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A Comparison of Student Misconceptions in Rotational and Rectilinear Motion

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For much of Professor Beichner's career he has focused his attention on redesigning introductory physics education and created the SCALE-UP (Student Centered Activities for Large Enrollment University Physics) project. SCALE-UP has been adopted at more than 250 universities and had spread to other content areas and into middle and high schools, necessitating a name change to Student Centered Active Learning Environment with Upside-down Pedagogies.

A comparison of student misconceptions in rotational and rectilinear motion

Abstract

The Test of Understanding Graphics in Kinematics (TUG-K) has been modified to produce a Test of Understanding Graphics in Rotational Kinematics (TUG-R) to probe student understanding of rotational kinematics. The seven objectives of the TUG-K were modified with three questions to explore each objective resulting in a 21 question TUG-R which closely parallels the original. For many questions the modification was a simple substitution of the equivalent rotational quantity for its linear counterpart in the question stem, answers and graph-axis labels. For the remainder of the questions the modification was straightforward. For instance, references to objects moving in a straight line were replaced by references to objects spinning about a fixed axis.

The TUG-R was administered to 198 students at a small, liberal arts college in New England. The use of a calculator was permitted and students were offered as much time as they wanted to complete the examination. No inducement or reward was offered to students to take the examination and it was not counted toward their grade in the class. In order to make a more direct comparison to the results of the TUG-K, the data were narrowed to consider only 93 students where the TUG-R was administered post-instruction in both linear and rotational kinematics. This group includes student instruction in a traditional, lecture-based format as well as active engagement classrooms. Approximately 80% of the students were enrolled in an algebra-based course, the remainder in a calculus-based course.

Post-instruction student responses on the TUG-K and TUG-R were compared. A 2 tailed *z*-test was performed to assess whether or not differences in sample size can account for the differences in results between the TUG-R and TUG-K which are reported. An objective by objective, question by question analysis of the results suggests the three basic types of misconceptions noted following post-instruction analysis of the TUG-K, namely graph type confusion, slope calculation and slope *vs.* area confusion, continue to be exhibited at some level by students taking the TUG-R. However, significant differences were noted, with TUG-R students performing better on every question in two of the seven objectives on 8 of the 21 questions and equally well on 9 of the remaining 13.

Further work will be conducted to verify that these observations and conclusions remain consistent as the testing sample is expanded across a broader spectrum of students of different levels, using different instructional techniques and at a larger cross section of institutions.

Introduction

Over the past few decades, the field of physics education has matured and grown. A reasonably comprehensive description of the state of the field can be found elsewhere[1]. The process of identifying misconceptions, creating curricula to address those misconceptions and then evaluating the efficacy of instruction has been applied to many areas of physics[2], perhaps nowhere more successfully than mechanics. In that arena, many well-validated and established instruments exist, including the Mechanics Baseline Test[3], Test of Understanding Graphics in Kinematics (TUG-K)[4] and the Force Concept Inventory[5] to name but a few. Physics educators have created a wide variety of research-based, pedagogically appropriate approaches and curricula including Peer Instruction[6], Workshop Physics[7], Real-time Physics[8] and Studio Physics[9].

But, what about circular mechanics? Arnold Aron's observes[10], "The kinematics of circular motion in a plane is usually glossed over very quickly because of the obvious parallelism to rectilinear motion. For students who have genuinely mastered the concepts and relations of rectilinear kinematics, this is appropriate since unnecessary repetition would waste their time." This philosophical approach has pervasively infiltrated introductory textbooks. Whole chapters are devoted individually to the topics of velocity, acceleration, etc. while all of rotational kinematics and sometimes even dynamics are crushed into the space of a single chapter or perhaps two[11].

Remarkably little work has been done in creating instruments of evaluation[12,13] and research-based curriculum exploring rotational mechanics. Without additional evidence, it would seem a valid conjecture that any student difficulties which exist concerning rectilinear motion would continue to be carried forward, further compounded by the inherent two-dimensionality of rotation about a stationary axis adding layers of complexity to an already murky understanding of that rectilinear motion.

The Instrument

We decided to address this question using the TUG-K developed in the early 1990's to explore student understanding and interpretation of graphs as they relate to kinematics. The process of creating and validating this examination is described in detail elsewhere[4]. This instrument is structured around a series of seven objectives with three questions assigned to probe student understanding of each objective yielding a 21 question test. The TUG-K was modified to probe a parallel understanding of rotational motion to create the Test of Understanding Graphs in Rotational Kinematics, the TUG-R. The seven original objectives and their rotational counterparts are shown in Table 1. We attempted to preserve the original conceptual content of the questions during the modification process. In a touch of irony, for many questions the modification was as simple as substitution of the equivalent rotational quantity for its linear counterpart in the question stem, answers and graph-axis labels. All units were also transformed as appropriate. That is, angle in radians is substituted for position in meters, angular velocity in radians/second for velocity in meters/second, etc. For the remainder of the questions the modification was straightforward, but not quite so routine. For instance, references to objects moving in a straight line needed to be replaced by objects spinning about a fixed axis. A more detailed discussion of these modifications is included, where relevant, in the analysis of the results.

Table 1: Objectives of the Test of Understanding Graphs-Kinematics and the Test of Understanding Graphs-Rotational Kinematics

	Rectilinear Motion		Rotational Motion	
	Given	Outcome	Given	Outcome
1	Position-Time Graph	Determine Velocity	Angle-Time Graph	Determine Angular Velocity
2	Velocity-Time Graph	Determine Acceleration	Angular Velocity-Time Graph	Determine Angular Acceleration
3	Velocity-Time Graph	Determine Displacement	Angular Velocity-Time Graph	Determine Angular Displacement
4	Acceleration-Time Graph	Determine Change in Velocity	Angular Acceleration-Time Graph	Determine Change in Angular Velocity
5	A Kinematics Graph	Select Another Corresponding Graph	A Rotational Kinematics Graph	Select Another Corresponding Graph
6	A Kinematics Graph	Select Textual Description	A Rotational Kinematics Graph	Select Textual Description
7	Textual Motion Description	Select Corresponding Graph	Textual Motion Description	Select Corresponding Graph

The TUG-R was administered to 198 students at a small, liberal arts college in New England. The use of a calculator was permitted and students were offered as much time as they wanted to complete the examination. No inducement or reward was offered to students to take the examination and it was not counted toward their grade in the class. In order to make a more direct comparison to the results of the TUG-K, the data were narrowed to consider only 93 students where the TUG-R was administered post-instruction in <u>both</u> linear and rotational kinematics. This group includes student instruction in a traditional, lecture-based format as well as active engagement classrooms. Approximately 80% of the students were enrolled in an algebra-based course, the remainder in a calculus-based course. In order to provide a direct comparison, the earlier data from the TUG-K has been added to the TUG-R results in Table 2.

Results and Discussion

For some of the questions, the percentage responses are remarkably similar. For instance, in Question 7, not only is the percentage responding with the correct answer essentially the same, the percentage selecting each distractor is also remarkably consistent between the two instruments. Other questions warrant closer inspection as either the percentage selecting the correct answer or some of the distractors are markedly different. Of particular note are questions where an incorrect answer is chosen with more frequency than the expected answer, even after instruction.

Table 2: Percentage of students choosing a particular answer for each examination item. The white columns on the left for each question refer to TUG-R and the grey shaded columns on the right for each question are adapted from Beichner[4] for the TUG-K. The correct answer is indicated in boldface.

Question	1		2		3		4		5		6		7	
A	11	41	0	2	22	8	6	2	2	3	22	45	**28**	**31**
B	**28**	**16**	10	10	0	0	23	14	4	2	**46**	**25**	24	20
C	1	4	15	24	27	20	32	23	**52**	**73**	5	6	12	10
D	28	22	4	2	**44**	**62**	**25**	**28**	38	18	5	6	25	28
E	32	17	**70**	**63**	8	10	13	32	4	4	22	16	11	10
blank	0	0	1	0	0	0	1	0	0	0	0	1	1	1

Question	8		9		10		11		12		13		14	
A	6	11	1	7	**47**	**30**	6	28	19	14	8	10	4	25
B	3	11	65	57	0	2	27	17	**70**	**67**	15	15	**72**	**48**
C	52	37	8	5	45	62	9	11	9	8	48	9	5	15
D	**34**	**37**	5	7	6	3	**48**	**36**	0	2	**27**	**61**	16	9
E	4	5	**22**	**24**	1	3	10	8	2	9	2	4	2	3
blank	0	0	0	0	0	0	0	0	0	0	0	0	2	0

Question	15		16		17		18		19		20		21	
A	**47**	**29**	0	1	**27**	**21**	9	7	24	19	17	11	**32**	**18**
B	8	24	35	39	19	46	**53**	**46**	9	9	34	6	43	72
C	12	13	24	31	12	8	28	32	**35**	**37**	2	10	8	2
D	9	8	**38**	**22**	5	7	3	4	8	12	1	2	15	5
E	24	26	2	7	35	19	6	10	24	23	**44**	**72**	1	0
blank	6	0	1	0	1	0	1	1	1	0	1	1	1	2

Discussion of Individual Questions

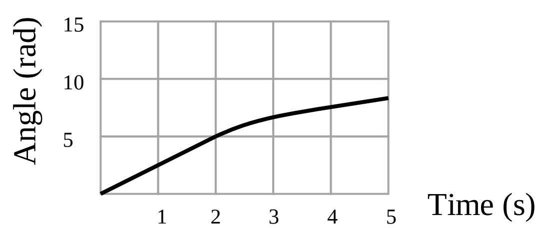
We will now undertake a question by question analysis of the instrument. Rather than proceed in order through the test, we will instead group questions by objective. The original TUG-K was constructed using common incorrect responses to fashion distractor answers. Unfortunately, this can make for a poor diagnostic examination as the same model thinking is not always present in the answers for different questions within each objective. Work is underway[14] to revise the TUG-K examination to present consistent models for incorrect answers and to better align the questions to be parallel between concepts. Throughout, the correct answers continue to be indicated using boldface with the corresponding pie wedge offset from the rest of the chart. The equivalent TUG-K version of the question can, in most cases, be determined by simply replacing rotation units with units associated with linear motion.

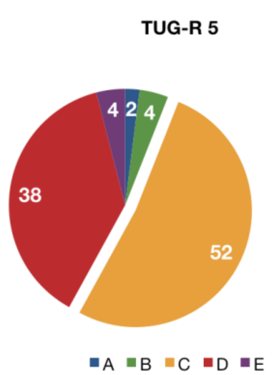
A 2 tailed *z*-test was performed on a question by question basis to compare student performance on the TUG-R and TUG-K. The *z*-test is used to assess whether or not the difference in sample size can account for the noted differences in results between the two tests. A two tailed *p*-value of 0.05 was used as the demarcation for significance, indicating certainty at the 95% confidence level that the results obtained did not occur by chance. The outcome of this analysis is summarized in Table 3 following the discussion of individual questions.

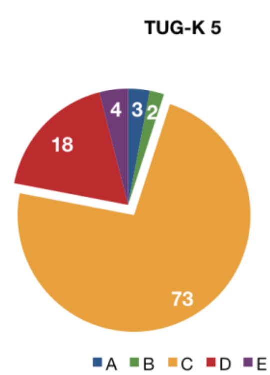
Objective 1: Given an angle-time graph, the student will determine the angular velocity.

5. The angular velocity at the 2 second point is:

(A) 0.4 rad/s 2%
(B) 2.0 rad/s 4%
(C) 2.5 rad/s 52%
(D) 5.0 rad/s 38%
(E) 10.0 rad/s 4%

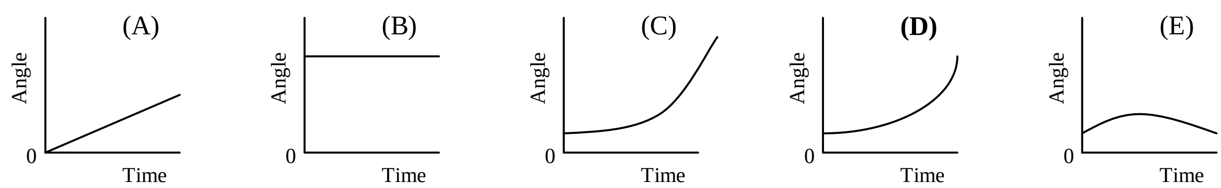




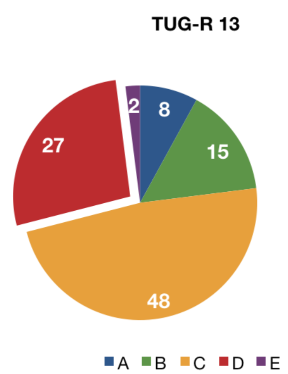


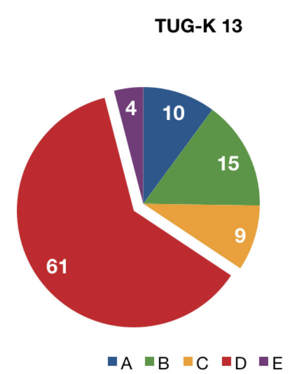
The linear motion version of this question is apparently easier for students. The most commonly selected incorrect answer is selection (D), where the student reads the value on the vertical at the 2 second point. One way of interpreting this type of axis reading error is essentially the same as a belief that switching between kinematic variables does not change the appearance of the graph. Thus, all one has to do to answer the question is read the value from the graph. Unfortunately, this answer also corresponds to a second model, one where the area under the curve from 0 to 2 seconds is calculated. An updated version of both tests should revise this item so that it is easier to determine which model was used by students.

13. Angle versus time graphs for five objects are shown below. All axes have the same scale. Which object had the highest instantaneous angular velocity during the interval?



8% 15% 48% **27%** 2%

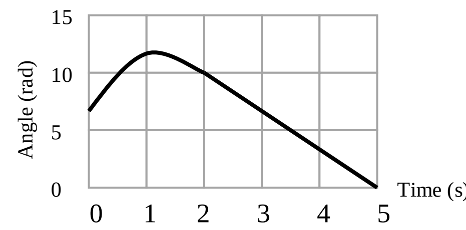


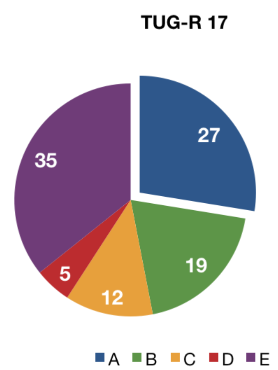


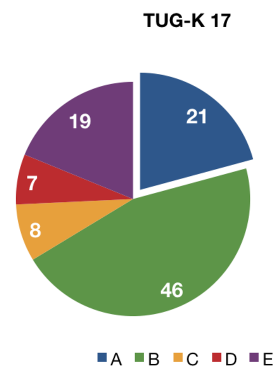
While not numerical, it is clear that this question is testing students' understanding of calculating an angular velocity from an angle-time graph. Students on the TUG-K did significantly better (*p*-value =0.0000 to four decimal places) on this question with 61% correct responses. For this question, not only did far fewer TUG-R respondents answer correctly, but it was a situation where almost twice as many TUG-R students were attracted to one of the distractors. Close inspection shows that the final value of the angle on answer C is slightly higher than the value of the angle for all other responses, suggesting the possibility that this is once again a variable switching issue. It is interesting that many students did not apply their correct linear choice to the case of rotational motion. This may indicate a memorization of a phrase they may have heard multiple times, "The slope of the position graph is velocity.", without an underlying understanding of the concept.

17. The angular velocity at the 3 second point is about:

(A) -3.3 rad/s 27%
(B) -2.0 rad/s 19%
(C) -0.67 rad/s 12%
(D) 5.0 rad/s 5%
(E) 7.0 rad/s 35%







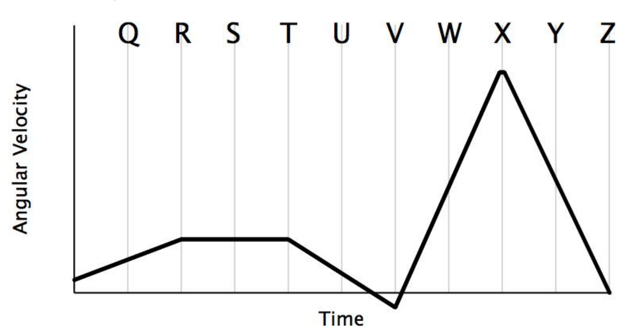
Similar percentages of both sets of students responded correctly on this question with no significant difference (*p*-value = 0.2263). The most popular response, correct or incorrect, was again the response where the student reads the value off of the vertical axis at the 3 second point. This is completely consistent with the results of Question 5 discussed above. Note, however, the slightly more uniform selection of wrong answers on the TUG-R. This may indicate that students were guessing, while the TUG-K version had a very attractive distractor corresponding to the variable switching error.

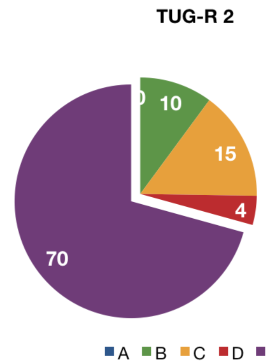
For this objective, students tend to perform somewhat better on the rectilinear questions than on the equivalent rotational motion items.

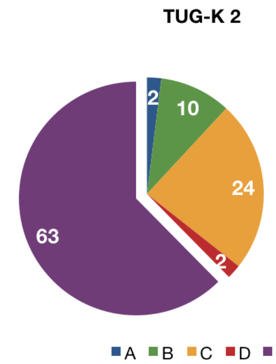
Objective 2: Given an angular velocity-time graph, the student will determine the angular acceleration.

2. When is the angular acceleration most negative?

(A) R to T 0%
(B) T to V 10%
(C) V 15%
(D) X 4%
(E) X to Z 70%



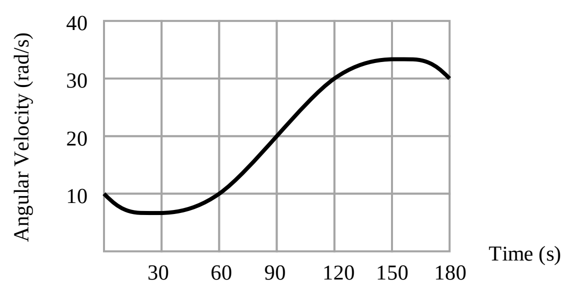


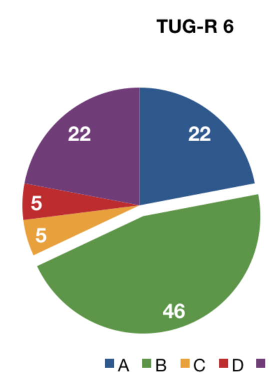


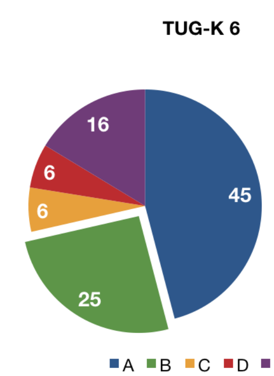
This question was answered well by both groups. The two most frequently selected incorrect answers are (B) where the angular acceleration is indeed negative and (C) which is the point where the graph attains its most negative value. Although (B) really doesn't correspond to an established poor model of the situation, one reason for selecting (B) would be because it ends at the most negative value. While not selected nearly as often as the correct response, the most common error is once again an axis reading error.

6. This graph shows angular velocity as a function of time for a grindstone of mass 3.0 x 10^2 kg and moment of inertia 35 kgm^2. What was the angular acceleration at the 90 s mark?

(A) 0.22 rad/s^2	22%
(B) 0.33 rad/s^2	**46%**
(C) 1.0 rad/s^2	5%
(D) 9.8 rad/s^2	5%
(E) 20 rad/s^2	22%



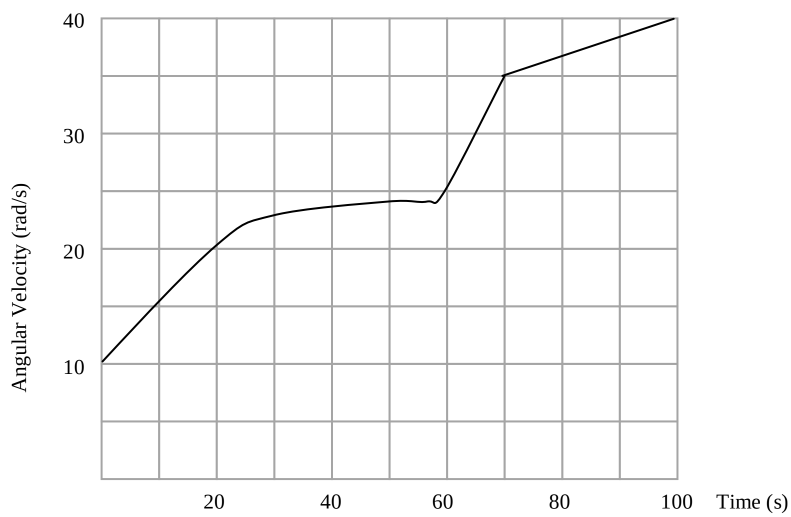


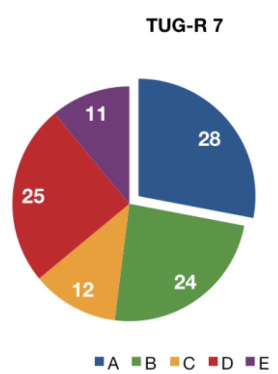


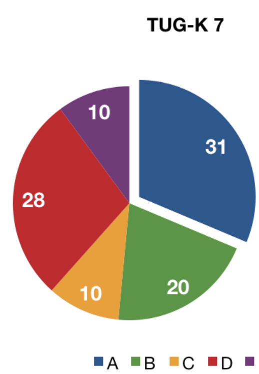
The TUG-R students were far more successful than the TUG-K students with this question. The two models which garnered equally large followings were (A) which is obtained by using *ω/t* at 90 seconds and (E) which is again an axis reading issue. Perhaps the students were more careful in their calculations with the rotation test because they were working with less familiar concepts. In other words, they may have been sloppy in the slope calculation on the TUG-K because they had seen similar problems where a straight line describing the motion happened to pass through the origin. Note that response (D) is a holdover from rectilinear motion and corresponds to the magnitude of the gravitational acceleration near the surface of the earth. A similar response has been included for many questions. The authors debated altering this answer to probe some other model, but opted to leave it for the sake of consistency between the TUG-K and this new examination.

7. The motion of an object spinning about a fixed axis is represented by the following graph. At time = 65 s, the magnitude of the instantaneous angular acceleration of the object was most nearly:

(A) 1 rad/s^2 28%
(B) 2 rad/s^2 24%
(C) 9.8 rad/s^2 12%
(D) 30 rad/s^2 25%
(E) 34 rad/s^2 11%







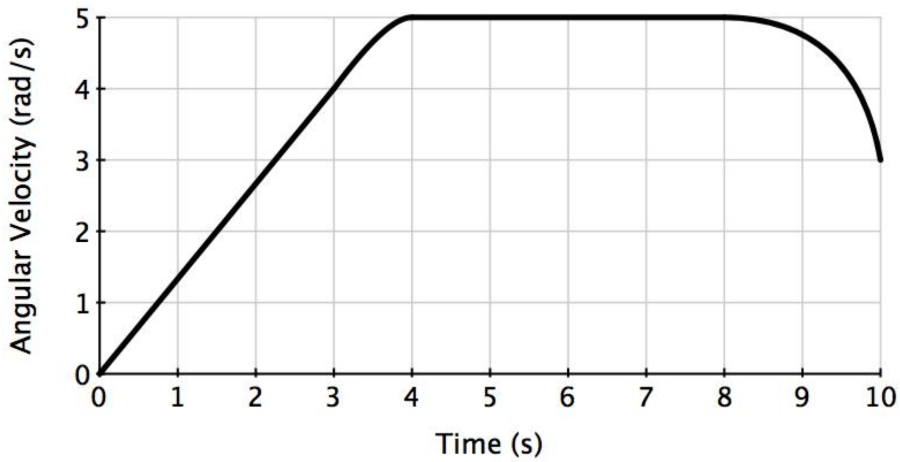
The results between the two groups are remarkably similar for this question, including the percentages choosing each of the distractor options. The two most commonly chosen incorrect responses had similar levels of attraction to the correct answer. Response (B) is a common mistake while reading graphs. In it the student counts boxes and sees a rise over run of 2 boxes over 1 box for the essentially linear region between 60 and 70 seconds, effectively ignoring the scaling factor of both axes. Response (D) is once again the result of reading directly from the axis. Note that while (C) is again the gravitational analog discussed above, it could in fact be arrived at by correctly noting the change on the vertical axis while counting boxes on the horizontal.

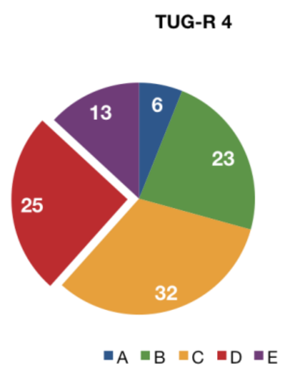
For this objectives, the results were somewhat mixed with no significant difference on two of the three questions. However, the TUG-R students did significantly better on the remaining question.

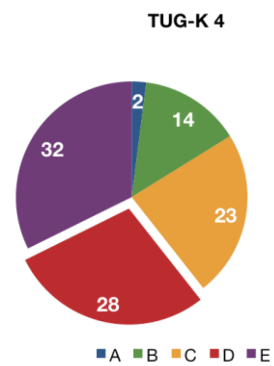
Objective 3: Given an angular velocity-time graph, the student will determine the angular displacement.

4. A pulley spins in place about a fixed axis. The mass of the pulley is 0.500 kg, its moment of inertia is 0.010 kg·m^2 and it moves with the angular velocity-time graph below. Through what angle does it spin during the first three seconds of motion?

(A) 0.75 rad 6%
(B) 1.33 rad 23%
(C) 4.0 rad 32%
(D) 6.0 rad 25%
(E) 12.0 rad 13%

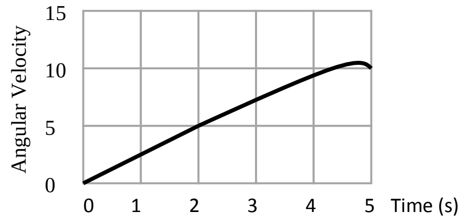




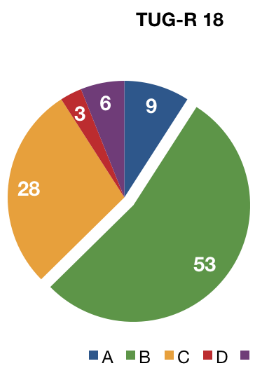


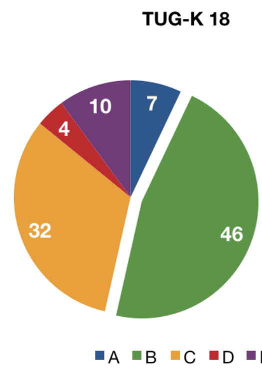
Students did not do well on either version of this question although the number chosing the correct response is not significantly different (p-value = 0.5423). For the TUG-R, the most common selection as well as the most common answer overall was (C), once again reading directly from the graph axis followed by (B), calculating the slope or angular acceleration rather than the area or angular displacement. This contrasts with the TUG-K where the most common answer and distractor was (E), calculating $\theta = \omega t$ which is appropriate only for motion with constant angular velocity.

18. If you wanted to know the angle through which an object rotated during the interval from $t = 0$ s to $t = 2$ s, from the graph below you would:



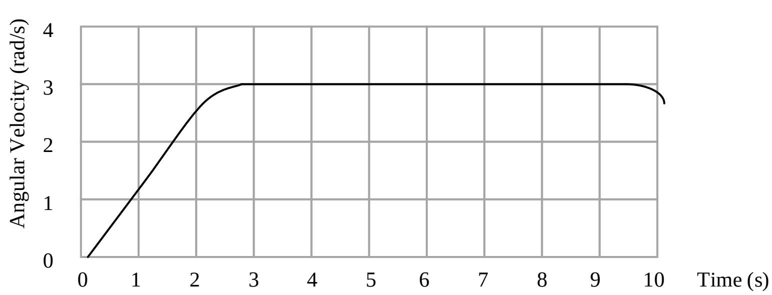
(A) read 5 directly off the vertical axis	9%
(B) find the area between that line segment and the time axis by calculating (5 x 2)/2	**53%**
(C) find the slope of that line segment by dividing 5 by 2.	28%
(D) find the slope of that line segment by dividing 15 by 5.	3%
(E) Not enough information to answer.	6%





This question gets to the heart of the issue, probing how the student feels they should proceed rather than inferring their thoughts based on the numerical answer chosen. There is no significant difference between the two groups (p-value = 0.2417) with approximately half of the students selecting the correct answer, in stark contrast to their performance on question 4. The most commonly selected distractor in both cases was (C) corresponding to calculating the slope rather than the area. Approximately one quarter of TUG-R respondents made this error in both Questions 4 and 18 providing evidence for slope *vs.* area confusion among approximately 25 % of respondents.

20. An object rotates according to the graph below:



Through what angle does it rotate during the interval from $t = 4$ s to $t = 8$ s?

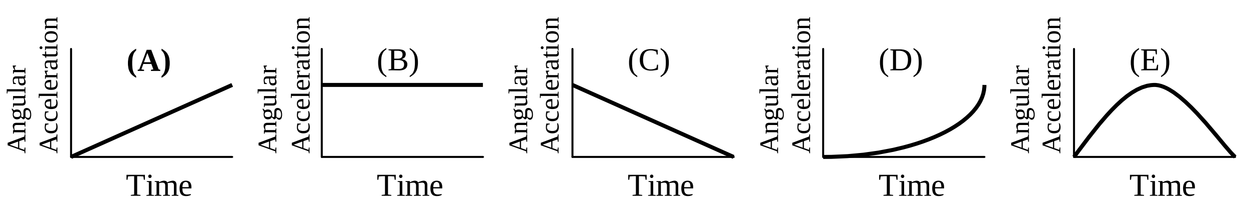
(A) 0.75 rad 17%
(B) 3.0 rad 34%
(C) 4.0 rad 2%
(D) 8.0 rad 1%
(E) 12.0 rad 44%

Students in the TUG-K group did significantly better on this question and no distractor stood out for that group. While the most common answer was the correct one, approximately one third of the TUG-R students chose selection (B), corresponding once again to the axis reading error. Note also that it is not possible to make a slope-calculation error for this question since the slope of zero (0) is not one of the distractors.

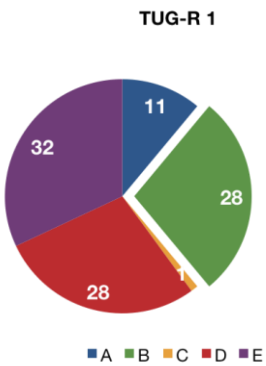
For this objectives, the results were somewhat mixed with no significant difference on two of the three questions. However, the TUG-R students did significantly worse on the remaining question.

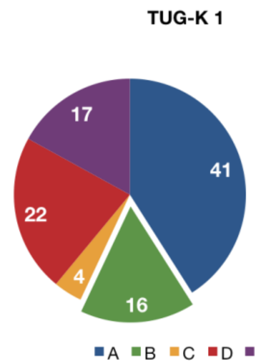
Objective 4: Given an angular acceleration-time graph, the student will determine the change in velocity.

1. Angular acceleration versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in angular velocity during the interval?



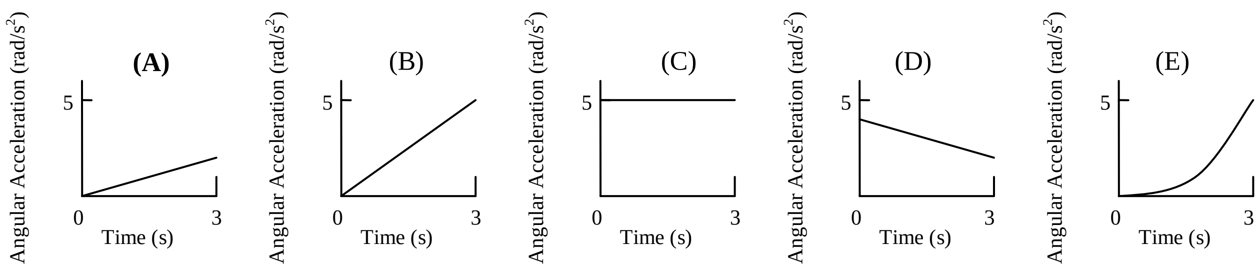
11%	**28%**	1%	28%	32%



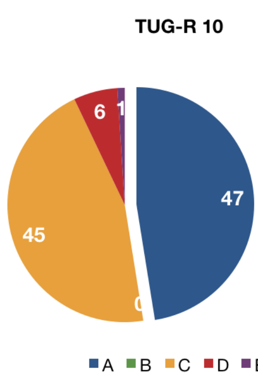


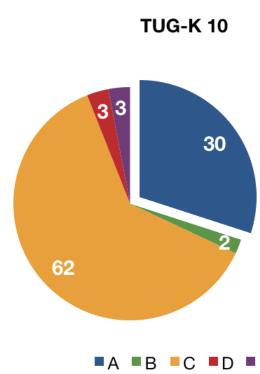
The TUG-R group did substantially better than the TUG-K group on this question (p-value = 0.0153). While the TUG-K group chose the constant positive slope of (A) as both their top overall choice and top distractor, the TUG-R group divided nearly equally between the correct selection (B) and the distractors (D) and (E). Both of these distractors have portions with large positive slopes, again suggesting confusion between the conceptual relevance of slope and area.

10. Five objects move according to the following angular acceleration versus time graphs. Which has the smallest change in angular velocity during the three second interval?



47% 0% 45% 6% 1%

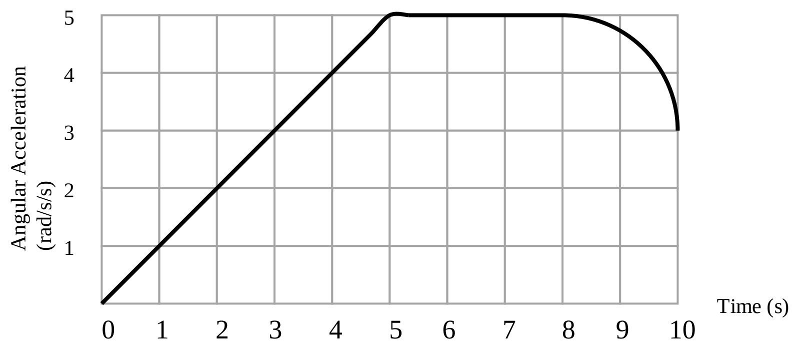


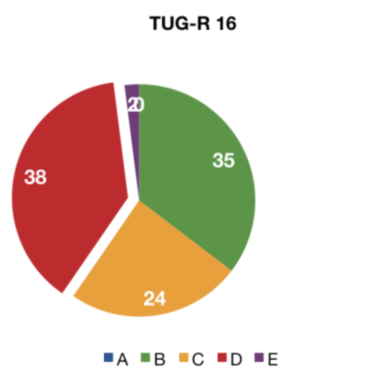


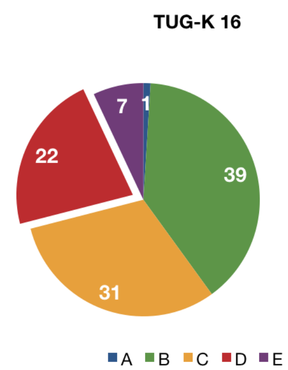
The TUG-R group did somewhat better on this question. Within both groups the most popular distractor was selection (C), for the TUG-K group this was even the most popular answer overall. Unfortunately, this answer corresponds to two potentially incorrect models. It clearly has the largest area and also has the line with the smallest, non-negative slope. Thus, it is conceivable that the same slope-area misconception is being displayed here as in Question 1.

16. An object moves according to the graph below. The object's change in angular velocity during the first three seconds of motion was:

(A) 0.66 rad/s 0%
(B) 1.0 rad/s 35%
(C) 3.0 rad/s 24%
(D) 4.5 rad/s 38%
(E) 9.8 rad/s 2%





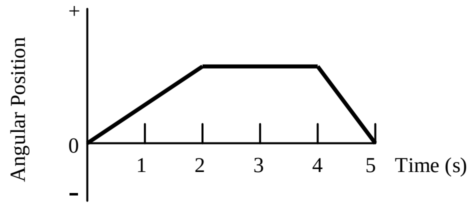


The TUG-R group did better on this question (p-value = 0.0024). Within both groups the most popular distractor was selection (B), for the TUG-K group this was the most popular answer overall. This answer corresponds to correctly calculating the slope or to counting squares to calculate the slope. Either model is a problem as the correct answer involves a calculation of area indicating potential area versus slope confusion. This form of failing to properly scale the problem was noted earlier as well. Another distractor which garnered a significant following was (C), chosen by approximately one quarter of the students. This again corresponds to an axis reading problem.

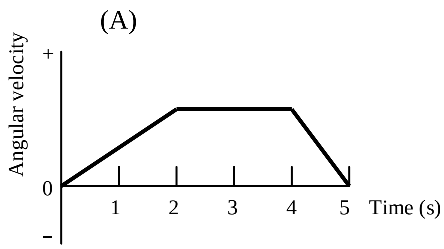
For this objective, performance on the TUG-R was consistently better than that on TUG-K.

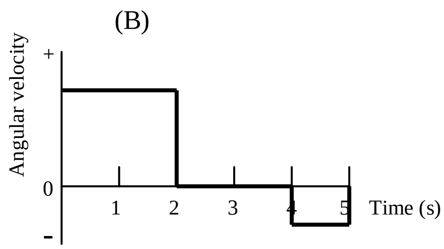
Objective 5: Given a rotational kinematics graph, the student will select another corresponding graph.

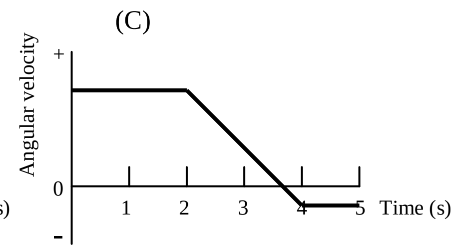
11. The following is an angle-time graph for a rotating object during a 5 s time interval.



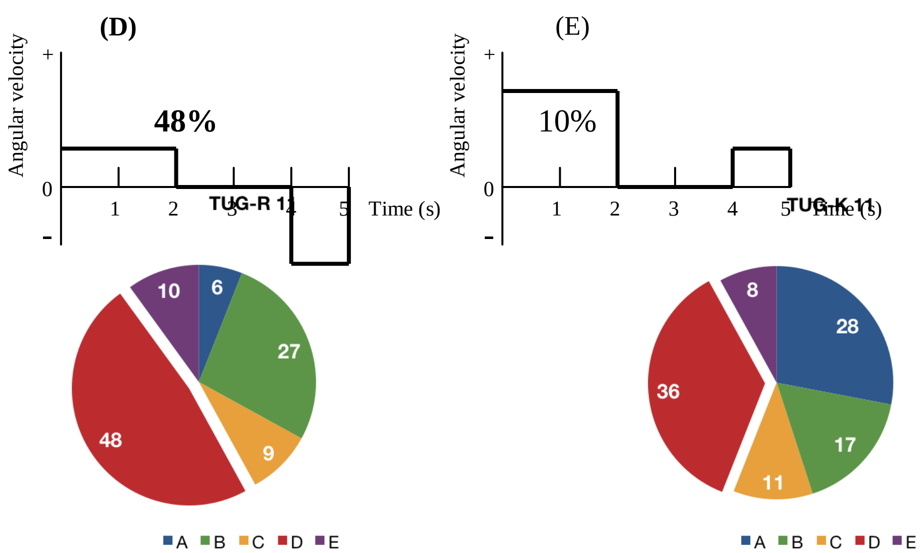
Which one of the following graphs of angular velocity versus time would best represent the object's rotational motion during the same time interval?





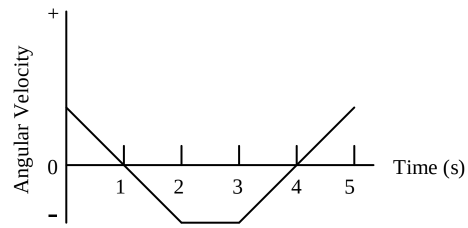


6% 27% 9%

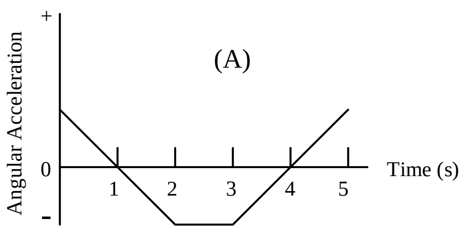


The TUG-R group did better on this question. While the most popular distractor for the TUG-K group was selection (A), the same graph, The TUG-R group's most common choice of distractor is (B). This choice is based upon a model where the relative steepness of the non-horizontal, linear portions is reversed while the numerical sign of each slope is correctly considered.

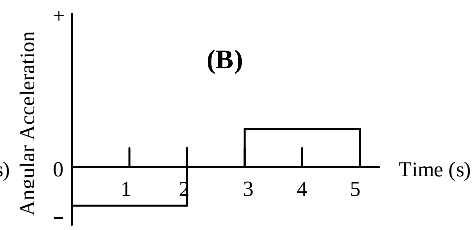
14. The following represents an angular velocity-time graph for an object during a 5 s time interval.



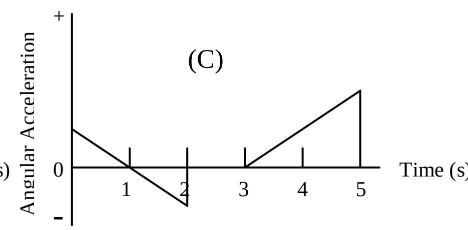
Which one of the following graphs of angular acceleration versus time would best represent the object's motion during the same time interval?



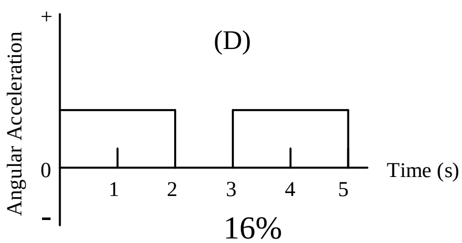
4%



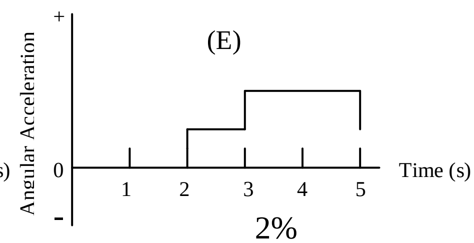
72%



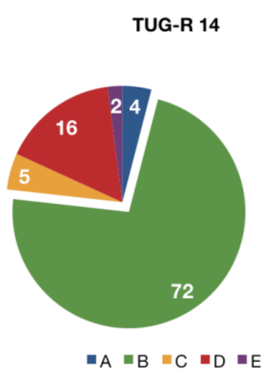
5%

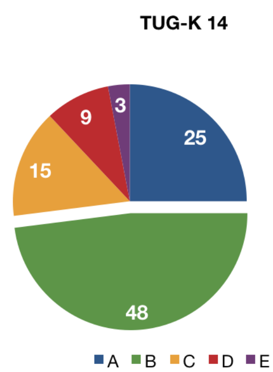


16%



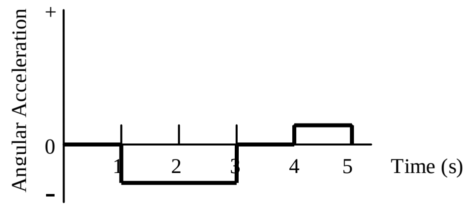
2%



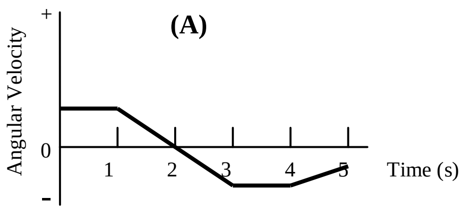


The TUG-R group did much better on this question. While selection (A), the same graph, continued to be an appealing choice for the TUG-K group, only selection (D) merits mention for the TUG-R group. The model for this selection is one in which the signs of the slopes are both taken to be positive, producing a graph which is the absolute value of the correct choice.

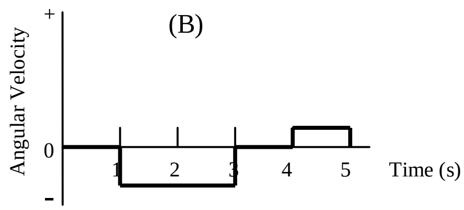
15. The following represents an angular acceleration graph for an object during a 5 s time interval.



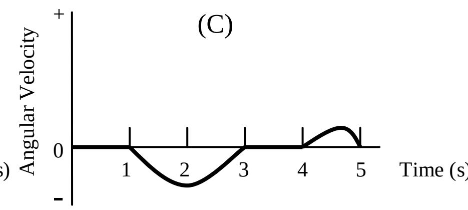
Which one of the following graphs of angular velocity versus time would best represent the object's motion during the same time interval?



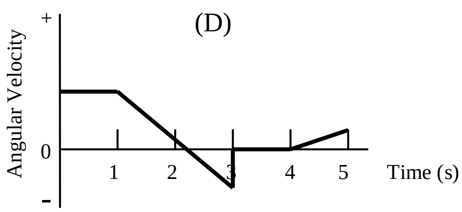
47%



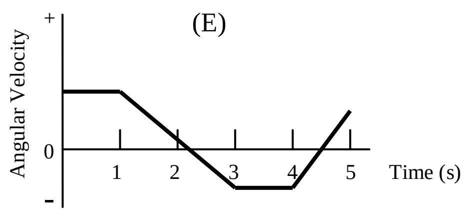
8%



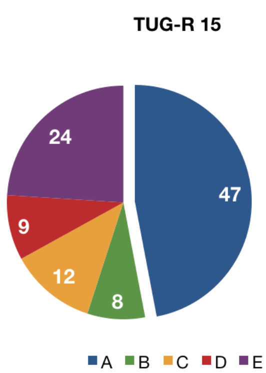
12%

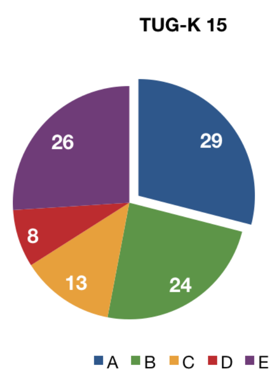


9%



24%



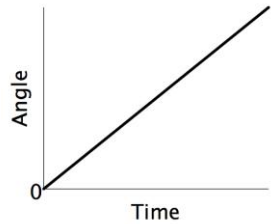


Again, the TUG-R group did better on this question. The most commonly chosen distractor for the TUG-R group was (D). The model for this choice is one where the relative slopes of the two non-horizontal, linear sections of the angular velocity are reversed. This selection was also appealing for the TUG-K group who chose it nearly equally with the same graph error and the correct answer.

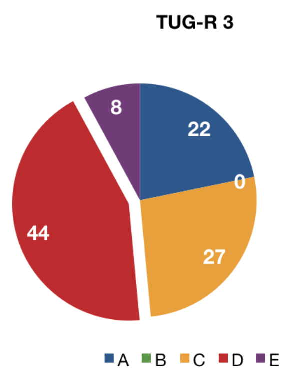
For this objective, performance on the TUG-R was consistently better than that on TUG-K. Perhaps the extra consideration required to work with graphs of rotational motion leads students to recognize that graphs of different variables should have different appearances.

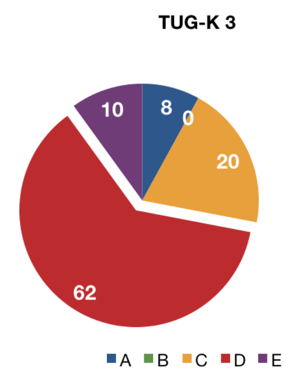
Objective 6: Given a rotational kinematics graph, the student will select a textual description.

3. Below is a graph of an object's rotational motion. Which sentence is the best interpretation?



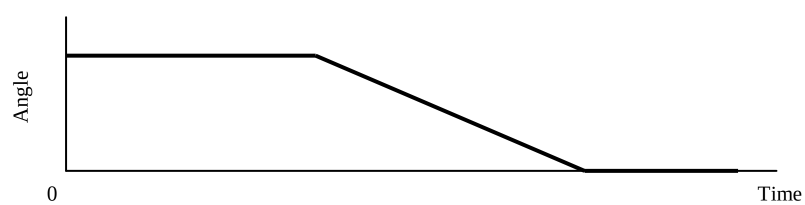
(A) The object is rotating with a constant, non-zero angular acceleration. 22%
(B) The object does not rotate. 0%
(C) The object is rotating with a uniformly increasing angular velocity. 27%
(D) The object is rotating with a constant angular velocity. 44%
(E) The object is rotating with a uniformly increasing angular acceleration. 8%



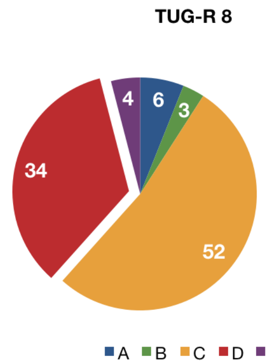


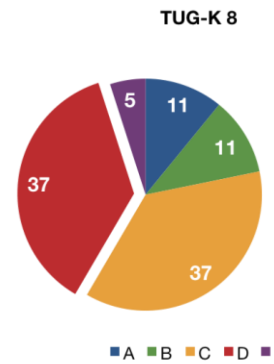
The TUG-K group did significantly better on this question. While selection (C), reading directly from graphs, was the most popular distractor for both groups, selection (A) gain significant support from the TUG-R group. This choice corresponds to a model in which rotational acceleration and velocity are confused for one another.

8. Here is a graph of a rotating object's motion. Which sentence is a correct interpretation?



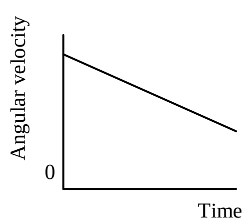
(A) The object spins along a flat surface. Then it rolls forward down a hill, and then finally stops. 6%
(B) The object doesn't spin at first. Then it rolls forward down a hill and finally stops. 3%
(C) The object is spinning at constant angular velocity. Then it slows down and stops. 52%
(D) The object doesn't spin at first. Then it spins clockwise and then finally stops 34%
(E) The object spins along a flat area, rolls backwards down a hill, and then it keeps spinning. 4%



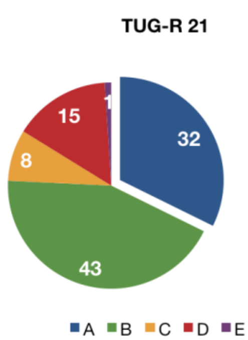


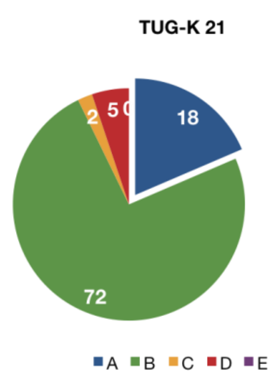
This question was, by far, the most difficult to modify to an equivalent form. The original question was meant to probe student misconceptions surrounding the physical situation, in this case a ball rolling down a hill, and the kinematic graphs that correspond to that motion. In fact, selection (E) actually looks like the hill described in the question stem. A total of 13% of TUG-R respondents chose any answer containing descriptions pertaining to rolling down a hill, apparently rejecting such choices almost immediately. There was no significant difference between correct answers for the two groups (p-value = 0.7112), and for both groups selection (C) was a very attractive distractor, the most popular choice for both groups and the choice of over half of the TUG-R students. Note that selection (C) is actually correct if the graph were an angular velocity-time graph rather than an angle-time graph: i.e. kinematic variable confusion.

21. Below is a graph of an object's motion. Which sentence is the best interpretation?



(A) The object is moving with a constant angular acceleration	**32%**
(B) The object is moving with a uniformly decreasing angular acceleration.	43%
(C) The object is moving with a uniformly increasing angular velocity.	8%
(D) The object is moving at a constant angular velocity.	15%
(E) The object does not rotate.	1%



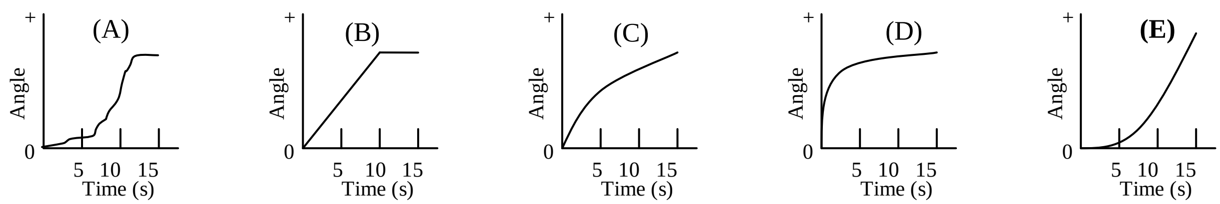


A much larger fraction of the TUG-R group responded correctly on this question. However, both groups chose selection (B) as their most popular answer, distractor or not. This distractor corresponds to a model where students are reading the graph directly without regard for the kinematic variable used. It is also possible that some form of kinematic variable confusion is being reflected.

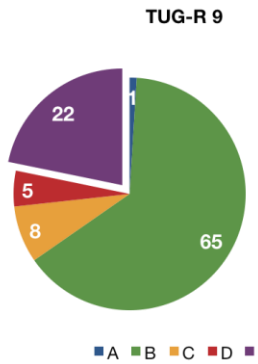
The outcome for this objective is mixed. Each group did significantly better than the other on one of the three questions within the objective and there was no significant dofference observed for the remaining question.

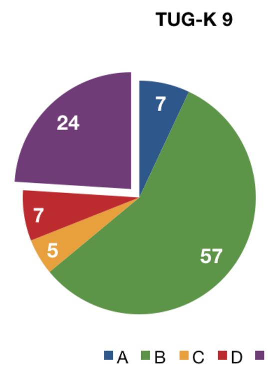
Objective 7: Given a textual motion description, the student will select corresponding graphs.

9. An object starts from rest and undergoes a positive, constant angular acceleration for ten seconds. It then continues on with a constant angular velocity. Which of the following graphs correctly describes this situation?



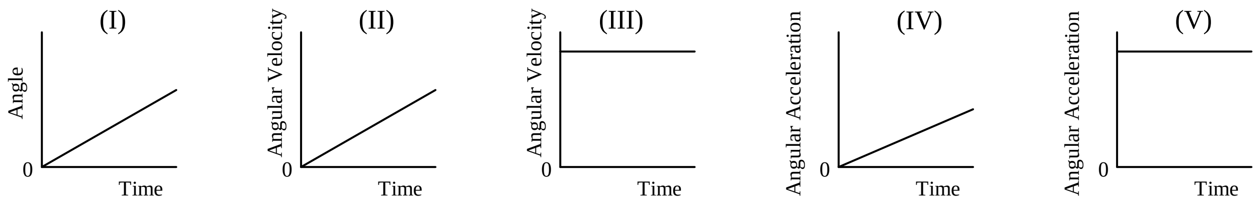
1% 65% 8% 5% **22%**





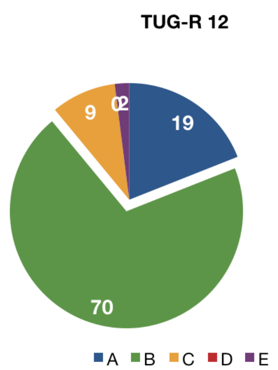
The response patterns on both tests are roughly the same, with fewer than ¼ of the students answering correctly. For both tests, the most popular response overall was the distractor selection (B). Selection (B) would be correct if the axis for the graph was rotational velocity rather than angle. So, once again, this could be confusion of the form of the all kinematic graphs are the same variety. However, some fraction of the incorrect responses could be simply a lack of care and concern.

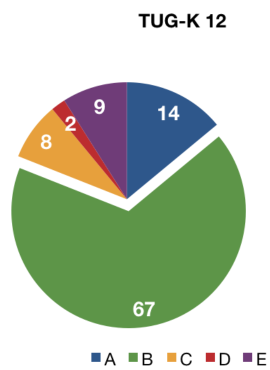
12. Consider the following graphs, noting the different axes:



Which of these represent(s) motion at constant angular velocity?

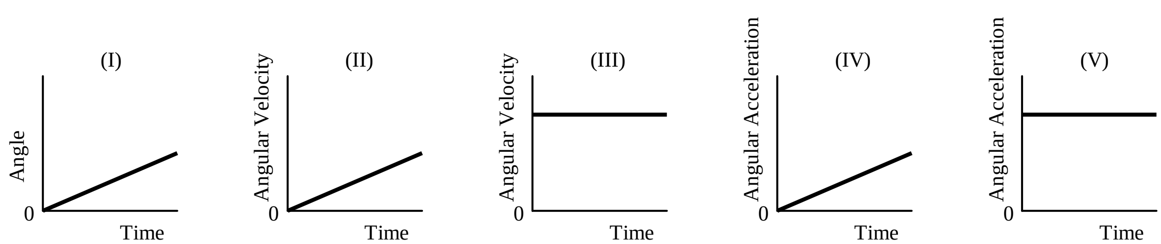
(A) I, II, and IV	19%
(B) I and III	**70%**
(C) II and V	9%
(D) IV only	0%
(E) V only	2%





The results of this question were remarkably similar for both groups of students. Two thirds or more of the students on both examinations answered this question correctly. The most frequently chosen distractor for the TUG-R group was selection (A). This selection does not actually correspond to any single poor model. It should also be noted that (B) is the only answer containing Option III. Thus, should a student determine that Option III is absolutely correct there would be no other choice to make regardless of whether or not they understood the validity of Option I.

19. Consider the following graphs, noting the different axes:



Which of these represent(s) rotation with constant, non-zero angular acceleration?

(A) I, II, and IV	24%
(B) I and III	9%
(C) II and V	**35%**
(D) IV only	8%
(E) V only	24%

Once again, as with the preceding question, the answers were remarkably similar for both groups of students. Popular distractors for the TUG-R group were selection (A) which again does not correspond to any single poor model and selection (E) which is correct, but incomplete.

For this objective, there was no real difference in performance on the linear and rotational items.

Comparison of Student Difficulties between TUG-K and TUG-R

Beichner found three basic, persistent types of misconceptions which were prevalent for post-instruction students taking the TUG-K[4]. All three types of misconceptions continue to be exhibited at some level by TUG-R respondents.

1) **The appearance of a graph is independent of the kinematic variable.** This misconception is probed directly in Objective 5 and indirectly in many other problems. Beichner identifies approximately 25% of TUG-K respondents with this issue. While far less than 25% of TUG-R respondents chose this model distractor across all questions for Objective 5, there is still evidence to support this as a continuing issue. In particular, in every question related to Objectives 1-4 where students work with given graphs of rotational motion to determine information about another kinematic variable, the most frequently encountered incorrect model was one where students read information directly from the given graph. In other words, they treat a graph involving the given rotational variable as if it was a graph of the desired variable. This direct axis reading model was used independent of whether the correct manipulation was to find the slope or area under the curve. Another potential explanation for these indirect observations is kinematic variable confusion. Kinematic variable confusion is evident in student responses for Objectives 6 and 7. In Questions 8 (52%), 9 (65%) and 21 (43%) the most frequently chosen answer corresponds to a distractor model where the answer chosen would be correct if the axes were labeled differently, i.e. students are displaying kinematic variable confusion.

2) **Students have difficulty calculating slopes.** While numerical calculations were generally done well by the TUG-R group, there is a definite existence of a "counting boxes" approach to calculating slope which persists for some students. Questions 7 (24%) and 17 (12%) found students using a counting boxes model which fails to properly scale the problem. This is a problem involving understanding and manipulation of graphs, which does not depend on the physical situation inherent in angular kinematics.

3) **Students confuse slope and area in extracting information from a particular angular kinematic graph.** In Question 4 (23%) and Question 16 (35%), both involving numerical manipulation, students selected the distractor suggesting they used a model where they calculated slope rather than area. In Question 1, 60% selected the response corresponding to largest area rather than largest slope. Finally, in perhaps the most direct probe, in Question 18 students select from descriptions in words the calculations they would do to answer the prompt rather than selecting a numerical answer which might correspond to several incorrect models. In this case, approximately one quarter of the TUG-R students indicated explicitly that they would calculate the slope to calculate the angle through which an object rotates when presented with an angular velocity-time graph.

Conclusions and Future Work

The 7 fundamental objectives of the exam can be divided into 3 groups based on the types of questions involved.

Objectives 1-4 involve numerical calculation, either actual or implied, of one kinematic quantity based on the graph of another. When actual calculations were not required, a comparison of the magnitudes of the implied calculation was expected. Misconceptions concerning kinematic variable confusion, slope calculation and slope-area confusion have been discussed above. In Objectives 2 and 3 TUG-R and TUG-K students did equally well, with no significant difference on 4 of the 6 questions. For Objective 1, TUG-K students did noticeably better on two of the three questions. Apparently students correctly applied the phrase "The slope of the position graph is the velocity" in the linear setting, but incorrectly when they were further removed from it in the rotational setting. This is troubling since it suggests a memorization of the phrase and corresponding calculation, rather than a deeper understanding of the fundamental principle. If true, this would be no better than the memorization of an equation which could be pulled out and used regardless if it is correct situation. For objective 4, TUG-R students did consistently better on all three questions. This is not to say that either group did particularly well on the three questions involved. In fact, neither group had even 50% correct response for any question. However, this is the first indication of systematic differences between the two groups of students.

Objective 5 requires students to move from one kinematic graph to the corresponding graph involving another kinematic variable. Here TUG-R students did consistently and significantly better than their TUG-K counterparts. The bulk of the difference in the percentages answering correctly appears to stem from a reduction in the amount of kinematic variable confusion. Evidence discussed elsewhere suggests that some of this type of confusion is still present; it doesn't appear to manifest itself as readily in choosing a graph which looks exactly like the graph they are initially presented with. It is possible that since students are less familiar with rotational motion, they are thinking more deeply and the extra consideration makes it more likely for them to realize that graphs of different variables should have different appearances and less likely to simply chose one which appears to be the same shape graph as the one they begin with.

Objectives 6 and 7 involve the interplay between graphs and textual descriptions. Neither group appears to have an advantage in this particular arena. For 4 of the 6 questions, including all of those related to Objective 7, both groups showed no significant difference. Again, this doesn't mean that either group did particularly well on these questions. With the notable exception of Question 12, neither group answered with 50% correct responses on any of these questions. On the remaining two questions, each group outperformed the other on one of the two.

Table 3: Comparative Performance on Objectives of the TUG-R and TUG-K. A 2 tailed *z*-test was performed to assess whether or not the difference in sample size can account for the noted differences in results between the TUG-R and TUG-K. A two tailed *p*-value of 0.05 was used as the demarcation for significance, indicating certainty at the 95% confidence level that the results obtained did not occur by chance. Red indicates rotational performance significantly worse than linear. Green indicates rotational performance is significantly better than linear. Black indicates TUG-R and TUG-K performance was similar. Percentages answering each question correctly are also included with TUG-R listed before TUG-K.

Objective	Conclusion	Individual Questions			Comments
1 From angle-time graph, determine angular velocity	R < K	Q5 52v73 p=0.0002	Q13 27v61 p=0.0000	Q17 27v21 p=0.2263	Performance on the TUG-R was significantly worse that than on the TUG-K on two of the three questions.
2 From angular velocity-time graph, determine angular acceleration.	R ~ K	Q2 70v63 p=0.1802	Q6 46v25 p=0.0001	Q7 28v31 p=0.5563	TUG-R students do significantly better on item #6, otherwise about the same.
3 From angular velocity-time graph, determine angular displacement.	R ~ K	Q4 25v28 p=0.5423	Q18 53v46 p=0.2147	Q20 44v72 p=0.0000	TUG-R students do significantly worse for item #20, otherwise about same.
4 From angular acceleration-time graph, determine change in velocity.	R > K	Q1 28v16 p=0.0153	Q10 47v30 p=0.0023	Q16 38v22 p=0.0029	Performance on the TUG-R was consistently better than that on TUG-K.
5 From rotational kinematics graph, select another corresponding graph.	R > K	Q11 48v36 p=0.0326	Q14 72v48 p=0.0000	Q15 47v29 p=0.0012	Performance on the TUG-R was consistently and substantially better than that on TUG-K.
6 From rotational kinematics graph, select a textual description.	R ~ K	Q3 44v62 p=0.0013	Q8 34v37 p=0.7112	Q21 32v18 p=0.0065	Mixed.
7 From textual description, select corresponding graphs.	R = K	Q7 22v24 p=0.5563	Q12 70v67 p=0.5641	Q19 35v37 p=0.7112	No significant difference in performance (or even answer patterns).
Overall Performance	R > K				

Overall, we conclude that the TUG-R students performed somewhat better than their TUG-K counterparts. They did significantly better on 8 of the 21 questions, including all of the questions in Objectives 5 and 6. Both groups did equally well on 9 of the 21 questions while the TUG-K students were more successful on the remaining 4 of the 21 questions. There was measurable improvement in both the tasks of calculating the change in (angular) velocity from a graph of (angular) acceleration and in selecting corresponding kinematic graphs from a given graph.

While percentages of students answering correctly and choosing distractors were often different between the TUG-K and TUG-R, it is clear from the discussion of individual problems, summarized in the section above, that all three general types of misconceptions found during development of the TUG-K continue to be present when students are asked to graphically address rotational kinematics. In retrospect, this should really not be a surprise as it is unlikely basic student difficulties with graphs will suddenly disappear after instruction in rotational motion. To this end, it could also be instructive to assess students between instruction in rectilinear and rotational motion and then again after all instruction has been completed to attempt to see if student misconceptions change as a result of instruction in rotational kinematics. Given the similarities described in this paper between the types of students misconceptions noted in the TUG-K and TUG-R, it would potentially be reasonable to administer the TUG-K as the intermediate instrument.

Lost in the discussion above is that we have been unable to unambiguously answer the question we initially posed concerning Arons's assertion of the usefulness of parallel mathematical construction between rectilinear and rotational kinematics for strong students. Our instrument is insufficient in that we have not included a metric by which to measure the strength of the student nor have we formulated any criteria by which to evaluate said strength. What is clear is that significant percentages of students still harbor difficulties in kinematics, even after instruction in both rectilinear and rotational settings. In the future, we would like to gather additional statistics by sampling a larger cross section of students, including students at other institutions, and including a wider range of instructional styles in order to verify that these conclusions remain valid.

In closing, we return again to Arons who writes in the sentence following the earlier quotation[10], "…many students do not master the concepts on the first go-around, and some form of spiraling back is essential. The altered context makes a somewhat more careful treatment very worthwhile for this group, and the pace can be a bit more rapid than previously." To this end, we have created a series of learner-centered activities similar in nature to the workshop, real-time and studio approaches to linear kinematics cited earlier. These activities make use of a rotary motion sensor rather than a motion detector. We have interwoven these activities into our previous instruction in kinematics at places that correspond to the equivalent rectilinear counterpart. We are evaluating the usefulness of these materials using pre and post-testing using the TUG-R instrument and compare with gains seen for classes using traditional instruction. We intend to report on this at a later time once sufficient data becomes available.

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[1] Beichner, R.J., "An Introduction to Physics Education Research" in *Getting Started in PER*, Henderson, C. and Harper, K. eds., American Association of Physics Teachers, College Park, MD, 2009.

[2] A list of some of the more popular assessments is available at http://www.ncsu.edu/per/TestInfo.html.

[3] Hestenes, D. and Wells, M., "A Mechanics Baseline Test", *Phys. Teach.* ***30***, 159-65 (1992).
[4] Beichner, R.J., "Testing student interpretation of kinematics graphs," *Am. J. Phys.* ***62*** (8), 750-762 (1994).
[5] Hestenes, D., Wells, M. and Swackhamer, G., "Force Concept Inventory," *Phys. Teach.* ***30*** (3), 141-158 (1992).
[6] Mazur, E., *Peer Instruction: A User's Manual*, Prentice-Hall, Upper Saddle River, NJ, 1997.
[7] See http://physics.dickinson.edu/~wp_web/wp_homepage.html for more information.
[8] Available from Wiley at http://www.wiley.com/WileyCDA/WileyTitle/productCd-0471487708.html.
[9] Wilson, J.M., "The CUPLE Physics Studio," *Phys. Teach.* ***32*** (9), 518-523 (1994). Also see R Beichner, J M Saul, D S Abbott, J. Morse, Duane Deardorff, Rhett J. Allain, S W Bonham, Melissa Dancy, and J. Risley, "Student-Centered Activities for Large Enrollment Undergraduate Programs (SCALE-UP) project," in *Research-Based Reform of University Physics*, edited by E F Redish and P. J. Cooney (American Association of Physics Teachers, College Park, MD, 2007)
[10] Arons, A.B., *A Guide to Introductory Physics Teaching*, John Wiley and Sons, New York, NY, 1990.
[11] See for example, Halliday, D., Resnick, R. and Walker, J., *Fundamentals of Physics*, Wiley, New York, NY, 2011.
[12] Singh, C. and Rimoldini, L.G., "Student understanding of rotational and rolling motion", *Phys. Rev. ST-PER* ***1***(1) (2005).
[13] Mashood, K.K., and Singh, V.A., "An inventory on rotational kinematics of a particle: unraveling misconceptions and pitfalls of reasoning", *European Journal of Physics, 33,* 1301-1312 (2012). Also see Mashood, K.K. and Singh, V.A., "Rotational kinematics of a particle in rectilinear motion: Perceptions and pitfalls", to be published in *American Journal of Physics*.
[14] Contact Genaro Zavala at Tecnológico de Monterrey for more information on this project.