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Weight, Weight, Don't Tell Me! A Single Measurement, Graphical Approach to the Study of the Motion of an Elevator

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AC 2010-87: WEIGHT, WEIGHT, DON'T TELL ME!: A SINGLE MEASUREMENT, GRAPHICAL APPROACH TO THE STUDY OF THE MOTION OF AN ELEVATOR

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Weight, Weight, Don't Tell Me!

A single measurement, graphical approach to the study of the motion of an elevator

An elevator in motion is often used as a one-dimensional example of uniformly accelerated motion. The free-body diagram of a person in an elevator is particularly simple; it involves only the earth's gravitational force acting downward and a supporting force from the elevator floor acting upward. It is common to mentally insert a scale between the person and the elevator floor and to consider the readings of the imaginary scale. But if you enter one of the elevators scattered around our campus, you might get to see real people standing on real scales: our physics students collecting data on the dynamics of their own motion!

Motivation

When this lab was originally conceived it was performed using an ordinary bathroom scale and a stopwatch. Students measured as best they could the rapid fluctuations in scale readings and timed the durations of various segments of the elevator ride. By estimating the distance that the elevator moved, they were able to create reasonable graphs of the actual motion of the elevator. A more time-consuming version of this activity involves the frame-by-frame analysis of the readings of a bathroom scale filmed in a moving elevator.¹ The development of computer-interfaced force probes simplified the data collection and allowed for forces on small objects (2 kg) to be measured in an elevator². As we describe below, the Vernier FP-BTA Force Plate, LabPro interface and LoggerPro 3 software³ allow us to take this experiment to an entirely new level by making the student the focus of the experiment and utilizing their kinesthetic sense to increase learning and motivation⁴. Not only is it possible for students to accurately measure the forces—and consequently the motion—of the elevator ride; now they can also see these graphs in real-time while *feeling* the forces and motion.

Pre-Lab Activities

The format of the activity is based on a cycle of prediction, action and examination. In order to facilitate the prediction process we begin by asking students to take a ride on an elevator. During this ride the students are asked to concentrate on the forces acting on their bodies—including when and how the forces change. They are often surprised that the forces felt during much of the ride are the same as the forces felt before the ride begins. It is only for a few brief moments that they feel something different. After completing the ride, they begin the task of constructing a free-body diagram of the forces acting on an elevator rider. Such a free-body diagram is shown in Figure 1. This is straightforward for many after having just experienced those forces. Along with the free-body diagram, students also need to consider the operation of a scale. They typically think of a bathroom scale as reading their weight; which is to say, telling them about the gravitational force the earth exerts on them. In reality, the scale can only respond to a force acting it. Only by careful application of Newton's first and third laws can we justify that the scale measures the magnitude of the gravitational force acting on us—and only

then in situations where Newton's first law is applicable. In the event that the elevator accelerates, the scale continues to measure the magnitude of the normal force from the floor, but it is no longer equal to the earth's gravitational force.

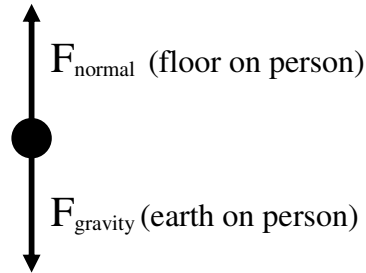


Figure 1: Free-body diagram of forces acting on an elevator rider.

At this point, students have successfully drawn a free-body diagram and demonstrated an understanding of what a scale reads in both constant velocity and accelerating frames of reference. Now they're prepared to apply this knowledge to predict both the forces acting on an elevator rider and the motion of the rider as functions of time. We scaffold this task by providing graph axes for six quantities that may vary with time. These include: the normal force of the elevator on the rider; the gravitational force on the rider; the net force acting on the rider; and the acceleration, velocity and position of the rider. Each of the variables is labeled on the vertical axis and time is labeled on the horizontal axis. It is the student's task to provide scales for each axis that correspond with the estimated duration and distance traveled by the elevator. In our inquiry-based course students will have already worked extensively with graphical representations of motion as a function of time. However, this is their first opportunity to graph force as a function of time.

There are several different paths for constructing the force and motion vs. time graphs. The most common approach used by our students is to begin with either the acceleration vs. time or the net force vs. time graphs. Because our course is algebra-based, our students typically choose to break the graphs into sections in which the acceleration is constant and therefore the velocity and position versus time graphs can be easily generated without calculus. In this process, all students are asked to create their own graphs; discuss and compare these graphs with their teammates and reconcile any differences; and present the agreed-upon team predictions to the instructor prior to continuing with the activity. We have found these group discussions to be an ideal opportunity for identifying and addressing common student misconceptions, such as graphing a (significantly) time-varying gravitational force, confusing acceleration and velocity, or assuming an object with increasing speed must have positive acceleration.

Data Collection and Analysis

A Vernier FP-BTA Force Plate, LabPro interface and LoggerPro 3 software are used to collect data for a student standing on the force plate in a moving elevator. The ability to easily create and graph new columns of data based upon existing columns within LoggerPro then allows students to generate the graphs they had attempted to predict earlier through the process described below. Note that all of the graphs are based on a single measurement of force *vs.* time rather than using accelerometer measurements as the basis for graphs of velocity and position⁵.

1. Ride the elevator and record the force plate reading *vs.* time. The force plate reading measures the normal force of the elevator on the rider. A typical force plate reading *vs.* time graph for an elevator ride is shown in Figure 2.

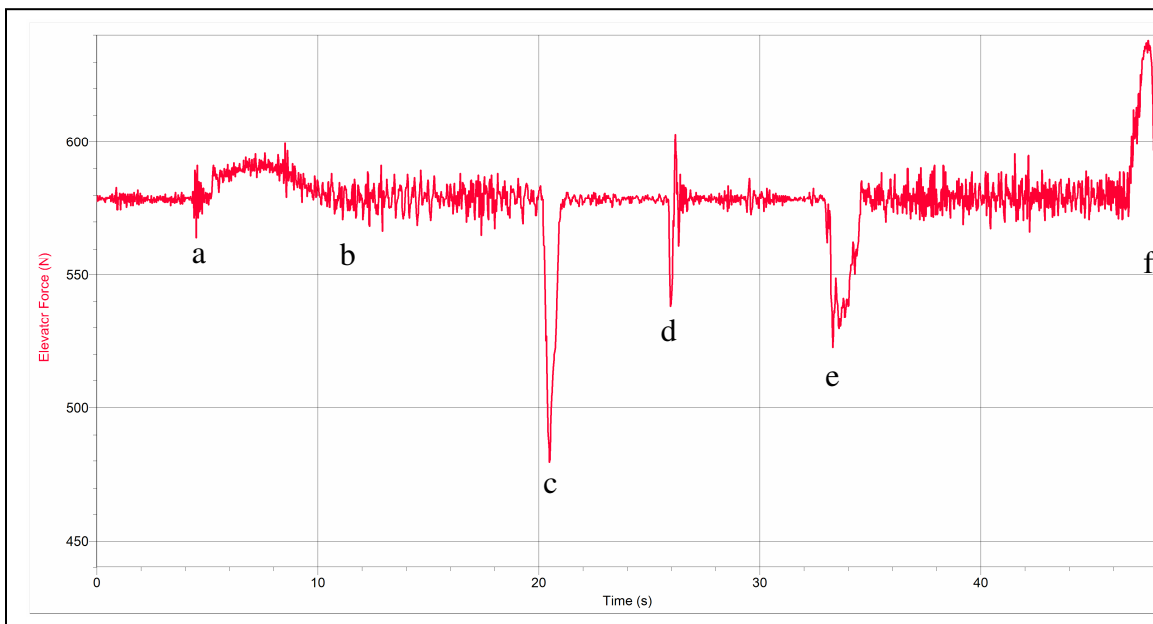


Figure 2: Typical force plate reading *vs.* time for elevator rider. The elevator traveled a total of three floors, first up and then down. At time *a* the elevator begins accelerating from rest. At *b* the elevator finishes accelerating upward and begins to travel at a constant velocity. At *c* the elevator undergoes the first of its decelerations as it approaches the uppermost floor of its trip. At *d* the elevator comes to a stop and the doors open. At *e* the elevator begins its downward trip. At *f* the elevator comes to a stop back at the point where it originally began.

2. Observe that the scale reading is constant whenever the elevator is at rest or moving with constant velocity. Based upon the pre-lab activities, students know that in this situation the force plate measures actual weight. They can then use LoggerPro to average the scale readings during intervals with constant velocity to estimate the gravitational force and therefore the mass of the rider.

- Plot the net force acting on the rider vs. time by subtracting the magnitude of the gravitational force measured in (2) from the normal force plotted in (1). A graph of the net force vs. time derived from the graph in Figure 2 is shown in Figure 3 with the remaining graphs of motion described in (4) and (5) below.

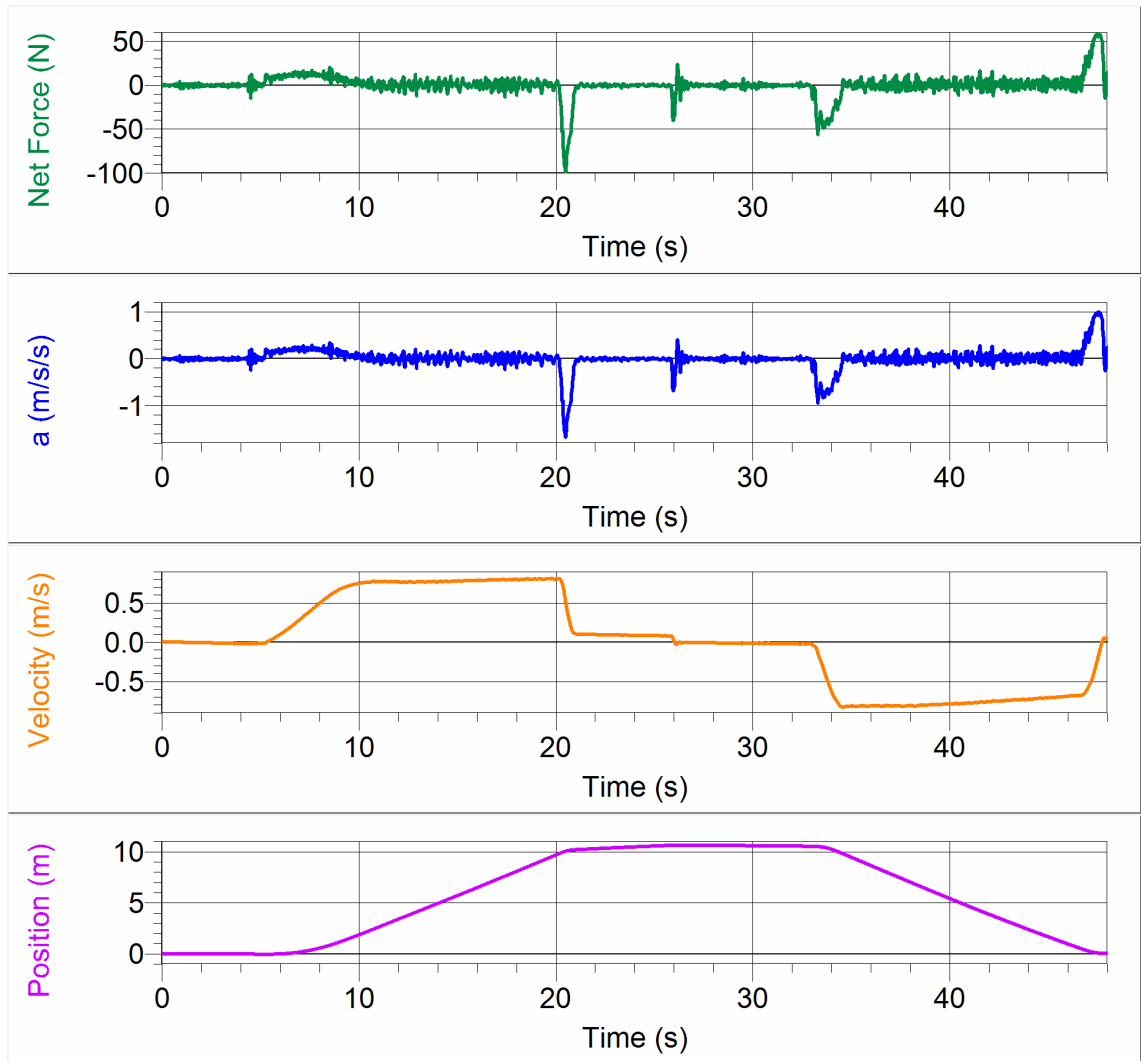


Figure 3: Graphs calculated from the force plate vs. time graph in Figure 2.

- Plot acceleration vs. time by applying Newton's 2nd law and dividing the net force found in (3) by the mass of the object. It is interesting to note that after considering so many situations in which all forces and accelerations are taken to be constant, it is sometimes surprising to students that Newton's second law also applies for time-varying forces.
- Finally, to obtain the position and velocity vs. time graphs, the students use LoggerPro's ability to create a new data column based upon the integral of an existing column. Thus integrating acceleration creates a velocity column and integrating velocity creates a position column that may be plotted. This action should make perfect sense to students who have been exposed to calculus or who

have learned kinematics through graphical approaches using motion sensors^{6,7,8}. Indeed, the LoggerPro software requires students to access the area between the curve and the time axis by using a tool referred to as “integral” in the pull-down menu, and so even students in algebra-based physics courses become exposed to the concept and terminology of integration.

To create the graphs in Figure 3, the net force was calculated by subtracting 587.825 N from the force plate reading and the acceleration was calculated by dividing by 59.06 kg. At first glance these values seem to be rather unusual. We have found that the original construction of the graphs frequently creates unacceptable results. Often the elevator appears to end with a non-zero velocity and at a place far different than it began. Both are conclusions the students know to be incorrect. At issue is typically the constant quantity (weight) subtracted from the force plate reading *vs.* time graph. A small difference in that quantity radically alters the portions of the graph which should average to zero allowing velocity and position to accumulate over time as the graphs are numerically integrated. Typically an adjustment of no more than 0.2-0.3 N will bring the graphs into acceptable alignment. There are two sources for error in arriving at the weight of the object. The first is that the average value of the nearly horizontal portions of the force plate reading *vs.* time can change slightly depending on exactly what region is chosen. The second is the small variations over time of the force plate reading itself. Even though students zero the force plate immediately before collecting data, the electronic offset does drift somewhat over time. Of course, adjusting the weight also means that the mass used in the calculation of the acceleration must also be adjusted.

In the data displayed in Figures 2 and 3, there are a number of interesting and unexpected features which can be considered. The initial upward acceleration was significantly smoother than the other three periods of acceleration as revealed by the lack of an abrupt, spiky feature. The data between 10 and 20 seconds has small fluctuations. This may be due to the other riders on the elevator shifting and moving which caused the rider on the scale to shift as well. It also may have been a characteristic of the elevator itself. In either case, it all averaged out in the final analysis; there is no net area under the acceleration *vs.* time graph during this period. The deceleration of the elevator at the top of its travel took place in two distinct pieces at approximately $t = 20$ and 26 seconds. The fluctuations in the graphs at $t = 26$ seconds are due in part to this second period of acceleration and also to the bump that took place as the elevator doors opened. Finally, the total upward distance the elevator moved is approximately 10.82 m. This correlates nicely with the characteristics of the building which has 2.5 m ceilings with approximately 1.0 m of mechanical space above the ceiling for each of 3 floors. A direct measurement in the stairwell of the building yields a distance of 10.92 m for the three floors traveled by the elevator.

Discussion

We have found that students are typically highly engaged in this lab and often find it remarkable that the position of an elevator can be inferred by simply standing on a scale. We feel that several points are critical for maximizing student learning.

First, it is essential for students to understand that the purpose of the lab is not to learn how to solve elevator problems, but instead to use the elevator as a context to learn about motion in general. Throughout the course, we explicitly build a conceptual framework with our students. As we progress through the activities, pieces are added to the framework. At the time the students undertake this activity the framework that has been developed is shown in Figure 4.⁹

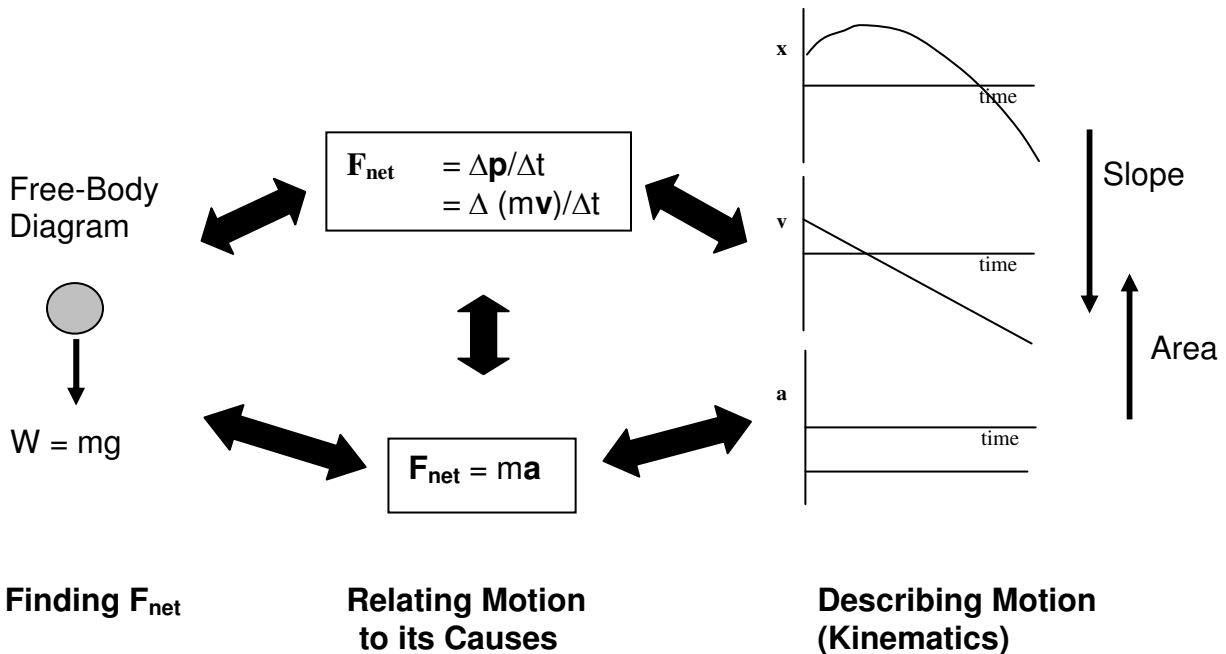


Figure 4: Algebra-based dynamics conceptual framework. A calculus-based version of the framework has also been developed (see Ellis and Turner⁹).

Typical problems that take advantage of this framework start with forces that are constant over time, derive the resulting net force and acceleration, and then end with consideration of uniformly accelerated motion. You can also reverse the input and output by starting with uniformly accelerated motion and ending with a consideration of forces. However, the elevator activity is designed to exploit the full breadth of this framework by considering forces which are not constant in time. In fact, the forces which act do not typically result in any periods with constant, non-zero acceleration. Since the forces change with time, students **MUST** consider time-based, kinematic graphs and **MUST** exploit the full power of the LoggerPro software to do so! In addition, it stresses that Newton's second law applies at all times, not just in situations which do not evolve with time. To reinforce this notion, after the elevator lab is completed we apply the same

concepts and techniques to a variety of new phenomena. These include using a force plate to investigate the dynamics of jumping and landing and using a motion detector to measure the motion and changing air force on falling coffee filters¹⁰.

Second, although we apply Newton's second law in our analysis above, we could have just as easily used an impulse-momentum or work-kinetic energy approach to analyze the elevator's motion. In fact, in our classes we purposely apply a variety of approaches to study the same physical phenomena with the goal of promoting transfer and reducing our students' tendency to compartmentalize their learning—i.e. Newton's second law is used to study elevators or momentum is used for studying collisions. We feel that such an approach helps students to see that while some approaches might be more appropriate to analyze certain phenomena, they are not restricted to these domains.

To this end, Figure 5 shows a graph of accumulating impulse as a function of time. It was created by integrating the net force column. The resulting graph has exactly the same shape as the velocity *vs.* time graph just as the net force graph has the same shape as the acceleration *vs.* time graph; each differs by a constant stretching factor of the mass of the object. The similarities between these graphs often surprise students. Figure 6 shows a graph of the net force *vs.* position for our elevator on only the upward portion of its journey. A similar graph of the entire motion is difficult to interpret as the elevator passes each position twice. Figure 7 shows the work done by the net force as it accumulates over time. It was created by integrating the net force *vs.* position graph. This is an acceptable strategy since the motion is linear with all forces parallel to the direction of motion and the initial position of the elevator is taken to be zero so that the position is effectively the displacement. The work-kinetic energy theorem requires that the work done equals the change in kinetic energy. But, since the initial velocity is zero and thus the initial kinetic energy is also zero, the work done by the net force equals the final kinetic energy.

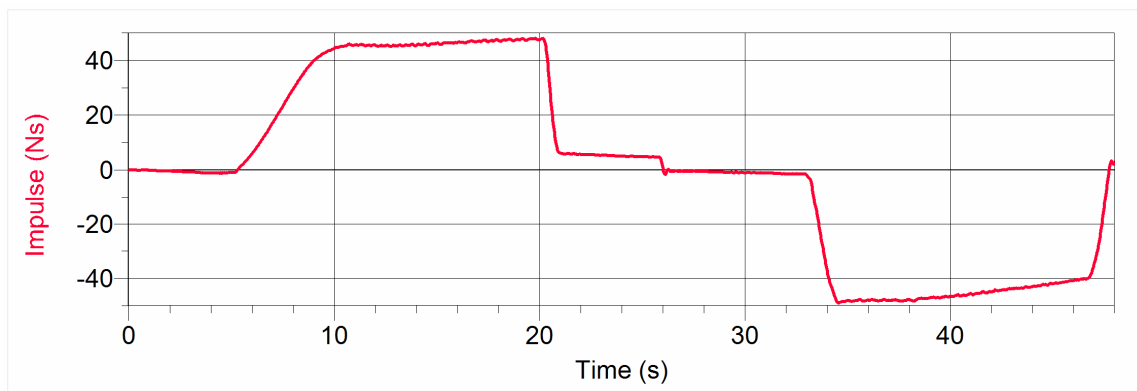


Figure 5: Graph of impulse calculated from the net force *vs.* time graph in Figure 3.

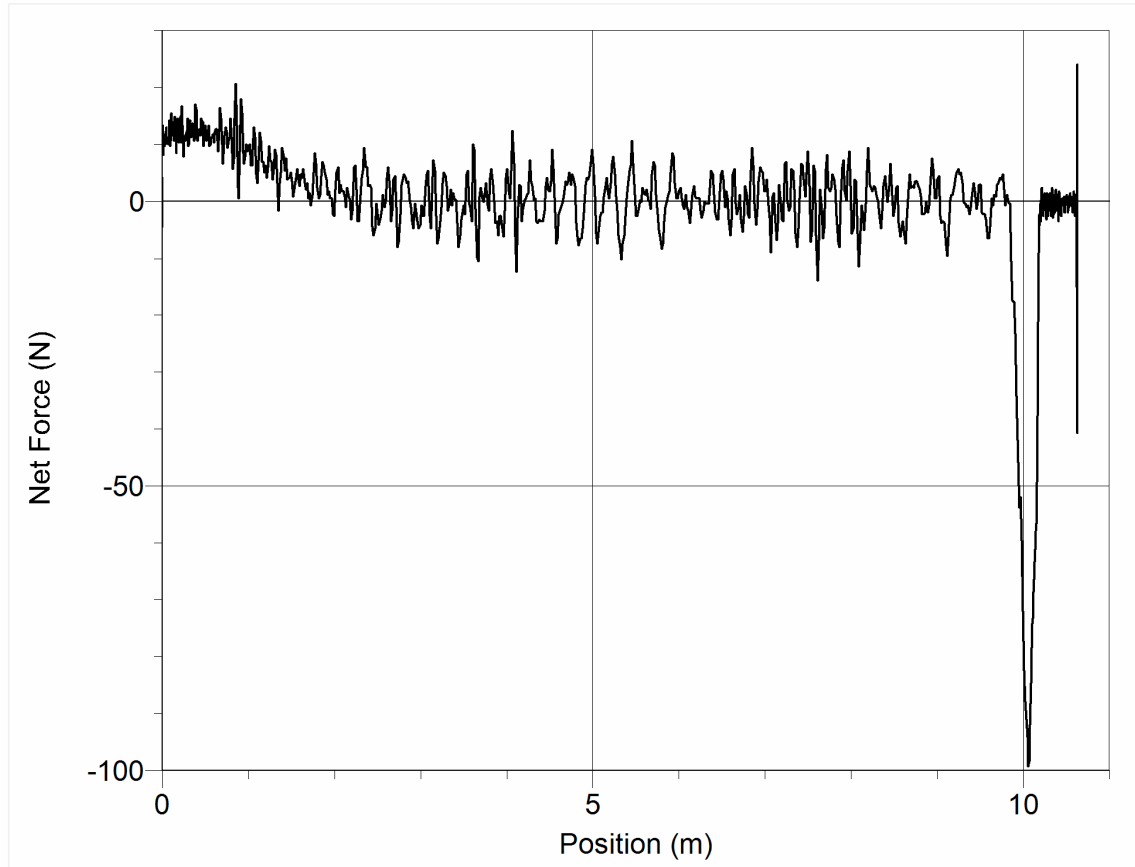


Figure 6: Graph of net force *vs.* position for the upward motion of the elevator.

The second graph in Figure 7 is the speed *vs.* time graph. It was created by taking the work done by the net force, dividing by the mass, and taking the square root of the result. At first glance this graph is identical to the velocity *vs.* time graph of Figure 3. However, when you compare the graphs carefully a small difference is noted in the first few seconds of the graph. The velocity graph is negative while the speed graph is positive. This serves as a strong reminder to our students that kinetic energy is linked to speed. The direction implicit in the minus sign must be added thoughtfully to the result. A final approach to the manipulation of the data would be to create graphs of kinetic and gravitational potential energy as functions of time. The sum of these two quantities is not constant over time which allows room for a fruitful discussion of work done by other forces, in this case the elevator force, in the conservation of energy formulation of the work-energy theorem.

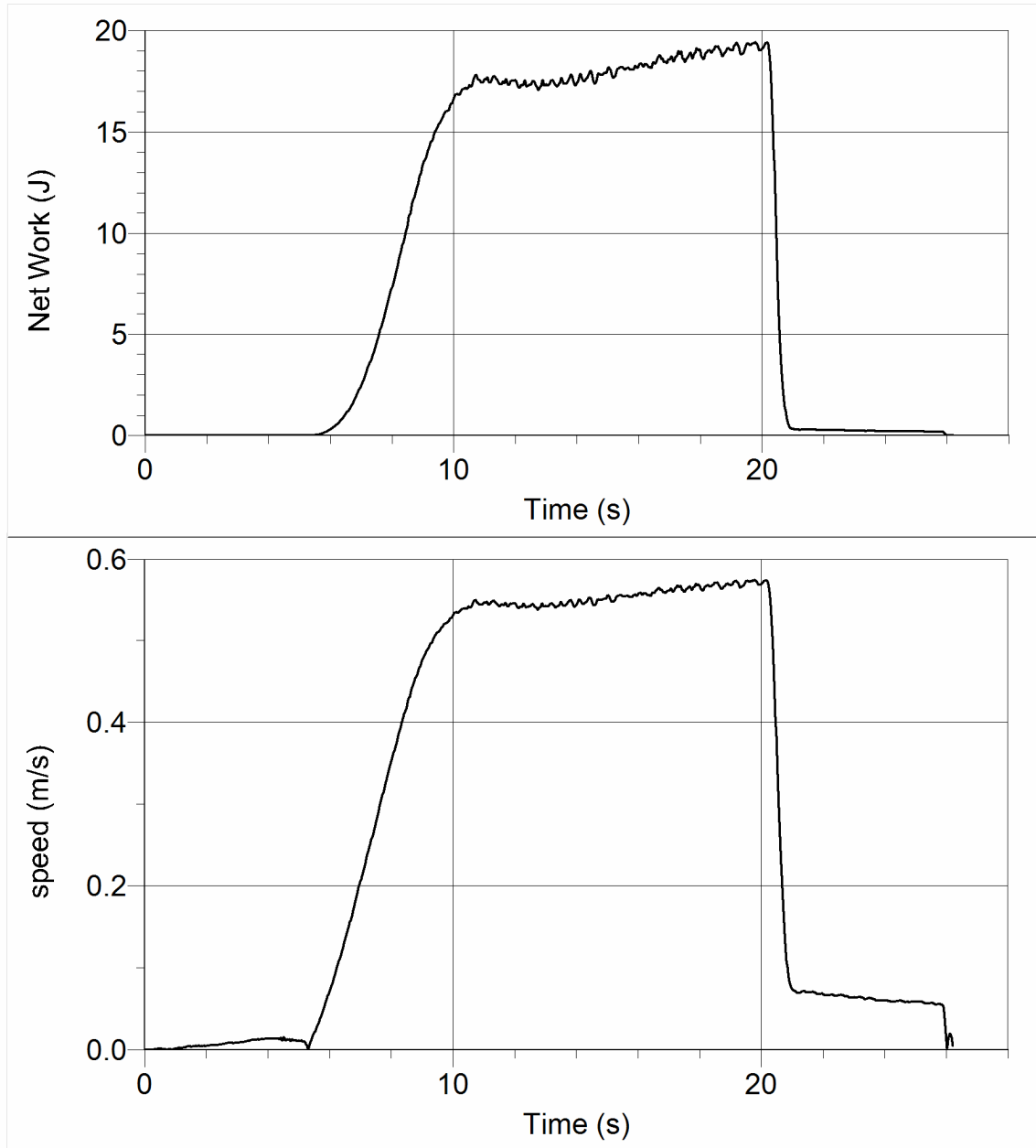


Figure 7: Graphs of net work and speed vs. time for the motion of the object in Figure 5.

Third, although physics texts typically contain problems that require analyzing a segment of an elevator’s motion, they seldom look at the entire motion of the elevator from the start to finish. We feel that studying the entire motion helps students achieve a more complete understanding. For example, students are often surprised to learn that the elevator only accelerates for a short period of time. Also, the physical meaning of the sign of velocity and acceleration is reinforced as students examine all combinations of positive and negative velocity and acceleration.

Assessment

This activity takes place within the context of a semester-long, comprehensive program of student-centered, interactive activities. We have used a pair of instruments in order to attempt to assess the effectiveness of this single component of that experience. The first assessment took place in two parts. Students were asked to respond to the same question, stated below, as a pre- and post-test. The question was designed to assess the students' ability to transfer what they had learned in the elevator lab to a different context. The pre-test was given the day before the elevator activity while the post-test was given at the end of the class period in which the activity was completed. It is important to note that the post-activity question was asked before any of the other activities intended to reinforce these concepts were completed.

You are in a rocket ship that takes off vertically. Describe your motion and relate it to the forces that act on you.

This question actually has two linked tasks. These are aligned with the mechanics framework shown in Figure 4. To be successful the student must (1) describe the forces present and (2) describe the resulting motion. These correspond to the dynamics and kinematics portions of the learning framework which are then connected through the concept of acceleration. Each response was evaluated separately with regard to each task—thereby reducing conflicts when a student's performance differs on the two tasks. For instance, a student who demonstrates a firm understanding of the forces acting on the object, but fails to describe the motion at all will score highly on one task but poorly for the other. This allows us to better track where any gains or losses in performance occur.

The rubric used in evaluating student responses is shown in Table 1. Each task was rated on a scale of 1 to 4 in order of increasing proficiency. A total of 35 students in two sections of algebra-based, general physics at Westfield State College participated in the activity. Participation in the two evaluation instruments was not required. As a result, there are fewer than 35 responses for each evaluation instrument. The results from the 30 pre-activity and 31 post-activity responses have been tabulated in Table 2. The combined average of the results for the two tasks for the pre-test is 1.83. The combined average of the results for the two tasks for the post-test is 2.37. The improvement in the overall average exactly mirrors the improvement on the individual tasks. Each improved, on average, by approximately $\frac{1}{2}$ of a proficiency rating with student performance on the description of the forces being approximately $\frac{1}{2}$ a proficiency rating above the performance on the description of the motion both before and after the elevator activity. A t-test performed on the data¹¹ yields a t-value of 3.6 for the results on forces and 2.3 for the results on the motion. Both are above a t-value of 2 indicating that the differences are significant. Since the question is not explicitly about the motion of an elevator, these results suggest that some transfer of knowledge has taken place.

Forces	Attributes
Distinguished 4	<ul style="list-style-type: none"> Multiple Free-Body diagrams representing the motion of the object at different times. All forces correctly described.
Proficient 3	<ul style="list-style-type: none"> Free-Body Diagram Correct All forces properly described
Apprentice 2	<ul style="list-style-type: none"> Free-Body Diagram present. May have incorrect number of forces. Forces described, but not all correctly
Novice 1	<ul style="list-style-type: none"> No Free-Body Diagram or Free-Body Diagram seriously flawed. Forces not described.
Motion	
Distinguished 4	<ul style="list-style-type: none"> Motion is described graphically. Motion of the object is discussed at different times in the process.
Proficient 3	<ul style="list-style-type: none"> Motion is described in terms of position, velocity and acceleration. Description of the motion has few if any minor flaws. Graphs may be used to describe the motion.
Apprentice 2	<ul style="list-style-type: none"> Motion is described but no attempt is made to place the description into the context of position, velocity and acceleration. Description of the motion may have minor flaws.
Novice 1	<ul style="list-style-type: none"> No description of the motion of the object or the description of the motion is seriously flawed.

Table 1: Rubric used for evaluating student pre- and post-activity responses.

Pre-Test	1	2	3	4	Average
Forces	4	19	7	0	2.10
Motion	16	11	3	0	1.57
Post-Test	1	2	3	4	Average
Forces	1	12	14	4	2.68
Motion	10	11	8	2	2.06

Table 2: Results from the evaluation of student responses to the pre- and post-activity question.

Figure 8 is a compilation of scans from student papers which are representative of each of the different proficiency ratings. An attempt was made to select papers which were equally representative of proficiency in the two tasks. These representative responses are a mixture of both pre- and post activity responses as was necessary since no student scored a proficiency rating of 4 on either task in their pre-activity response.

Figure 8: Representative student papers from each of the four different proficiency ratings.

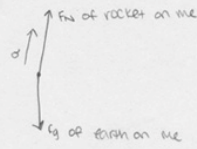
You are in a rocket ship that takes off vertically. Describe your motion and relate it to the forces that act on you.

Your motion is straight up.

Force of gravity of Earth on Rocket

(1) Novice

You are in a rocket ship that takes off vertically. Describe your motion and relate it to the forces that act on you.



- start @ rest.
- accelerate up @ const acc. until reach desired / max vel.
- once reach desired vel, acc = 0 m/s² + velocity stays const.
- the normal force of the rocket on me changes as the rocket moves, b/c

$$F_{net} = ma$$

mass stays the same, acc changes

$$\therefore F_{net} \Delta$$


$$F_{net} = F_N - F_g$$

since the same

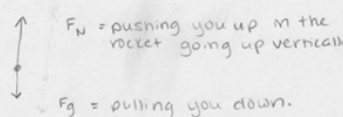
$$\therefore F_N \Delta$$

(4) Distinguished

You are in a rocket ship that takes off vertically. Describe your motion and relate it to the forces that act on you.



The motion is moving up with the rocket.



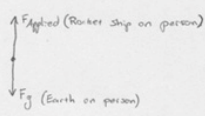
$F_N =$ pushing you up in the rocket going up vertically

$F_g =$ pulling you down.

(2) Apprentice

You are in a rocket ship that takes off vertically. Describe your motion and relate it to the forces that act on you.

As the rocket ship takes off vertically, it accelerates at 10 m/s^2 . Being in the ship, you will also accelerate at 10 m/s^2 . Because your body will remain at rest unless acted upon by an outside force, you will feel as if you are being pressed onto the seat in which you are sitting. However, it is actually the force of the rocket ship pushing up; in addition to gravity pulling down.



$F_{Applied}$ (Rocket ship on person)

F_g (Earth on person)

(3) Proficient

The second assessment was a student survey that was administered following the elevator laboratory. In this survey 31 students responded to 13 questions on a five-point Likert scale. The results of the survey are shown in Table 3. Student responses to questions 1-4 not only showed that they enjoyed the lab (77%), but also indicated that they found the lab to be an effective component in their learning process. 87% agreed that the lab changed the way they think about motion and forces and 84% reported a better understanding of motion and forces after the laboratory. A smaller majority (58%) also agreed that they could transfer what they had learned to new situations.

	SA	A	N	D	SD
1. I enjoyed the elevator lab.	7	17	7	0	0
2. Because of the elevator lab, the way that I think about motion and forces has changed.	4	23	4	0	0
3. Because of the elevator lab, I now have a better understanding of motion and forces.	7	19	5	0	0
4. I feel that I can apply the principles learned in the elevator lab to new situations.	3	15	11	2	0
5. During the elevator lab I spent too much time in discussion with my lab partners.	3	3	14	8	3
6. My predictions and my partners' predictions of the motion and forces in the elevator lab completely agreed.	2	15	4	8	2
7. By the completion of the lab, my partners and I reconciled any differences in our predictions about motion and forces.	5	20	5	1	0
8. I learned a lot by talking with my lab partners.	9	13	7	2	0
9. There were no elements of the elevator's motion that surprised me.	0	6	5	17	3
10. The elevator lab helped clear up misconceptions that I had about motion and forces.	0	16	13	2	0
11. The elevator lab helped me pull together a number of concepts.	5	16	10	0	0
12. The elevator lab helped me to better see the big picture of how motion and forces are related.	5	19	7	0	0
13. Because of the elevator lab I now think about motion and forces more often outside of my physics class.	4	14	10	3	0

Table 3: Results from student surveys. Response scale: Strongly Agree (SA), Agree (A), Neither Agree nor Disagree, D (Disagree), SD (Strongly Disagree).

The National Research Council (NRC) writes that instruction can be viewed as “helping the students unravel individual strands of belief, label them, and then weave them into a fabric of more complete understanding.” This requires designing classroom experiences and formative assessments that help “make students’ thinking visible to themselves, their peers, and their teacher.”¹² Requiring students to make predictions, reconcile them with their teammates, discuss the agreed upon predictions with their instructor and then test their predictions is a key element of the elevator lab. Not only have the instructors observed that this process is effective in identifying misconceptions, but the student survey supports this observation also. As shown in Table 3, only 19% of the students reported that they were not surprised by some element of the elevator’s motion. Further, the survey shows that the lab encouraged discourse that helped students improve upon their initial understanding—52% of students reported that the lab helped clear up their preconceptions about forces and motion and 71% of agreed that they learned a lot by talking with their lab partners. Social cognitive theory suggests that engagement often happens in a context in which students encounter the thinking of others. This may help explain the high rate of students reporting that they enjoyed the lab (77%) and think about motion and forces more often outside of my physics class because of the lab (58%).

In addition to reports of student satisfaction with this particular activity, normalized gains for pre- and post-course test on both the Force Concept Inventory and Test of Understanding Graphs – Kinematics fall in the 30-40% range that is typical for classes taught using this approach.

Conclusions

An elevator laboratory based on a cycle of prediction, action and examination was developed to help students improve their understanding of motion and forces. Based upon the laboratory experience, improvement was noted in the students’ ability to transfer their understanding of forces and motion to a new context. A survey indicated that students enjoyed the lab, identified and addressed preconceptions, and improved their understanding of how dynamics concepts fit together.

¹ L.Jensen, “Apparent weight changes in an elevator”, *Physics Teacher* **14**, 436 (1976).

² C. R. Rhyner, “Studying the Motion of an Elevator”, *Physics Teacher* **36**, 111 (1998).

³ Vernier Software & Technology, Beaverton, Oregon, LoggerPro 3.5.0.

⁴ R. J. Beichner, “The Effect of Simultaneous Motion Presentation and Graph Generation in a Kinematics Lab,” *Journal of Research in Science Teaching* **27**, 803 (1990).

⁵ “Force Plate in an Elevator”, *Caliper*, **19** (2002).

⁶ D.R.Sokoloff, *Real Time Physics: Active Learning Laboratories Modules 1 – 4*, (John Wiley & Sons, Inc., Hoboken NJ, 2004).

⁷ P.W. Laws, *Workshop Physics Activity Guide, The Core Volume: Mechanics I: Kinematics and Newtonian Dynamics (Units 1-7), Module 1, 2nd Edition*, (John Wiley & Sons, Inc., Hoboken NJ, 2004).

⁸ G.W. Ellis and W.A. Turner, Improving the conceptual understanding of kinematics through graphical analysis, *Proceedings of the 2002 American Society for Engineering Education Annual Conference and Exposition, Montreal, Canada, June 15-19* (2002).

⁹ G.W. Ellis and W.A. Turner, Helping students organize and retrieve their understanding of dynamics, *Proceedings of the 2003 American Society for Engineering Education Annual Conference and Exposition, Nashville, TN, June 22-25* (2003).

¹⁰ K. Appel, J. Gastineau, C. Bakken and David Vernier, *Physics with Vernier*, (Vernier Software & Technology, Beaverton OR, 2007).

¹¹ <http://www.dimensionresearch.com/resources/calculators/ttest.html>.

¹² NRC Commission on Behavioral and Social Sciences and Education, *How People Learn*, National Academy Press, Washington, D.C. (2000).