a polynomial. The result is a series of rectified calibrated images with wavelength on one axis and slit position along the other, with most night-sky

TABLE 1Summary of Observations

QSO Field	^Z QSO	Z _{covered}	λ(Å)	Exposure (s)	Date of Observation
0731+653	3.035	2.7-3.2	4483-5135	2 × 3600	1987 Nov 23
0301-005	3.205	3.1-3.6	4934-5573	2×3600	1988 Oct 16
0014+813	3.41	3.1-3.6	4934-5573	2×4000	1988 Oct 17
0301-005	3.205	4.2–4.7	6285-6918	2 × 5000	1988 Oct 17

emission lines virtually eliminated. The level of RMS noise fluctuation in regions of blank sky was measured to be $1-4 \times 10^{-18}$ ergs s⁻¹ cm⁻² arcsec⁻², including readout noise from the CCD and a minimal amount of noise introduced during the data processing, but dominated by Poisson photon noise from the night-sky emission. (The long exposure times were chosen to ensure that sky noise dominated even between night-sky lines.)

III. ANALYSIS OF BLANK SKY

We first analyzed the data to determine limits on any possible population of unresolved $Ly\alpha$ emitting clouds at random locations and redshifts in each field. Any real $Ly\alpha$ emission in the observed fields would appear at the same position relative to the QSO spectrum, and at the same intensity, in both frames in each pair. Any real unresolved cloud would also be expected to have an image profile corresponding to the instrumental spectral resolution and to the seeing, i.e., an elliptical Gaussian with a FWHM of a few pixels. As in Cowie's (1988) search, no emission features of any size or brightness are visible to the naked eye in any of the eight frames. However, to verify a null detection and to quantify its statistical significance, we conducted a series of numerical tests with the data frames.

Each frame was searched for candidate objects by convolving the image with a Gaussian filter matching the instrumental and seeing profiles, and then searching the convolved image for local maxima that exceeded a specified threshold. This procedure enhances the detection probability of real objects, while suppressing that of cosmic rays and cosmetic defects on the CCD. (The long integrations allowed several hundred cosmic rays to contaminate each frame, but they are easy to recognize by their extreme intensity and sharpness.) The candidates found in this way were then examined in closer detail. An accurate centroid and an integrated intensity with respect to background for each candidate object were computed, and the lists of candidates from the two frames of each field were compared to identify any objects that appeared in both frames at the same relative separation from the QSO spectrum, at the same wavelength, and with approximately the same intensity.

By lowering the detection threshold, the number of candidate objects from a single frame could be increased to over 1000. Most of these were merely low-level noise events, although a few cosmic rays slipped through the Gaussian filter. Given enough candidates, a match between two features from different images of the same field is inevitable; it is then important to evaluate the significance of the detection. Here the slight shift between images of a given field was again helpful. The detection threshold was adjusted so that just enough candidate objects appeared to produce a few matches between the two frames. Features from the two candidate lists were then searched for correlations at separations different from the real shift; in no case was the number of matches at the correct separation statistically distinguishable from the number of spurious correlations, indicating that matches were between random noise spikes, not interesting detections.

To determine the level of Ly α emission that could be excluded by our nondetection, we tested our search method with synthetic signals. For each of a range of intensities, 100 features that matched the Gaussian seeing and instrumental profiles were added to both image frames of a given field. Running the search routine on the synthetic data then yielded a detection probability p_d for a single feature of the tested intensity. Given p_d for a range of intensities, it remained to No. 1, 1990

calculate the average space density of $Ly\alpha$ emitters as a function of intensity that could produce a null detection (i.e., a high average density of dim objects could entirely escape detection, while a single bright object per field could not). The probability of a null detection is

$$P_{x=0} = \sum_{N=0}^{N \ge \langle n \rangle} P_{\text{have}}(\langle n \rangle, N) \times P_{\text{detect}}(x=0, N) +$$

where the first term is the probability of having N objects in the field of view given an average density of objects $\langle n \rangle$ per field of view, and the second term is the probability of detecting 0 objects given N objects in the field. P_{have} is given by the Poisson probability distribution, while P_{detect} is given by the binomial distribution, calibrated by our synthetic data. With x = 0, the probability of a null detection is then

$$P_{x=0} = \sum_{N=0}^{N \geqslant \langle n \rangle} \frac{\langle n \rangle^N}{N!} e^{-\langle n \rangle} (1-p_d)^N$$

Constraining this to be 0.05, one can calculate the values of $\langle n \rangle$ that can be excluded with 95% confidence for a given intensity of Ly α emitters. Figure 1 shows the limits placed by our observations on the space density of Ly α emitters in an $\Omega = 1$ universe as a function of their flux. We plot our results in units of space number density rather than surface number density on the sky, since the latter requires modeling the total redshift space occupied by Ly α emitters. The limits are calculated for each redshift range individually; our null result considered over the entire volume searched would lead to even stricter limits. Although visual inspection of the frames with synthetic features added produced a higher number of identifications than the automated procedure, it also produced many spurious candidates, indicating that our search algorithm, which returned null results, is conservative but rigorous.

Also plotted in Figure 1 are the limits corresponding to the 2σ noise levels in our data, for 1 arcsec^2 sources. Observed through broad-band filters, pure emission line sources at our 2σ flux limits would have brightnesses, from lowest to highest redshift, of $B \sim 30.2$, $V \sim 29.7$, and $R \sim 31.0$. If we assume a rest equivalent width in Ly α of 70 Å (maximal for a "normal" young stellar population; Partridge and Peebles 1967; Koo 1986), we can also calculate the total flux expected in a given broad bandpass, including both line and continuum emission. Our 2σ detection limits would then correspond to brightnesses, from lowest to highest redshift, of $B \sim 28.8$, $V \sim 28.5$, and $R \sim 29.1$, making our search the most sensitive to date. It should be stressed, however, that our simulations indicate constraints at the 95% confidence level that are about one order of magnitude brighter.

IV. LIMIT ON DIFFUSE IONIZING RADIATION

Our data can also be analyzed from a different point of view, namely, given the fact that H I gas is known to exist at certain places in our search volumes, a lack of $Ly\alpha$ emission from the gas constrains the incident UV flux (Hogan and Weymann 1987). We focus on one particular absorption system in the 0731 + 653 field, a Ly α absorber at z = 2.912 known to have a continuum optical depth greater than 3 (Sargent, Steidel, and Boksenberg 1989). Assuming that a fraction β of incident ionizing radiation re-emerges as Lya, the observed Lya flux density is just $J_{\nu\alpha 0} = (1 + z)^{-3}\beta J_{\nu}$, where J_{ν} is the flux density of the ionizing background at redshift z. Based on commonabsorption-line studies (e.g., Foltz et al. 1984), the angular size of the absorbing cloud is likely to be larger than our spatial resolution element; $\theta = 1''$ (R/3.8 h^{-1} kpc) at this redshift. Therefore, a limit on the Ly α emission surface brightness in the absorption line core translates directly into a limit of the diffuse ionizing flux at high redshift. Since the $Ly\alpha$ emitting gas



FIG. 1.—Curved lines show statistical 95% confidence upper limits on the comoving number density of unresolved Ly α emitters in various redshift ranges, as a function of their flux in the line. Straight lines show 2 σ flux limits for 1 arcsec² sources. Regions to the upper right of both sets of lines are excluded.

lies close to the surface of the cloud (at $\tau_{\rm UV} \lesssim 2$), quenching by dust is not a problem in this situation.

We obtain this limit somewhat differently from the previous blank-sky analysis, because we now know exactly where the emission should occur in our frame. To begin with, we can now use co-added data from two frames to reduce the noise. We then add flux, matching the instrumental point-spread function, to the appropriate place in the data frame to see how much is needed to be confidently detected; this turns out to correspond to a 2 σ flux in the peak pixel, and a total intensity in the line $B = 4.8 \times 10^{-17}$ ergs cm⁻² s⁻¹ arcsec⁻², or $J_{\nu\alpha0} < 3.2 \times 10^{-21}$ ergs cm⁻² s⁻¹ sr⁻¹ Hz⁻¹. This translates directly into an upper limit on J_{y} at the redshift of the system:

$$J_{\nu} < 19 \times 10^{-20} \beta^{-1} \,\mathrm{ergs} \,\mathrm{cm}^{-2} \,\mathrm{s}^{-1} \,\mathrm{sr}^{-1} \,\mathrm{Hz}^{-1}.$$

Theoretical expectations for this quantity vary widely (Bechtold et al. 1987); however, they do not generally go as high as our limit. This limit is also not as sensitive as measurements based on modeling the "inverse effect" (Bajtlik, Duncan, and Ostriker 1988) which yield $J_{\nu} \simeq 10^{-21\pm0.5}$. However, it is possible that the inverse effect is entirely unrelated to photoionization by the QSO, in which case J_{y} could be much higher. Therefore our limit is not surprising, but is probably represents the most direct and in some respects least model-dependent information about the diffuse ionizing flux at high z.

V. DISCUSSION

Although the negative results of most recent searches for direct evidence of galaxy formation have been interpreted as excluding recent (z = 1-3) primeval galaxies (Koo 1986), some current models of galaxy formation have not been constrained by the observed nondetection levels. The primeval galaxy models of Baron and White (1987), for example, which simulate the dissipational inhomogeneous collapse of protogalaxies in a cold dark matter scenario, predict relatively recent formation of the bulk of stars, possibly continuing until z = 1-2, taking place over an extended period of time, and not in a more spectacular burst, as had been suggested earlier. The resulting objects tend to be large, dim, and diffuse, but with several peaks of emission corresponding to 28 mag arcsec⁻² across $\sim 2''$. Could we detect such an object, or the brightest peaks?

White (1989) provides scaling laws for the appearance of the primeval galaxies modeled in Baron and White (1987). Assuming a constant mass, and that the primeval galaxies are observed at the time of collapse, the angular diameter is constant with z, while the surface brightness scales as $(1 + z)^{-1/2}$ [including $(1 + z)^{-4}$ from cosmological factors and $(1 + z)^{7/2}$ from evolutionary factors]. Assuming a rest equivalent width

of Ly α of 70 Å (corresponding to the bulk of stellar ionizing radiation emerging as Lya), the bright 2" diameter peaks would then have a total Ly α flux $\sim 3.4 \times 10^{-17}$ ergs s⁻¹ cm⁻² for z = 3.0, or $\sim 3.0 \times 10^{-17}$ ergs s⁻¹ cm⁻² for z = 4.5. From Figure 1 it can be seen that the low-redshift values are not excluded for any number density by our observations, but the emission peaks would escape detection only marginally. Furthermore, the high-redshift curve in Figure 1 excludes objects of the calculated flux with an average space density $\langle n \rangle > 1$ $Mpc^{-3} h_{100}^3$. Baron and White, however, predict fewer than 0.02 emission peaks Mpc^{-3} , assuming an average of five such peaks per primeval galaxy and a uniform distribution of objects from z = 3.9 to z = 6.4, so the model is not constrained by our data, because it covers an insufficient volume.

Although we cannot exclude with any certainty the specific model proposed by Baron and White, the proximity of our sensitivity level to the predicted characteristics of a very conservative primeval galaxy model is encouraging. Our 95% confidence limits indicate that we would have detected sources at all redshifts surveyed emitting $> 10^8 L_{\odot} h_{100}^{-2}$ in the Ly α line, or less than $\sim 2 \times 10^9 L_{\odot} h_{100}^{-2}$ total luminosity, values corresponding to only moderately luminous galaxies today. If galaxies collapse with a greater dissipation rate than that modeled, if the initial gas fraction is higher, or if there are more massive stars in the IMF, the objects will appear brighter (White 1989). On the other hand, even a small amount of dust present in a collapsing gas cloud should reduce Lya emission by a large factor below the 70 Å assumed above. Although it is encouraging that large equivalent widths of Lya emission are actually observed in such high-redshift objects as the Lya companion to 3C 326.1 (McCarthy et al. 1987), there is no guarantee that this would hold for most galaxies.

What is more interesting, and not dependent on the dust abundance, is the possibility of reaching the level at which all optically thick absorbers would glow detectably in $Ly\alpha$ due to the UV background. If the UV background is only as large as inferred from the inverse effect, an increase in sensitivity by a factor of 100 would be needed to detect Ly α from all optically thick hydrogen clouds. Such a large increase does not seem immediately practicable, although two-dimensional Lya line mapping of gas distributions in the environs of QSOs, where UV fluxes are larger, certainly appears possible.

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