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Entrepreneurial Capital, Inequality, and Asset Prices

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Abstract

This paper investigates the contribution of entrepreneurship to increasing U.S. wealth inequality. Using data from the Survey of Consumer Finances (SCF), I document that, since 2000, the increase in the wealth shares of the top 0.1% and 1% groups of households is almost exclusively driven by entrepreneurs, identified empirically as private business owner-managers. Additional evidence from the SCF points to an increase in the average returns to entrepreneurial ventures as a likely driver of these patterns. I develop analytical characterizations of summary measures of inequality in the context of a model of wealth accumulation featuring heterogeneity in investment returns and in labor earnings across households in order to examine the restrictions that the wealth distribution imposes on the underlying return heterogeneity. To match the relative position of entrepreneurs across the wealth distribution and the level of top concentration in the SCF data, as well as changing inequality from the 1990s to the 2010s, the model requires high persistence of entrepreneurial status across households and a substantial increase in the average excess return to entrepreneurial investments. The associated slow transition dynamics of the wealth distribution in the model imply that, if not reversed, recent structural shifts may lead to widening inequality for many years to come.

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1 Introduction

In recent years, the United States and many other countries around the world have experienced a sustained rise in wealth inequality, particularly benefiting those at the very top of the distribution. As I show in this paper, the notable rise in top wealth concentration in the U.S. has been accompanied by an increase in the share of aggregate wealth held by entrepreneurs, both in the aggregate and at the top of the wealth distribution, especially since 2000. I investigate the drivers and implications of these shifts in the cross-sectional structure of inequality using a model of wealth accumulation featuring heterogeneity in investment returns and in labor earnings across households. A calibration of the model to U.S. data shows that a substantial increase in the returns to entrepreneurship is necessary to explain the growth in wealth inequality at the top and in the relative wealth of entrepreneurs in recent years, and it implies that wealth inequality is likely to continue to grow for decades if the returns to entrepreneurship remain high.

Using the US Survey of Consumer Finances (SCF) from 1989 to 2016, I document that the increase in top wealth concentration since 2000 appears to be driven almost exclusively by entrepreneurial households, defined empirically as private business owners who actively manage their businesses. In particular, entrepreneurs within the top 0.1% group by net worth account for the entire 4 percentage-point increase in the aggregate net worth share of the top 0.1% group from the 2001 survey wave to the 2016 wave. Similarly, entrepreneurs within the top 1% group by net worth account for 85% of the 6 percentage-point increase in the top 1% net worth share during the same period. The aggregate share of net worth held by entrepreneurs also experienced an economically and statistically significant increase from 41% in the 1990s to 45% in the 2010s even though entrepreneurs have remained stable as a fraction of the population, at around 12% according to my empirical definition.[1]

The reasons behind the growing importance of entrepreneurs in the wealth distribution are unclear. Although changes in the return characteristics of entrepreneurial ventures are a natural candidate and the focus of this paper, these characteristics are hard to estimate precisely due to the lack of high-quality, representative micro-level data on private business returns. Moreover, superior average returns to entrepreneurial ventures relative to other financial assets are not the only reason for the prevalence of entrepreneurs at the top of the wealth distribution. Private business owner-managers tend to receive labor earnings (wage income declared in tax returns) almost twice as high on average as the average household, so one would expect them to be wealthier on average even without any heterogeneity in the

[1] These patterns are consistent with the findings of Guvenen and Kaplan (2017) and Smith et al. (2017), who find using administrative tax and social security data that the increase since 2000 in top U.S. income inequality is almost entirely explained by an increase in pass-through business income. In the SCF, the increase since 2000 in the aggregate income shares of the top 0.1% and 1% groups by income is also driven almost entirely by entrepreneurial households.

returns to invested wealth across households. Entrepreneurs may also be more risk tolerant than non-entrepreneurs on average, holding a greater fraction of their net worth in risky assets, equities in particular (whether public or private), both across the entire population and within top groups. For example, in 2016, entrepreneurs in the top 1% group by net worth held 62% of their (gross) assets in equities relative to 47% for non-entrepreneurs in the top 1% group, although the entire difference is accounted for by inside private equity, that is, equity in private businesses actively managed by an entrepreneurial household. According to standard financial theory, entrepreneurs should be compensated for their greater risk-taking via a higher average return on their wealth portfolio, even if average returns to privately-held equity are no different from those to financial assets with similar aggregate risk exposure.[2]

Although all of these factors contribute to the prevalence of entrepreneurs at the top of the wealth distribution, my analysis, centered on a calibrated partial-equilibrium model of wealth accumulation, points to an increase in the average return to actively-managed privately-held equity as the most likely driver of the recent increase in the relative wealth of entrepreneurs and in top wealth inequality.

My model features two household types: non-entrepreneurs, who receive labor earnings and income from liquid financial investments, and entrepreneurs, who additionally have access to an investment technology that is subject to undiversifiable idiosyncratic risk. A key theoretical contribution of the paper is an analytical characterization of the long-run level of inequality, including inequality between entrepreneurs and non-entrepreneurs across the wealth distribution, and the speed at which inequality evolves following a transitory or permanent structural shift. These analytical results highlight the distinct impact of cross-sectional heterogeneity in labor earnings and heterogeneity in the returns on wealth. In the model, the impact of the latter is driven by two key features of entrepreneurship dynamics at the household level, the inside equity premium, that is, the average excess return to entrepreneurial investments per unit of idiosyncratic entrepreneurial risk exposure (Sharpe ratio), and the cross-sectional persistence of entrepreneurial status. The model also highlights the contribution of idiosyncratic entrepreneurial risk to wealth inequality both as a source of cross-sectional dispersion in realized returns on wealth and also through households' choice of the scale of their entrepreneurial investments.

The model can reproduce the structure of top wealth inequality and its recent secular

[2] Another theoretical possibility is that other sources of expected-return-on-wealth heterogeneity unrelated to entrepreneurial ventures, such as the level of sophistication and diversification of the financial portfolios of households, are cross-sectionally correlated with entrepreneurial status and contribute to the prevalence of entrepreneurs in top wealth groups. It is, however, unlikely that this source of return heterogeneity has become more important in recent years, as the diversification advantage of larger portfolios has probably declined recently, with more investors using mutual funds and indexed exchange-traded funds to diversify even small portfolios at low cost.

shifts quantitatively as well as qualitatively. I use information from the cross-sectional structure of wealth inequality and its secular shifts from the 1990s to the 2010s in the SCF data to estimate the model by the simulated method of moments and infer the structural increase in the inside equity premium that can account for the increase in the relative wealth of entrepreneurs and in top wealth concentration. The calibration also takes into account the concurrent increase in within-labor-earnings heterogeneity during this period. In the baseline calibration, an increase in the inside equity premium (Sharpe ratio) from 0.22 to 0.27, corresponding to a sizeable increase in the expected return-on-wealth differential between entrepreneurs and non-entrepreneurs from 1.7% to 2.5%, is needed to match the increase in the (top-weighted) net worth share of entrepreneurs from the 1990s to the 2010s. The calibration accounts for the majority of the increase in the top 1% and top 0.1% net worth shares during this period.

The increase in the return differential is essential for matching the increase in top wealth concentration. In the SCF data, the share of aggregate labor earnings held by the group of entrepreneurs as a whole has declined slightly from the 1990s to the 2010s. This is the case both for the classification of total household income into wage income and capital income in surveyed households' tax returns, and also for an alternative factor decomposition of income after a regression-based imputation for the labor earnings of entrepreneurs that do not report regular wages in their tax returns. The model calibration takes into account this decline in entrepreneurs' aggregate share of labor earnings and, as a result, by itself the calibrated increase in within-labor-earnings inequality during this period can account for only a small fraction of the increase in top wealth inequality and cannot explain the rise in the relative wealth of entrepreneurs.

The sizeable inferred increase in the return differential between entrepreneurs and non-entrepreneurs is a robust feature of empirically plausible calibrations of the model. The model requires high cross-sectional persistence of entrepreneurial status in order to jointly match the long-run levels of both top wealth concentration (the aggregate net worth shares of top groups of households by net worth) and the (top-weighted) net worth share of entrepreneurs in the data.[3] In turn, the degree of entrepreneurial persistence is the key determinant of the speed of transition of the wealth distribution following structural shifts, with high persistence implying slow transitions. In the baseline calibration, the (asymptotic) transition half-life for the aggregate wealth share of entrepreneurs is 46 years. Given these slow transition dynamics, the model must assume a sizeable increase in the inside equity premium in order to match the sizeable recent shifts in the structure of top inequality that have occurred during a period of only 20 years.

[3] High entrepreneurial persistence, that is, the fact that entrepreneurship tends to be a life-long profession for part of the population despite high business failure rates, has also been documented empirically in the U.S. (Quadrini, 2000).

In support of this key implication of the model calibration, I provide micro-level evidence for an increase in the average returns to private businesses, conditional on their survival in private form, relative to returns on other (passive) financial investments. Using SCF data on the initial investment and the current estimated market value of private businesses reported by surveyed households, I construct a measure of the long-term return to a household's primary actively-managed private business in excess of a liquid index (the S&P 500) over the life of the business. Although this cross-section cannot capture the impact of business failure, the most important source of risk for a private business, the analysis reveals a large increase in the conditional cross-sectional average excess return to private businesses since 2000, while the conditional cross-sectional volatility of returns has remained unchanged. These results are qualitatively consistent with those of Kartashova (2014), who uses SCF and aggregate accounting data to construct estimates of the aggregate returns to U.S. private equity following the methodology of Moskowitz and Vissing-Jørgensen (2002), and finds a significant increase in the aggregate premium of private equity over public equity since 2000.[4]

The model abstracts from changes in sources of return heterogeneity other than entrepreneurship, such as changes in the risk characteristics of the (passively-managed) financial portfolios of entrepreneurs and non-entrepreneurs, which could also have contributed to an increase in the average return differential between the two groups of households. However, measures of the differential wealth exposure to risky assets between entrepreneurs and non-entrepreneurs in the SCF offer no evidence of an increase in the average risk taking of entrepreneurs relative to non-entrepreneurs. In fact, the changes from the 1990s to the 2010s in the average equity portfolio share differentials are slightly negative and not statistically significant, including within top net worth groups.

The slow transition dynamics of the wealth distribution in any realistic calibration of the model, driven by the high inferred cross-sectional persistence of entrepreneurial status, also imply that recent structural shifts may have a protracted impact on inequality in the future. For example, if the shifts in the inside equity premium and in within-labor-earnings heterogeneity are permanent and holding all else constant in a model simulation under the baseline calibration, the top 1% net worth share will increase by another 2.9% over the next 20 years from the 2010s to the 2030s, almost as much as its 3.9% increase from the 1990s to the 2010s. Even if the structural shifts are fully reversed going forward, the economy will only slowly revert to its lower 1990s levels of inequality (the original steady state of the model), over several decades of transition.

[4] The conclusions are also consistent with those of Smith et al. (2017), who use U.S. administrative tax data linking pass-through firms to their owners. They find that more than 80% of the increase from 2001 to 2014 in the income of S-corporations owned by individuals in the top 1% group by income is due to rising profitability per unit of scale (worker) rather than rising scale, a finding strongly suggestive of an increase in the returns to these firms.

Related Literature This paper contributes to the literature in macroeconomics and finance on the drivers of wealth and income inequality related to cross-sectional heterogeneity in rates of return to wealth, and in particular the part of this literature that emphasizes entrepreneurship as a key source of this heterogeneity.

The empirical literature on inequality has documented that a rise in within-labor-earnings inequality accounts for most of the increase in inequality in the US since the 1980s until about the 2000s (Piketty and Saez, 2003). Using SCF data, this paper documents the important contribution of entrepreneurship to the increase in top wealth inequality, especially since the 2000s. Guvenen and Kaplan (2017) and Smith et al. (2017) reach a similar conclusion for income inequality using tax data.[5]

A number of papers develop models of entrepreneurship highlighting the ability of idiosyncratic entrepreneurial risk to generate a Pareto tail in wealth and income and to match the large observed levels of wealth concentration at the top of the wealth distribution. Quadrini (2000) and Cagetti and De Nardi (2006) emphasize the impact of financial frictions (borrowing constraints) faced by entrepreneurs, inducing an endogenous selection of entrepreneurs among wealthy people in the first place. Pástor and Veronesi (2016) focus on the impact of redistributive taxation on self-selection into entrepreneurship. Jones and Kim (2017) emphasize heterogeneity within the group of entrepreneurs and highlight creative destruction as a stabilizing force limiting the income growth of high-growth entrepreneurs. Aoki and Nirei (2017) develop a neoclassical growth model with entrepreneurs that can generate both Zipf's law of the firm size distribution as well as a Pareto tail in incomes. Relative to these models, the key contribution of my theoretical framework, which abstracts from the issues of entrepreneurial self-selection and within-entrepreneur heterogeneity in expected returns, is to offer a theoretical and quantitative analysis of the relationship between key properties of the dynamics of entrepreneurship, in particular the expected excess returns to entrepreneurial ventures and entrepreneurial persistence, and observable features of the cross-sectional wealth distribution, especially the prevalence of entrepreneurs across the distribution.[6]

The theoretical results of this paper and the insights on the important role of entrepreneurial persistence for the evolution of inequality are closely related to the contribution of Gabaix et al. (2016), who introduce mathematical tools from ergodic theory and the theory of partial differential equations to characterize the transitional dynamics of the cross-sectional income distribution in response to structural shifts in continuous-time settings. Luttmer

[5] See footnotes 1 and 4.

[6] The baseline version of my model employs a reduced-form representation of entrepreneurial investment that is symmetric to investment in a liquid financial asset. In a general-equilibrium, endogenous-production extension of my model presented in Appendix E, I show that even in a setting where entrepreneurial investments are realistically modelled as illiquid, nontradable assets, the equilibrium implications of entrepreneurial risk and return for portfolio choice and inequality are qualitatively very similar.

(2007) also emphasizes the slow speed of transition for aggregates in an economy with a power law firm size distribution.

The empirical studies of Quadrini (2000) and Gentry and Hubbard (2004) on the dynamics of entrepreneurship find much larger saving rates for entrepreneurs relative to workers, while Quadrini (2000) also documents a high level of entrepreneurial persistence, with past private business owners reentering into entrepreneurial ventures at much higher rates than households without entrepreneurial experience. The quantitative analysis of my model replicates these two important empirical facts on the dynamics of entrepreneurship.

In an important recent empirical contribution, Fagereng et al. (2018) use panel tax data from Norway, which administers a wealth tax and thus collects information on households' asset holdings, to establish the presence of a large degree of persistence in the heterogeneity of returns on wealth across households.[7] Moreover, they show that entrepreneurs play an important role in the estimated degree of persistent heterogeneity in returns and for the correlation of returns with wealth, consistent with my model's inference of a high degree of cross-sectional persistence in the return differentials between entrepreneurial and non-entrepreneurial households.

A large theoretical literature studies mechanisms that can generate the empirically observed Pareto tail in the wealth distribution.[8] The theoretical and quantitative analyses of Benhabib, Bisin, and Zhu (2011, 2015, 2016), Benhabib, Bisin, and Luo (2015), and Cao and Luo (2017) highlight that it is capital income risk and heterogeneity in returns on wealth that drive the thickness in the right tail of the wealth distribution, rather than heterogeneity in labor income, consistent with the results of my quantitative analysis in Section 4.[9]

Although the quantitative analysis of my model focuses on entrepreneurship as the source of cross-sectional return heterogeneity, the key insights of the analysis regarding the protracted impact of shifts in the return differentials across different groups of the population also apply to other sources of return heterogeneity, as long as they are cross-sectionally persistent. The literature has highlighted a number of other empirically relevant drivers of return heterogeneity, including heterogeneous financial exposures to aggregate risk (Bach, Calvet, and Sodini, 2017), differing levels of investor sophistication and skill (Kacperczyk, Nosal, and Stevens, 2018), under-diversification (Campbell, Ramadorai, and Ranish, 2018),

[7] In particular, an individual permanent component accounts for 60% of the explained variation in the returns on wealth in their sample, and this permanent component also accounts for the bulk of the cross-sectional correlation between returns and the level of wealth. The authors also find that return heterogeneity is mildly persistent across generations.

[8] See, e.g., Stiglitz (1969), Moll (2012), Toda (2014), Piketty and Zucman (2015), and Achdou et al. (2015) among many others, and the excellent literature surveys by Gabaix (2009) and Benhabib and Bisin (2016).

[9] In a similar spirit, the literature survey by Benhabib and Bisin (2016) notes that quantitative models of inequality focusing exclusively on labor income inequality, such as Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and Kindermann and Krueger (2014) need to assume a counterfactually large degree of within-labor-earnings inequality.

and limited stock market participation (Guvenen, 2009; Favilukis, 2013).

A distinct literature has focused on the challenging empirical task of estimating the average returns to entrepreneurship (Moskowitz and Vissing-Jørgensen, 2002; Hamilton, 2000; Hall and Woodward, 2010; Kartashova, 2014). In an important contribution, Moskowitz and Vissing-Jørgensen (2002), henceforth MV2002, estimate the time series for the aggregate private equity premium, that is, the aggregate returns to private equity in excess of publicly traded equity, using mainly SCF data and also aggregate US accounting data. I follow several aspects of their methodology in my empirical analysis of the SCF data, especially on the construction of household-level long-term private business returns. MV2002 question the existence of a positive private equity premium but Kartashova (2014) repeats the procedure of MV2002 to estimate the aggregate private equity premium in an updated SCF sample and finds a substantial improvement in the aggregate performance of private equity in the 2000s and a positive historical average premium, consistent with both the empirical findings and the theoretical predictions of this paper.[10]

Paper Outline The paper is structured as follows. Section 2 presents empirical findings regarding the role of entrepreneurs in the recent increase in top U.S. wealth inequality. Section 3 develops a model of entrepreneurship and inequality and offers analytical characterizations of the level and transitional dynamics of top wealth inequality. Section 4 presents the quantitative calibration of the model to U.S. data and additional evidence from the SCF consistent with the conclusions of the model calibration. Section 5 concludes.

2 Entrepreneurship and Wealth Inequality in the United States: An Empirical Investigation

Section 2.1 discusses the empirical definition of entrepreneurship used in this paper and documents the prevalence of entrepreneurs at the top of the wealth distribution. Section 2.2 investigates the growing importance of entrepreneurs in recent years by studying the evolution of shares of U.S. aggregates held by entrepreneurs as a group. Section 2.3 documents the key role of entrepreneurial households in the recent increase in top U.S. wealth inequality.

2.1 The Prevalence of Entrepreneurs at the Top

This subsection introduces and addresses concerns regarding my empirical definition of entrepreneurs, and documents their prevalence at the top of the wealth distribution.

[10] Using their tax data from Norway (2004 to 2015), Fagereng et al. (2018), discussed above, estimate a large average premium of private businesses over directly held listed stocks of around 6%.

The empirical analysis of entrepreneurship in the present paper centers on micro-level data from the Survey of Consumer Finances (SCF), conducted by the Federal Reserve Board triennially since 1989. The survey interviews a random sample of families (households) in the US on various aspects of their finances, including assets, sources of income, economic expectations, and other demographic characteristics. In its tenth and most recent survey wave in 2016, 6,248 households were interviewed. The SCF does a much better job of capturing characteristics at the top of the wealth and income distributions relative to other US surveys because it employs a sophisticated sample selection design involving a subsample of very wealthy individuals (see Appendix A.4.1 for more details).

Moreover, the SCF possesses two critical advantages relative to US tax return data with regard to the study of entrepreneurship and inequality. First, it offers detailed household-level estimates on economic stocks (assets and loans, including estimates of the value of entrepreneurial investments) rather than just flows. In particular, the level of detail in the survey allows for the construction of fairly comprehensive measures for the net worth, that is, non-labor wealth (simply referred to as "wealth" in the present section), as well as the total income of each household.[11,12] Second, it contains information on key characteristics of households' entrepreneurial investments, including their ownership share in a business and whether the household actively manages that business. Because the survey is conducted independently of the US tax authority, the latter features also set this data source apart from tax data in countries that collect information on households' wealth for tax collection purposes.[13]

I define entrepreneurs empirically as households that partly or wholly own *and actively manage* a private business.[14] According to this definition, entrepreneurial households con-

[11]In the discussion of the theoretical framework in Section 3, I distinguish between a household's total wealth, which includes the household's capitalized labor income stream, and net worth, which refers to the non-labor components of total wealth. However, I use "wealth" and "net worth" interchangeably in the discussion of the empirical results in this section.

[12]Throughout the analysis, I use the SCF Bulletin measures of household net worth and total income. Appendix A.4.2 offers some details on these measures.

[13]A number of recent papers, notably Fagereng et al. (2018) and Bach, Calvet, and Sodini (2017), use datasets derived from the tax records of Scandinavian countries, which (unlike the United States) administer a personal wealth tax in addition to a personal income tax and thus collect information on households' (net) assets as well as flows. Although these datasets possess distinct advantages relative to SCF data (for example, the dataset of Fagereng et al. (2018) derived from Norwegian tax data has a strong panel dimension), the value of private businesses reported by households to their tax authority is very close to the book value of these firms, consistent with tax-minimization motives. The book value is typically weakly related to their market value. In contrast, the group administering the SCF goes at great lengths to assure the individuals surveyed that the data collected in the survey will in no way be given to the IRS for tax verification purposes (tax audits). As a result, there is no reason to believe that SCF households systematically underreport their perceived private business valuations.

[14]More specifically, a household is classified as an entrepreneurial household if at least one adult in the household (a member of the household's "primary economic unit") reports owning (wholly or in part) as well as actively managing at least one private business. I exclude from the group of entrepreneurs a small number of households that satisfy these criteria yet report zero business earnings (for the year prior to the survey year) *and* zero business net worth (for the survey year).

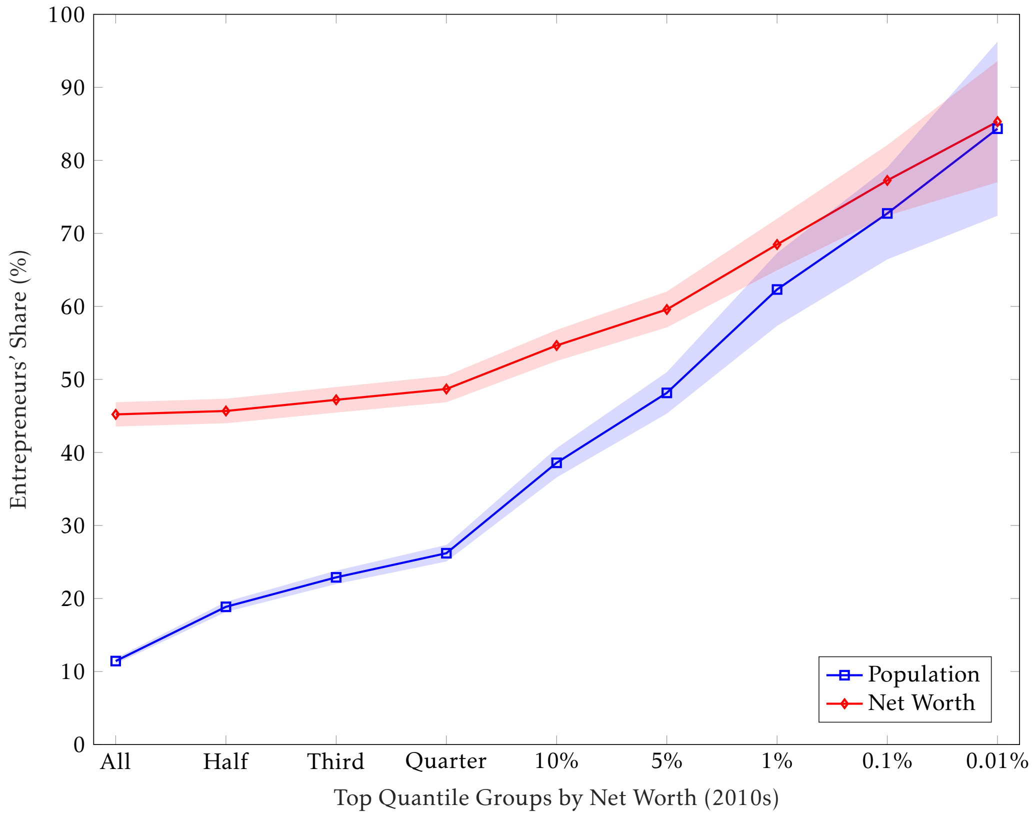

Figure 1: Entrepreneurs and Top Wealth Inequality

Notes: This figure plots the shares of entrepreneurs in top quantile groups of the net worth distribution for households in the Survey of Consumer Finances (SCF). Estimates are averaged over the three latest survey waves (2010, 2013, and 2016). The colored bands represent 95% confidence intervals for the estimates (see Appendix A.4.3 on statistical inference in the SCF).

stitute about 12% of the total population of US households in the 2016 survey. Yet, as a group they are far more important in economic terms, holding 45% of aggregate US net worth and 28% of personal income.

The prevalence of entrepreneurs at the top of the wealth distribution is striking. Figure 1 plots the population and net worth shares of entrepreneurs within top quantile groups of the net worth distribution. Averaging estimates over the three most recent survey waves (the "2010s"), entrepreneurs comprise less than 12% of the total population but their share in top quantile groups in terms of net worth increases sharply as we move towards the top of the distribution in the top panel of Figure 1.[15] In particular, entrepreneurial households account for 68% of the wealthiest 1% of households (the "top 1%"), 77% of the wealthiest 0.1%, and

[15]Throughout this paper SCF population weights are used to construct representative group-level and economy-wide totals.

85% of the wealthiest 0.01%.[16,17] A similar pattern holds for the net-worth-weighted share of entrepreneurs in top groups, that is the fraction of the total net worth of the top group that is held by the members of that group classified as entrepreneurs. It is, therefore, evident that any empirical or theoretical analysis of wealth inequality at the top should first and foremost address the role of entrepreneurship.

At the same time, there is large wealth heterogeneity within the group of private business owner-managers. Figure A.1 plots the fraction of entrepreneurs who are in each net worth decile for the latest (2016) survey wave, where the deciles refer to the distribution of net worth across all households, including non-entrepreneurs. Entrepreneurs can be found in all parts of the wealth distribution. In particular, a majority of them are part of the "middle class", defined by Piketty (2014) to comprise households in the 50th through 90th percentiles.

As the model of Section 3 makes clear, two key features of entrepreneurship are the focus of this paper: first, an entrepreneur's payoff is explicitly tied to firm-level performance and is therefore exposed to business risk; second, an entrepreneur makes firm investment decisions, implicitly choosing his or her exposure to firm-level risk. Relative to this theoretical definition of an entrepreneur, the empirical definition used in this paper, common in studies of entrepreneurship and inequality (e.g. Cagetti and De Nardi (2006)), has certain limitations. One set of concerns relates to the fact that households owning a private business may not be managers of businesses in a conventional sense, even if they explicitly report actively managing one or more businesses (as they do in the SCF). First, a non-manager "superstar", such as an athlete or singer, may register a private business under his or her name as a way to receive the stream of rents to his or her scarce skill and talent (e.g. album profits or payments for sponsorship deals). These profit streams, the argument goes, should not be interpreted as the payoffs to scalable capital investments but as disguised labor earnings. Second, a wealthy household may set up a private business as a "side hobby", that is, its private business may not be an important source of its income and wealth. Third, a wealthy household may set up a private company simply to manage its financial portfolio.

Even though these are valid concerns, they are unlikely to apply to a large fraction of private business owners-managers at the top of the wealth distribution. Although the public version of the SCF does not offer information on occupations, a comparison with the study of Bakija, Cole, and Heim (2012) based on confidential IRS tax data suggests that most wealthy

[16] The uncertainty of the estimates, captured by the width of the 95% confidence intervals in the figure (these intervals are constructed using bootstrap standard errors; see Appendix A.4.3 for details on inference in the SCF), increases as we move further along the top of the distribution towards smaller and smaller groups of households. For example, the top 0.01% of the wealthiest families includes only 12,600 families in 2016. Still, the prevalence of entrepreneurs in the top groups is hard to dispute even under the most conservative estimates

[17] The SCF by design excludes the members of the Forbes 400 list of wealthiest individuals, almost all of whom would be classified as entrepreneurs under the empirical definition of this paper.

private business owner-managers identify as such when asked about their primary occupation. In particular, Bakija et al. (2012) show that individuals identifying themselves in tax returns as executives, managers, supervisors, and finance professionals ("managers", for short) account for 60% of the top 0.1% group by income in 2005 and for 70% of the increase in the top 0.1% income share from 1979 to 2005.[18] A comparison of the relative population and income-weighted fractions of this group of managers within top income groups in 2004 with the ones for private business owner-managers in the 2004 SCF survey is telling. For the top 0.1% income group, this fraction is 59.6% by population (65.7% by income) for managers (see tables 3 and 7 in Bakija et al. (2012)) and 65.5% by population (66.5% by income) for private business owner-managers in the SCF. The discrepancy is similarly small for the top 1% income group.[19]

A second response to these concerns is that privately-held equity constitutes a large share of total household assets for the majority of private business owner-managers in top wealth groups, as can be seen in Figure A.2. This implies a large wealth exposure to firm-level risk for these households, given that a household's holdings of inside private equity are typically concentrated in a very small number of businesses. Relatedly, even if the source of success and superior investment returns for a private business is the artistic, medical, or legal skill of its owners rather than "conventional" managerial talent, the owners still make firm investment decisions and a large part of their wealth is exposed to the risk that business operations and investments entail. For the same reason, to the extent that a wealth management company ("home office") set up by a very wealth individual takes on idiosyncratic portfolio investment risk, it can be interpreted as an entrepreneurial venture.

A distinct potential limitation of the empirical definition is that it excludes the employed top managers of public firms, as I am unable to identify households by occupation in the public version of the SCF. Although executive pay for large public firms is subject to a distinct set of issues, managers of public firms can be interpreted to satisfy the two key features of entrepreneurship discussed above: their payoffs are exposed to idiosyncratic, firm-level risk through their performance-based compensation schemes; and they implicitly decide over the exposure of their wealth to this firm-level risk by choosing their firms' corporate investment policies, which in turn shape the risk of their performance-based pay.[20] However, Kaplan and Rauh (2010) show that top executives of public firms are few in number, comprising a small fraction of conventional top income groups, around 3% of the top 0.1% income group and less than 7% of even the top 0.001% income group in 2004.

[18]"Arts, media, and sports" comprise only 3% of the top 0.1% income group in 2005.

[19]For the top 1% income group, the fraction of managers is 44.0% by population (52.8% by income) (see tables 2 and 6 in Bakija et al. (2012)), while the fraction of private business owner-managers in the SCF is 54.2% by population (58.1% by income).

[20]Consistent with this interpretation, Panousi and Papanikolaou (2012) find that the sensitivity to idiosyncratic risk of the investment of publicly traded firms increases when managers own a larger fraction of the firm.

2.2 Entrepreneurs and U.S. Aggregates

This subsection documents a rise since the 1990s in the aggregate shares of net worth and total household income held by entrepreneurs, which has not been accompanied by an increase in entrepreneurs' aggregate share of labor earnings.

During the 27-year period spanned by SCF survey waves (1989-2016) there has been a sizable and statistically significant increase in the economic importance of entrepreneurs, measured by their shares of key US aggregates. Table 1 reports the shares of US aggregates held by entrepreneurial households in the SCF over the last three decades. The second, third, and fourth columns report average estimates and corresponding standard errors from survey waves occurring within each of the last three decades: the 1990s (waves 1989, 1992, 1995, and 1998), the 2000s (waves 2001, 2004, and 2007), and the 2010s (waves 2010, 2013, and 2016), respectively. The last column reports the absolute change in the estimate from the 1990s average to the 2010s average. The aggregate shares of entrepreneurs for net worth and income have both experienced a relative increase of about 10% from the 1990s to the 2000s that is statistically significant at the 1% significance level. In particular, entrepreneurs' share of aggregate net worth has increased by almost 4 percentage points, from 41.4% in the 1990s to 45.2% in the 2010s, and their share of aggregate income has increased by about 3 percentage points, from 24.6% to 27.4%. A similar increase has occurred for aggregate non-business net worth, that is, all components of net worth other than inside (actively managed) private equity (by construction, entrepreneurs own 100% of inside private business equity in all years).

In contrast, entrepreneurs' aggregate share of labor earnings has not increased over the same period. The fifth row of Table 1 reports the aggregate share of labor earnings for the group of entrepreneurs, where the measure of labor earnings for each household is its reported wage income (inclusive of bonuses), extracted through survey questions that closely follow key lines of US personal tax returns. The average labor earnings across entrepreneurial households are approximately twice as high as average earnings across all households, but the aggregate share of entrepreneurs appears to have fallen slightly from the 1990s to the 2000s.

A limitation of this measure of labor earnings is that a substantial fraction of entrepreneurs, especially sole proprietors, report no wage income; this was especially true in earlier survey waves. To address this shortcoming, I impute an estimate for the labor income of these entrepreneurs, following a regression-based imputation method by Moskowitz and Vissing-Jørgensen (2002) that uses information in the SCF on the hourly wages of employed individuals and on the hours worked in a year of both employed and self-employed individuals. Appendix A.4.4 describes the method in detail. This adjustment to the "raw" factor decomposition of personal income based on tax returns is intended to capture a notion

Aggregate	Decade Averages (%)			Change (%)
	90s	00s	10s	90s → 10s
Population	11.66	11.98	11.43	-0.22
	(0.21)	(0.23)	(0.15)	(0.26)
Net Worth	41.40	42.99	45.21	3.81*
	(0.63)	(0.88)	(0.69)	(0.94)
↳ Non-Bus	30.04	31.42	33.67	3.63*
	(0.60)	(0.87)	(0.61)	(0.85)
Income	24.58	26.93	27.43	2.85*
	(0.64)	(0.70)	(0.69)	(0.95)
↳ Labor	19.19	19.92	17.85	-1.35
	(0.51)	(0.55)	(0.49)	(0.70)
↳ Labor (adj)	24.30	24.79	22.45	-1.85*
	(0.45)	(0.68)	(0.52)	(0.68)

Table 1: Entrepreneurs' Share of US Aggregates Over Time

Notes: This table reports shares of US aggregates held by entrepreneurial households over time in the SCF. Entrepreneurs' share of aggregate labor income is reported both before and after an adjustment for the unpaid labor of self-employed entrepreneurs, discussed in the text and detailed in Appendix A.4.4. Standard errors, accounting for both sampling variability and imputation uncertainty in the SCF, are reported in parentheses (see Appendix A.4.3). A star next to an estimate in the last column indicates rejection of the null hypothesis of no change at the 1% significance level.

of labor income as compensation for hours worked that is not directly dependent on firm performance, in juxtaposition with capital income in both active (actively-managed) and passive forms, which is directly scaled by the level of investment in a (literal or notional) asset. This conceptual distinction between labor income and active capital income may not be relevant for normative analysis,[21] but I show in Section 3 that it may be important for our understanding of the implications of different types of structural shifts for overall wealth inequality. Conceptually similar adjustments of labor share measures for the labor income of entrepreneurs and self-employed individuals are common in the literature and have been shown to yield more consistent estimates of the labor share across time and countries (Gollin, 2002).

Entrepreneurs' aggregate share of labor earnings also declines from the 1990s to the 2010s under the adjusted factor decomposition of personal income, and the decline in this share by 1.85 percentage points is now statistically significant. Moreover, the labor share of aggregate income also declined over the same period according to the SCF, from 81% on

[21] For example, Smith et al. (2017) argue that because the superior returns to private business owner-managers are the result of their skill, the increase in top inequality in the 21st century is still driven by the "working rich" as it was during the 20th century, in contrast to the "passive rentier" view of the wealthy in recent years advocated by Piketty (2014).

average in the 1990s to 73% in the 2000s to 70% in the 2010s.[22,23] These trends suggest that within-labor-earnings heterogeneity is not a likely reason for the increase in the relative wealth of the group of entrepreneurs.

The slight decline in the relative labor earnings of entrepreneurs and the large decline in the labor income share may partly reflect a tax-motivated reclassification of managerial compensation from wage income to capital income in pass-through entities, S-corporations in particular, which have grown in number in recent years. This effect is not fully accounted for through the adjusted factor decomposition discussed above. However, the magnitude of this effect is likely to be limited.[24] In any case, a mere reclassification of managerial income cannot by itself explain the increase in entrepreneurs' aggregate share of total income.

2.3 Entrepreneurs and Top Wealth Inequality

In this subsection, I decompose the increase in the top 0.1% and 1% net worth shares by entrepreneurial status and find that it is driven almost exclusively by entrepreneurial households since 2000. I also show that most of the overall increase in top wealth inequality since the 1990s took place from the 2000s to the 2010s. In Section 4.1, I confirm these trends through two new summary measures of top inequality that play an important role in the theoretical analysis of this paper.

Concentration of wealth at the top has increased substantially over the past three decades. Figure A.3 plots the aggregate net worth share of top 1% group of households by net worth from the first SCF survey wave in 1989 to the latest wave in 2016. Over this 27-year period, the share of aggregate net worth held by the top 1% group has risen by 8.6 percentage points, from 30.0% in 1989 to 38.6% in 2016. Similarly, the top 0.1% net worth share has risen by 3.7 percentage points, from 11.1% in 1989 to 14.8% in 2016.[25] Although there is some disagreement on the exact magnitude of the wealth inequality increase, owing to different methods employed in the literature for estimating the wealth distribution in the U.S., the results from the SCF are on the lower end of the different estimates (Saez and Zucman, 2016; Kopczuk, 2015, 2016) and yet point to a substantial increase. This can be seen in Figure A.3, which also plots an estimate of the time series for the top 1% net worth share by

[22] These estimates are for the adjusted measure of labor income, that is, after the adjustment for unpaid entrepreneurial labor income. The labor share under the tax-returns-based factor decomposition of income also declines by about 10 percentage points, from 76% in the 90s to 68% in the 2000s to 66% in the 2010s.

[23] This declining estimate for the labor share is largely consistent with other better-known estimates of labor share dynamics from national tax and income accounting data. For example, Karabarbounis and Neiman (2014) document a 5-percentage-point decline in the labor share of the corporate sector globally using an international data set compiled from country tax and income accounting data.

[24] According to a back-of-the-envelope calculation by Smith et al. (2017), after accounting for this effect, the true decline in the U.S. corporate sector's labor share from 1980 to 2012 is 6.3% rather than 7.5%, which is still a substantial decline.

[25] The increase in these measures is slightly greater once one takes into account the increase in the wealth of the richest 400 individuals in the Forbes 400 list, which are by design excluded from the SCF.

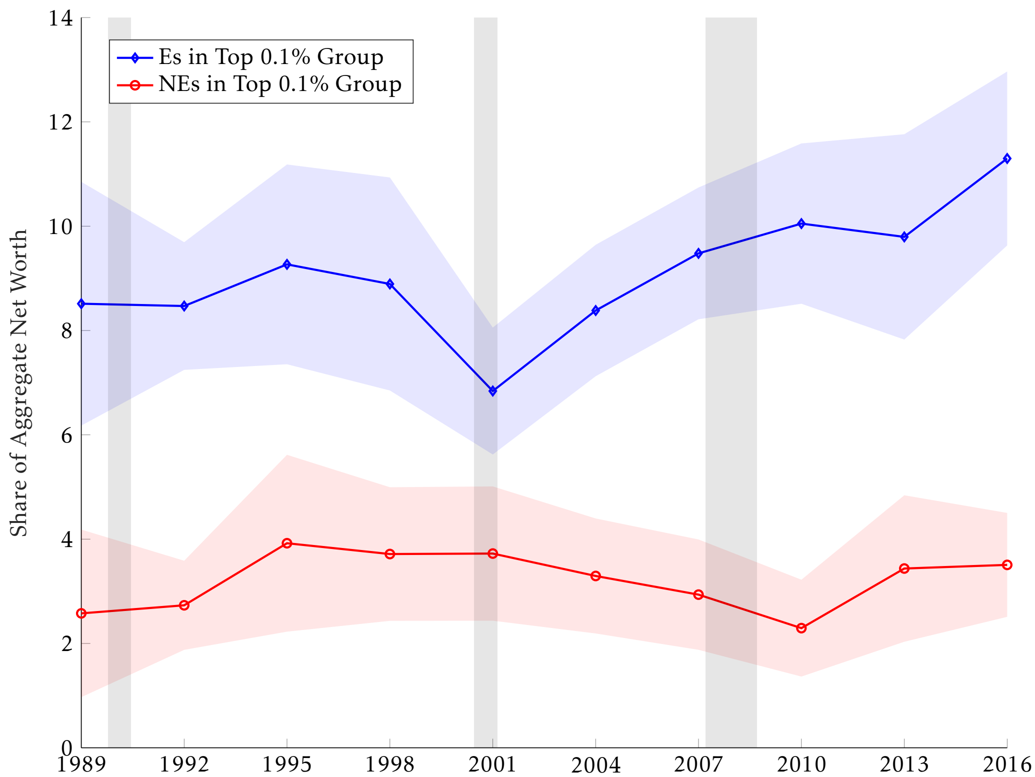

Figure 2: Decomposition of Top 0.1% Net Worth Share by Entrepreneurial Status over Time
Notes: This figure plots the share of aggregate net worth held by the entrepreneurs (E) and non-entrepreneurs (NE) that are part of the top 0.1% group by net worth. The colored bands represent 95% confidence intervals for the SCF estimates based on bootstrap standard errors. Gray shaded areas denote NBER-designated recessions.

Saez and Zucman (2016), who use an income capitalization method based on tax data.

The increase in top wealth concentration has not taken place uniformly over this 27-year period. Instead, it has mostly taken place since 2000s. The top 1% net worth share increased by 3% on average from the 2000s to the 2010s but it increased only by 0.8% from the 1990s to the 2000s (first column of Table A.1). The top 0.1% share in fact slightly declined on average from the 1990s to the 2000s (Table A.2)).

Which types of households have contributed most to the increase in top wealth concentration? First, consider a group decomposition of the top 1% share as $T = \widetilde{T}^E + \widetilde{T}^{NE}$, where $\widetilde{T}^E$ ($\widetilde{T}^{NE}$) is the aggregate value share of the entrepreneurs (non-entrepreneurs) that belong to the top 1% group. Appendix Section A.2 discusses this decomposition for the top 1% value share for several variables at a point in time (in the 2010s), showing that entrepreneurs within the top 1% groups hold a larger fraction of aggregate net worth and capital income relative to non-entrepreneurs, while non-entrepreneurs hold a larger fraction of labor income.

Figure 2 plots the net worth shares of the two groups of households within the top 0.1% group over time, showing that the notable increase in top wealth concentration since 2000 is entirely driven by entrepreneurs. In fact, the aggregate net worth share of non-entrepreneurs within the top 0.1% group does not increase at all over the period from 2001 to 2016; the point estimate declines from 3.7% in the 2001 survey wave to 3.5% in the 2016 wave. The picture is similar for the evolution of the top 1% net worth share decomposition, plotted in Figure A.4 and also summarized in terms of decade averages in the left panel of Table A.1. Entrepreneurs within the top 1% group account for about 85% of the cumulative increase in the top 1% share since 2000. Although these point estimates are accompanied by large standard errors, since they correspond to a very small fraction of the population, they do suggest that a shift occurred in the early 2000s, following a period in the 1990s where the gap between the wealth of entrepreneurs and non-entrepreneurs in the top groups was declining.

Appendix Section A.3 presents a decomposition of the increase in top income inequality (the income share of the top 1% income group) since the 1990s both by group and by income factor (labor and capital). This decomposition shows that the increase in the labor income of the top 1% income group mainly occurred during the 1990s and almost exclusively through non-entrepreneurs. In contrast, the increase in capital income is exclusively due to entrepreneurs and has mostly taken place since 2000.

An analysis of US tax data also suggests that an important shift in the composition of top income inequality and its increase occurred around the early 2000s, as emphasized by Guvenen and Kaplan (2017). Figure A.5 plots the components of the income of the top 0.1% and the top 1% groups by income over time, using the classification of pre-tax income into wage income (wages, salaries, and bonuses, including exercised employee stock options), financial income (interest income, dividends, and rents) and business income (referred to as "entrepreneurial income" in Piketty and Saez (2003)), the latter category defined to include the profits of partnerships, sole proprietorships, and type-S (pass-through) corporations, and royalties. Although part of the increase in the business income of top groups since the 1980s is due to the U.S. tax reform of 1986, which created incentives for firms to register as pass-through entities, the figure shows that the growth in the wage income for the top groups has stopped or, for the top 0.1% group, reversed since 2000, while business income has experienced a sustained increase until the end of the data series in 2015, with a significant uptick in the early 2000s.[26]

3 A Model of Entrepreneurship and Inequality

This section introduces a parsimonious partial-equilibrium model of wealth accumulation in order to examine the drivers and implications of the recent observed shifts in the cross-sectional structure of inequality. Section 3.1 introduces the model setting and discusses key

[26]Income classified as financial income also experienced a similar uptick in the 2000s, but that was reversed in large part during the Great Recession of 2007-2009.

features of agents' optimal policies. Section 3.2 introduces two analytically tractable summary measures of top wealth concentration and of the prevalence of entrepreneurs at the top of the wealth distribution. Section 3.3 offers analytical characterizations of the level and transitional dynamics of these inequality measures and other key aspects of the equilibrium cross-sectional wealth distribution in the model. Section 3.4 discusses a general-equilibrium, endogenous-production extension of the model, presented in detail in Appendix E. Appendix B.2 contains additional details on the model and generalizations of the propositions of this section. The quantitative calibration and analysis of the model is presented in Section 4.

3.1 Setting and Optimal Policies

Overlapping generations There is a unit mass of households. All households die randomly at an exponential rate ω and a mass ωdt of offspring households, one for each deceased household, is born every instant.

Preferences Households have identical scale-independent, recursive-utility preferences over their consumption stream as well as the wealth bequeathed to their offspring, formally described in Appendix C. This is a new specification of preferences that extends the continuous-time version of Epstein-Zin utility, the Kreps-Porteus case of the stochastic differential utility class of Duffie and Epstein (1992), to allow for utility from bequests. Appendix C is dedicated to their derivation and characterization. As with the standard Epstein-Zin specification, household preferences are characterized by the pure rate of time preference ρ, the elasticity of intertemporal substitution (EIS) of consumption ψ, and the coefficient of relative risk aversion γ. The new specification features two additional parameters: parameter V_D that controls the strength of the bequest motive (the marginal value from bequeathed wealth), and parameter $\widetilde{\psi}$ that can be interpreted as the elasticity of intergenerational substitution (EGS) of consumption.

Financial markets All households can frictionlessly invest in an instantaneously riskfree asset at rate r_f, and a risky financial asset with excess return over the riskfree rate $dR^e_{Bt} = \pi_B dt + dB_t$, where B_t is a Brownian motion representing aggregate risk. This risky asset is only exposed to aggregate risk, and its Sharpe ratio π_B corresponds the price of aggregate risk, that is, the risk premium (expected excess return) per unit of exposure to the aggregate source of risk B_t.

Households can also invest in an annuity asset. A household with wealth W_{it} pledges a fraction θ_{Dt} of its wealth at its random time of death to the annuity fund in exchange for a flow of income equal to $\omega\theta_{Dt}W_{it}dt$ while alive.[27] A negative position $\theta_D < 0$ in this market can be interpreted as holding a life insurance policy. Bequeathed wealth at the time of death equals $W_{Dit} = (1-\tau_D)(1-\theta_{Dt})W_{it}$, where τ_D is the estate (bequest) tax rate.

All household income (including labor) is taxed at a proportional tax rate τ.

Entrepreneurial investment At any given point in time, an exogenous mass m^E of households are entrepreneurs (Es) and a mass $1-m^E$ are non-entrepreneurs (NEs). Entrepreneurial households have access to inside (entrepreneurial) equity, an asset with excess return over the riskfree asset $dR^e_{Zit} = \pi_Z dt + dZ_{it}$, where household-specific Brownian motion Z_{it} is a source of purely idiosyncratic risk that cancels out on average across entrepreneurs. π_Z is the price of risk (Sharpe ratio) of inside equity or risk premium per unit of idiosyncratic risk exposure, a key model parameter that is taken as exogenous in the partial-equilibrium setting of the present section.

Type switching NE households become Es at an exponential rate ν^{NE} and E households become NEs at a rate ν^E. The offspring of deceased households retain the entrepreneurial type of their parents. The inflow-outflow balance condition for the mass of the two types, ensuring that the mass of each type remains constant over time, is

$$(1-m^E)\nu^{NE} = m^E\nu^E. \tag{1}$$

Labor earnings and newborn wealth At the time of birth, household i is endowed with a flow of $L_{it} = L_t \exp(\underset{\sim}{l}_i)$ units of permanent labor earnings, interpreted to include any government transfer income, which they receive continuously until their death. Newborn household i's draw of log relative earnings, $\underset{\sim}{l}_i \equiv \log(L_{it}/L_t)$, is from a (scaled) distribution $f^s_{\underset{\sim}{l}}$, for $s \in \{E, NE\}$, which may depend on entrepreneurial status at the time of birth.[28] Let $\underset{\sim}{\alpha}^E \equiv \underset{\sim}{L}^E_t/L_t$ denote the ratio of average earnings across newborn E households over average earnings across all households.[29] Average earnings L_t evolve at a rate g_{Lt} and have proportional exposure σ_{Lt} to aggregate risk B_t. I consider equilibria featuring balanced growth in the long-run, so that, in the model's steady state, g_L coincides with the growth rate of aggregate income and wealth g, and σ_L coincides with the proportional exposure of aggregate wealth to aggregate risk, σ.[30]

[27] This expression assumes that the annuity market is perfectly competitive, as in Blanchard (1985).

[28] Distributions $f^E_{\underset{\sim}{l}}$ and $f^{NE}_{\underset{\sim}{l}}$ sum to m^E and $1-m^E$, respectively, so that $f^E_{\underset{\sim}{l}} + f^{NE}_{\underset{\sim}{l}} = f_{\underset{\sim}{l}}$, the (proper) distribution of log labor earnings across all newborn households.

[29] I assume that average earnings across newborn households equal average earnings across the population.

[30] The endogenous-production model extension discussed in Section 3.4 and Appendix E offers a microfoundation of this balanced-growth assumption.

Labor earning streams of living households are assumed to be fully pledgeable, so that, at the time of their birth, newborn households pledge their future earning streams in exchange for a capitalized stock of "labor wealth" W_{Lit}. In steady state, $W_{Lit} = (1-\tau)L_t \exp(l_i)/(r_f + \pi_B\sigma_L - (g_L - \omega))$. Hence, the initial wealth of a newborn household is the sum of the wealth inherited from his parent household and his own stock of labor wealth:

$$W_{it}^{\text{newborn}} = W_{Djt} + W_{Lit}, \tag{2}$$

where j refers to the parent of household i.

The assumption of fully pledgeable labor income streams is made for the sake of tractability, as it drastically simplifies the characterization of optimal policies under scale-independent preferences, while still retaining the additive nature of labor earnings in the wealth accumulation process across generations. Note that, under the perfect pledgeability assumption, any transitory labor earnings risk over the life of a household would be diversified away. A number of empirical studies have noted the importance of inequality in the permanent component of labor earnings for overall inequality in labor incomes relative to heterogeneity driven by transitory earnings shocks.[31]

The evolution of household wealth The wealth of surviving households evolves as

$$\frac{dW_{it}}{W_{it}} = \mu_{Wit}dt + (1-\tau)\theta_{Bit}dB_t + (1-\tau)\theta_{Zit}dZ_{it}, \tag{3}$$

where τ is the income tax rate and

$$\mu_{Wit} = (1-\tau)r_{Wit} - c_{it} \tag{4}$$

is the expected growth rate of wealth. Here, $c_{it} = C_{it}/W_{it}$ is the consumption-wealth ratio of the household, and

$$r_{Wit} \equiv r_f + \omega\theta_{Dit} + \pi_B\theta_{Bit} + \pi_Z\theta_{Zit} \tag{5}$$

is the expected (pre-tax) rate of return on the wealth of household i, and θ_{Bit}, and θ_{Zit} denote the optimally chosen proportional exposures of household wealth to the (pre-tax) returns of the risky financial asset and inside entrepreneurial equity, respectively, with $\theta_{Zit} = 0$ if the household is a non-entrepreneur.

[31] Guvenen et al. (2017) offer direct empirical evidence on the importance of lifetime labor earnings inequality in the US, and Kopczuk, Saez, and Song (2010) show that almost all of the increase in the variance in annual (log) earnings in the US since 1970 is due to an increase in the variance of permanent earnings (as opposed to transitory earnings).

Net worth and labor wealth Total household wealth W_{it} is the theoretically appropriate concept of household wealth, capturing all resources over which a household has a claim that can be used to finance present and future consumption, but it is not directly observable. The empirical measure of household net worth (net assets) is best interpreted in this model as the non-labor component of total household wealth, or total wealth less the measure W_{Lit} of the capitalized future labor income stream of the household:

$$N_{it} = W_{it} - W_{Lit} = W_{it} - W_{Lt}\exp(l_{it}), \tag{6}$$

where W_{Lt} denotes aggregate labor wealth.[32]

In steady state,

$$W_{Lt} = \frac{(1-\tau)L_t}{r_f + \pi_B\sigma - (g-\omega)}, \tag{7}$$

and the share of aggregate labor income in total expected income, a proxy for the aggregate labor income share, is

$$ly = \frac{L_t}{r_W W_t} = w_L \frac{r_f + \pi_B\sigma - (g-\omega)}{(1-\tau)r_W}, \tag{8}$$

where

$$w_L \equiv \frac{W_{Lt}}{W_t} \tag{9}$$

is the ratio of aggregate labor wealth to total wealth.

Household portfolio choice and saving The model features tractable household-level policies, given in appendix Proposition B.1, which only depend on the household's type (E or NE) after scaling by the household's total wealth. In particular, households choose consumption-wealth ratios $c(s) \equiv C_{it}/W_{it}$, bequeathed-to-surviving wealth ratios $w_D(s) \equiv W_{Dit}/W_{it} = (1-\tau_D)(1-\theta_D(s))$, and proportional wealth exposures θ_B and θ_Z^E to the risky asset returns that are independent of the level of wealth and are only a function of household type $s \in \{E, NE\}$. Under unit elasticity of intertemporal substitution of consumption, $\psi = 1$, all households choose the same consumption-wealth ratio; similarly, under unit elasticity of intergenerational substitution of consumption, all households choose the same bequeathed-to-surviving wealth ratio.

Because all households have identical preferences, and in particular the same risk aversion coefficient γ, they always choose the same proportional exposure to aggregate risk

[32] In a version of the model with finite lifetimes, the capitalized stock of future labor income approaches zero as the household nears the end of its (working) life. In the present setting, the capitalized value of future labor earnings only depends on a household's initial permanent earnings draw and not on the household's age, an implication of the perpetual-youth structure of the setting. An age-dependent profile of labor earnings could easily be added to the present model to generate more realistic age-labor income dynamics with minimal impact on the key model predictions.

$\theta_B = \pi_B/((1-\tau)\gamma)$ regardless of type, as shown in Proposition B.1. The inclusion of the risky asset in the model simply serves to disentangle the average return to wealth from the riskfree rate in the quantitative calibration of the model.

Entrepreneurs choose to invest a fraction

$$\theta_Z^E = \frac{\pi_Z}{(1-\tau)\gamma} \tag{10}$$

of their wealth in inside equity, which is proportional to the inside equity premium π_Z and inversely proportional to risk aversion γ. As a result, the expected excess pre-tax return earned by the average entrepreneur on inside equity is given by

$$\Pi_Z = \pi_Z \theta_Z^E = \frac{\pi_Z^2}{(1-\tau)\gamma}. \tag{11}$$

The expected excess return on inside equity Π_Z is the key determinant of the expected-return-on-wealth differential between entrepreneurs and non-entrepreneurs, $r_W^E - r_W^{NE}$. In the benchmark case where households have unit elasticities of intertemporal and intergenerational substitution, $\psi = \widetilde{\psi} = 1$, households have the same consumption and bequest policies regardless of type and, as a result, equations (3)–(5) imply $r_W^E - r_W^{NE} = \Pi_Z$.

Defining the saving rate of an agent as the expected change in total wealth as a fraction of expected (after-tax) income,

$$sy^s \equiv \frac{\mu_W^s}{(1-\tau)r_W^s} = 1 - \frac{cw^s}{(1-\tau)r_W^s}, \tag{12}$$

for $s = \{E, NE\}$, the model reproduces the strong empirical regularity that entrepreneurial households have much higher saving rates than non-entrepreneurs (Gentry and Hubbard, 2004; Quadrini, 2000), as a result of the superior average return on wealth earned by entrepreneurial households.[33]

3.2 Top-Weighted Inequality Measures

In this subsection, I introduce two summary measures of top wealth inequality, top-weighted average wealth and the top-weighted wealth share of entrepreneurs, which I characterize

[33] This result holds for empirically plausible levels of consumption elasticities not too close to zero. In the simple case with unit elasticities of intertemporal and intergenerational substitution of consumption, $\psi = \widetilde{\psi} = 1$, the saving rate differential is directly proportional to the return differential:

$$sy^E - sy^{NE} = \frac{\rho\left(r_W^E - r_W^{NE}\right)}{(1-\tau)r_W^E r_W^{NE}} = \frac{\rho \Pi_Z}{(1-\tau)r_W^E r_W^{NE}}. \tag{13}$$

More generally, the saving rate differential is increasing in the elasticities ψ and $\widetilde{\psi}$, all else equal.

analytically in the next subsection and investigate empirically in Section 4.1.

I define *top-weighted average wealth* as the cross-sectional expectation (average) of individual wealth relative to average wealth raised to a power $\zeta \geq 1$:[34]

$$\mathcal{F}_t(\zeta) \equiv \mathbb{E}^* \left[\left(\frac{W_{it}}{W_t} \right)^{\zeta} \right], \tag{14}$$

where $\mathbb{E}^*$ denotes the cross-sectional expectation operator.

By definition, $\mathcal{F}_t(1) = 1$. In the case of perfect equality (a degenerate wealth distribution), this function would be constant and equal to unity for all ζ, but, for any non-degenerate distribution of wealth, $\mathcal{F}_t(\zeta)$ is a strictly increasing function for $\zeta > 1$. When $\zeta > 1$, the expectation overweighs richer households, so that $\mathcal{F}_t(\zeta)$ can be interpreted as a measure of top inequality, for a given exponent ζ, with higher ζ implying further overweighting of the top of the distribution. When the wealth distribution has a right Pareto tail, as is the case empirically, with Pareto exponent ζ^*, it can be shown that top-weighted average wealth is finite for $1 \leq \zeta < \zeta^*$, and diverges to infinity as $\zeta \to \zeta^*$.[35]

Figure B.1 plots the schedule of top-weighted average wealth as a function of ζ when the wealth distribution is exactly Pareto.[36] Because a lower Pareto exponent ζ^* corresponds to a more unequal distribution, top-weighted average wealth for a fixed $\zeta > 1$ can be used to compare the degree of top inequality across different distributions.

An additive group decomposition of the schedule of top-weighted average wealth can be used to study the prevalence of entrepreneurs across the wealth distribution. In particular, decompose top-weighted average wealth as $\mathcal{F}_t(\zeta) = \mathcal{F}_t^E(\zeta) + \mathcal{F}_t^{NE}(\zeta)$, where

$$\mathcal{F}_t^E(\zeta) = m^E \mathbb{E}^* \left[\left(\frac{W_{it}}{W_t} \right)^{\zeta} \middle| i \in \mathrm{E} \right] \tag{16}$$

$$\mathcal{F}_t^{NE}(\zeta) = \left(1 - m^E\right) \mathbb{E}^* \left[\left(\frac{W_{it}}{W_t} \right)^{\zeta} \middle| i \in \mathrm{NE} \right], \tag{17}$$

and m^E is the population share of entrepreneurs. Define the *top-weighted average wealth share*

[34] When the empirical measure of wealth can take negative values, as in the case of net worth, I restrict attention to households with strictly positive wealth when taking the average.

[35] Of course, in any finite sample, this cross-sectional expectation is finite for any $\zeta > 1$.

[36] That is, log relative wealth $w = \log(W_{it}/W_t)$ has an exponential distribution, $f(w) = c\exp(-\zeta^* w)$ for $w \geq \underline{w}$ ($f(w) = 0$ otherwise) for some constant $c > 0$. In this case, top-weighted average wealth is given by

$$\mathcal{F}(\zeta) = \frac{\zeta^* \left(\frac{\zeta^* - 1}{\zeta^*} \right)^{\zeta}}{\zeta^* - \zeta}. \tag{15}$$

of entrepreneurs as

$$\varphi_t^E(\zeta) \equiv \frac{\mathcal{F}_t^E(\zeta)}{\mathcal{F}_t(\zeta)}, \tag{18}$$

for $\zeta \geq 1$. When $\zeta = 1$, $\varphi^E(1) = \varphi^E$ is simply the aggregate wealth share of entrepreneurs. For $\zeta > 1$, we can interpret $\varphi^E(\zeta)$ as a tractable indicator for the prevalence of entrepreneurs in top wealth groups. If the wealth distribution within the two groups was identical, up to scaling, $\phi^E(\zeta)$ would be flat as a function of ζ and equal to the population fraction of entrepreneurs.

These measures can be used to study the cross-sectional structure of inequality in any variable, not just wealth. In particular, I define top-weighted average labor earnings and entrepreneurs' top-weighted share of labor earnings as:

$$\mathcal{F}_{lt}(\zeta) \equiv \mathbb{E}^*\left[\left(\frac{L_{it}}{L_t}\right)^{\zeta}\right] \tag{19}$$

$$\varphi_{Lt}^E(\zeta) \equiv \frac{\mathcal{F}_{lt}^E(\zeta)}{\mathcal{F}_{lt}(\zeta)}, \tag{20}$$

respectively, where $\mathcal{F}_{lt}^E(\zeta)$ and $\mathcal{F}_{lt}^{NE}(\zeta)$ are defined as in equations (16) and (17).

3.3 The Structure of Inequality

In this subsection, I present some key analytical results regarding the implications of the model for inequality, both in the long-run and during transitions following structural changes in the economy. For expositional simplicity, I consider the simple case of no bequests, $V_D = 0$ in this section. Appendix B.2 extends these results to the general case with bequests.

To examine analytically the implications of this framework for inequality, it is useful to characterize the evolution of log relative household wealth $w_{it} \equiv \log(W_{it}/W_t)$, that is, the log of the ratio of individual wealth to average wealth. The log relative wealth of surviving E and NE households evolves, respectively, as[37,38]

$$dw_{it}^E = \left(\mu_t^E + \omega(1-w_L) - \left(\theta^E\right)^2/2\right)dt + \theta^E dZ_{it} \tag{21}$$

$$dw_{it}^{NE} = \left(\mu_t^{NE} + \omega(1-w_L)\right)dt, \tag{22}$$

where $\mu_t^E \equiv \mu_W^E - \widetilde{g}_t = (\mu_W^E - \mu_W^{NE})(1-\varphi^E)$ is the mean excess wealth growth rate of surviving Es relative to the average growth rate of all surviving households, $\widetilde{g}_t$, and $\theta^E \equiv (1-\tau)\theta_Z^E$ is

[37] These expressions follow from the law of motion of household wealth (3) and Ito's Lemma.

[38] For ease of exposition, I omit the time subscript from variables that I take to be constant both in steady state and during the transition following a structural shift (except possibly for an instantaneous jump). The assumptions regarding the transition are discussed in Section 4.

the proportional wealth exposure of E households to the after-tax return to inside equity. Similarly, $\mu_t^{NE} \equiv \mu_W^{NE} - \widetilde{g}_t = -\mu^E \varphi^E/(1-\varphi^E)$ is the excess wealth growth of surviving NEs. μ^E (μ^{NE}) is increasing (decreasing) in the return differential $r_W^E - r_W^{NE}$, and thus the inside equity premium π_Z, all else equal. By the law of large numbers, the proportional wealth exposure of E households to entrepreneurial risk Z, θ^E, is also the cross-sectional volatility of the instantaneous, after-tax returns to wealth across entrepreneurs. By the optimal choice of the entrepreneurial investment scale, $\theta^E = \pi_Z/\gamma$ is also increasing in the inside equity premium π_Z.

Inequality in the long-run Lemma 1 below characterizes the aggregate total wealth and net worth shares of Es in terms of structural parameters in the steady state of the model.

Lemma 1 (Entrepreneurs' Share of Aggregates)**.** *In steady state, the aggregate share of total wealth held by Es is*

$$\varphi^E = m^E \frac{\underset{\sim}{\alpha}^E \omega w_L + \nu^{NE}/m^E}{\omega w_L + \nu^{NE}/m^E - \mu^E}. \tag{23}$$

The aggregate share of net worth held by Es is

$$\varphi_N^E = \frac{\varphi^E - \varphi_L^E w_L}{1 - w_L}, \tag{24}$$

and their aggregate labor earnings share is

$$\varphi_L^E = m^E \frac{\underset{\sim}{\alpha}^E \omega + \nu^{NE}/m^E}{\omega + \nu^{NE}/m^E}. \tag{25}$$

Equations (23) and (24) capture the two key distinct sources of heterogeneity affecting the aggregate wealth and net worth shares of Es and NEs. First, if Es receive higher labor earnings on average than NEs, $\underset{\sim}{\alpha}^E > 1$ and thus $\varphi_L^E > m^E$, their aggregate total wealth and net worth shares, ϕ^E and ϕ_N^E, will both be greater than their population share m^E even if they have identical returns on wealth, $\mu^E = 0$.[39] In the data, $\varphi_L^E/m^E \approx 2$ (see Table 1). Second, if Es receive superior returns on average, $\mu^E > 0$, their wealth and net worth shares will also exceed their population shares even if they receive the same earnings on average, $\underset{\sim}{\alpha}^E = 1$ and thus $\varphi_L^E = m^E$. Moreover, under $\mu^E > 0$, their aggregate total wealth and net worth shares are increasing both in the level of return differential through μ^E and in the cross-sectional persistence of entrepreneurial status (decreasing in the type-switching rate, ν^{NE}), since high persistence implies that a given entrepreneurial household has access to the superior-average-return entrepreneurial technology for a longer period of time on average.

[39]This is true for the net worth share under the empirically relevant case of positive aggregate net worth.

Next, I characterize inequality throughout the wealth distribution. Denote the cross-sectional distribution (probability density function) of log relative wealth by $f_t(w)$, and the (scaled) distributions for E and NE households by $f_t^E(w)$ and $f_t^{NE}(w)$, respectively, which satisfy $f_t(w) = f_t^E(w) + f_t^{NE}(w)$. The dynamics of the cross-sectional distributions $f_t^E(w)$ and $f_t^{NE}(w)$ obey a system of partial differential equations known as the Forward Kolmogorov Equations, given by appendix equations (55) and (56). The steady-state distributions $f^E(w)$ and $f^{NE}(w)$ for the model under the baseline calibration, discussed in the next subsection, are plotted in appendix Figure B.2.

Similarly, denote the cross-sectional distribution of log relative labor earnings by $f_{lt}(w)$, and the corresponding (scaled) distributions for E and NE households by $f_{lt}^E(w)$ and $f_{lt}^{NE}(w)$. These distributions also obey a system of Forward Kolmogorov Equations, given by appendix equations (59) and (60).

The measure of top-weighted average wealth, introduced in the previous subsection, corresponds mathematically to the moment-generating function of the distribution of log relative wealth $f(w)$,[40]

$$\mathcal{F}_t(\zeta) = \int_{-\infty}^{\infty} \exp(\zeta w) f_t(w) dw, \tag{26}$$

and similarly for the (scaled) top-weighted average wealth across Es and NEs, $\mathcal{F}^E(\zeta)$ and $\mathcal{F}^{NE}(\zeta)$, respectively. Similarly, top-weighted average labor earnings correspond to the moment-generating function of the distribution of log relative earnings $f_l(l)$:[41]

$$\mathcal{F}_{lt}(\zeta) = \int_{-\infty}^{\infty} \exp(\zeta l) f_{lt}(l) dl. \tag{27}$$

Proposition 1 offers analytical characterizations for the long-run levels of the top-weighted wealth inequality measures.

Proposition 1 (Top-Weighted Moments)**.** *In steady state, the top-weighted aggregate wealth share of entrepreneurs* $\varphi^E(\zeta)$ *and top-weighted average wealth* $\mathcal{F}(\zeta)$ *satisfy*

$$\varphi^E(\zeta) = m^E \frac{w_L^{\zeta}(\mathcal{F}_l(\zeta)/\mathcal{F}(\zeta))(\omega + \nu^{NE}/m^E)\varphi_L^E(\zeta)/m^E + \left(1 - w_L^{\zeta}(\mathcal{F}_l(\zeta)/\mathcal{F}(\zeta))\right)\nu^{NE}/m^E}{\lambda^E(\zeta)} \tag{28}$$

[40] Gabaix et al. (2016) use essentially the same measure to derive theoretical results regarding the speed of convergence for cross-sectional distributions in continuous-time settings like the present one. They state their results in terms of the Laplacian transform of the log relative wealth (income) distribution, which is simply the horizontal reflection of the moment-generating function around the y-axis.

[41] As shown in Appendix Lemma B.1, in the steady state of the model, the cross-sectional distribution of labor earnings across all households $f_l(l)$ coincides with the (exogenous) earnings distribution across newborn households, $f_{\underset{\sim}{l}}(l)$. For this reason, it is also the case that $\mathcal{F}_l(\zeta) = \mathcal{F}_{\underset{\sim}{l}}(\zeta)$ in steady state. However, because of type switching, $f_l^s(l) \neq f_{\underset{\sim}{l}}^s(l)$, for $s \in \{E, NE\}$, that is, the equilibrium earnings distribution for each type does not coincide with the earnings distribution across newborn households of that type.

and

$$\mathcal{F}(\zeta) = \frac{\omega w_L^{\zeta}}{\lambda(\zeta)} \mathcal{F}_l(\zeta), \tag{29}$$

where

$$\lambda^E(\zeta) \equiv \omega\left(1 - \zeta(1 - w_L)\right) + \nu^{NE}/m^E - \mu^E \zeta - (\theta^E)^2 \zeta(\zeta - 1)/2 \tag{30}$$

$$\lambda^{NE}(\zeta) \equiv \omega\left(1 - \zeta(1 - w_L)\right) + \nu^{NE}/m^E - \mu^{NE} \zeta, \tag{31}$$

and

$$\begin{aligned}\lambda(\zeta) &\equiv \lambda^E(\zeta)\varphi^E(\zeta) + \lambda^{NE}(\zeta)(1 - \varphi^E(\zeta)) - \nu^{NE}/m^E \\ &= \omega\left(1 - \zeta(1 - w_L)\right) - \mu^E \frac{\varphi^E(\zeta) - \varphi^E}{1 - \varphi^E} \zeta - \varphi^E(\zeta)\left(\theta^E\right)^2 \zeta(\zeta - 1)/2.\end{aligned} \tag{32}$$

For given ζ and for $\mu^E > 0$, both wealth inequality measures are increasing in the mean excess wealth growth rate of entrepreneurs μ^E and, when $\varphi^E(\zeta) > \varphi^E_L(\zeta)$, in entrepreneurial type persistence (decreasing in the type-switching rate ν^{NE}). For $\zeta > 1$, they are also increasing in the volatility of entrepreneurial returns θ^E.

The prevalence of entrepreneurs at the top of the wealth distribution, captured by $\varphi^E(\zeta)$ for high $\zeta > 1$, is increasing in the return differential between Es and NEs and in the cross-sectional persistence of this differential, as in the special case of the aggregate wealth share, $\varphi^E = \varphi^E(1)$, presented in Lemma 1.The fraction of Es at the top is, however, also increasing directly in the volatility of returns across Es, θ^E. Even in a world with no labor earnings heterogeneity between Es and NEs, $\varphi^E = m^E$, and no superior average returns to entrepreneurship, $\mu^E = \mu^{NE} = 0$, if there is, for unmodeled reasons, dispersion in realized entrepreneurial returns, $\theta^E > 0$, successful entrepreneurs will tend to dominate the top of the distribution merely as a consequence of the idiosyncratic risk that they take. This highlights the importance of explaining the entire schedule of the top-weighted entrepreneurial wealth share, including both its intercept at $\zeta = 1$ and its slope.

Overall top wealth inequality, as captured by the measure of top-weighted average wealth $\mathcal{F}(\zeta)$ for $\zeta > 1$, is increasing in top concentration within the labor earnings distribution, as captured by top-weighted average earnings $\mathcal{F}_l(\zeta)$. By equation (32), it is also increasing in the expected return differential between Es and NEs, and the magnitude of this effect is increasing with the slope of Es' top-weighted share schedule.

A key insight from equations (29) and (32) is that, after controlling for the schedule of the top-weighted wealth share of Es, the long-run level of top-weighted average wealth does not depend directly on the churning rate ν^{NE}. As I discuss in Section 4.5, this implies that entrepreneurial persistence can be separately identified from the average return differential between entrepreneurs and non-entrepreneurs by imposing the joint restrictions of observed top wealth concentration (the schedule $\mathcal{F}_t(\zeta)$ or top group wealth shares) and the observed relative wealth of entrepreneurs across the distribution (the schedule $\phi^E(\zeta)$).

Even in a world with homogeneous investment returns, entrepreneurs will be prevalent at the top of the wealth distribution, $\varphi^E(\zeta) > m^E$, as long as they are prevalent at the top of the labor earnings distribution, $\varphi_L^E(\zeta) > m^E$. However, in realistic model calibrations, the impact of Es' top-weighted labor earnings share $\phi_L^E(\zeta)$ on their top-weighted wealth share $\varphi^E(\zeta)$ declines as one moves along the top of the wealth distribution by considering increasing values of ζ. The reason is that, empirically, the Pareto tail of wealth is thicker than the Pareto tail of labor earnings. As a result, $\mathcal{F}_l(\zeta)/\mathcal{F}(\zeta)$ declines to zero as ζ approaches the wealth Pareto tail exponent ζ^* from below.[42] As $\zeta \nearrow \zeta^*$, the top-weighted wealth share of entrepreneurs becomes independent of labor-earnings heterogeneity. The next proposition, which characterizes the asymptotic right Pareto tail of the wealth distribution, further illustrates this point.

Proposition 2 (Pareto Tail in Steady State)**.** *Wealth and net worth exhibit a right Pareto tail. In particular,* $f^E(w) \to c^E \exp(-\zeta^* w)$ *and* $f^{NE}(w) \to c^{NE} \exp(-\zeta^* w)$ *as* $w \to \infty$*, where* $\zeta^* > 1$ *is the Pareto exponent of the tail and* $c^E, c^{NE} > 0$ *are constants.*

In any equilibrium where the Pareto tail of wealth is thicker than the Pareto tail of labor earnings (the empirically relevant case), the Pareto tail exponent solves the equation

$$\lambda^E(\zeta^*)\lambda^{NE}(\zeta^*) - \nu^E \lambda^E(\zeta^*) - \nu^{NE} \lambda^{NE}(\zeta^*) = 0, \tag{33}$$

where $\lambda^E(\zeta)$ *and* $\lambda^{NE}(\zeta)$ *are given by equations* (30) *and* (31)*, respectively.*[43]

The (population and wealth-weighted) share of Es in the asymptotic top wealth (and net worth) group is

$$\frac{c^E}{c^E + c^{NE}} = \frac{\nu^{NE}}{\lambda^E(\zeta^*)}. \tag{34}$$

Inequality at the very top of the distribution as captured by the Pareto tail exponent, with a lower exponent corresponding to higher inequality (thicker tail), is determined exclusively by the process of wealth accumulation and is independent of the distribution of labor earnings.[44] These asymptotic expressions hinge on the empirically realistic assump-

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[42]Recall from the discussion of Section 3.2 that top-weighted average wealth diverges to infinity as $\zeta \nearrow \zeta^*$. The same is true for top-weighted average earnings as $\zeta \nearrow \zeta_L^*$, where ζ_L^* is the Pareto tail exponent of earnings. However, the fact that the Pareto tail of wealth is thicker than that of earnings implies $\zeta^* < \zeta_L^*$, so that $\mathcal{F}_l(\zeta)/\mathcal{F}(\zeta) \searrow 0$ as $\zeta \nearrow \zeta^*$.

[43]Note that equation (33) is equivalent to $\lambda(\zeta^*) = 0$, where $\lambda(\zeta)$ is given by equation (32).

[44]The result that the Pareto tail of wealth is either dominated by the Pareto tail of labor earnings or completely independent of labor earnings inequality is a celebrated result in formal theories of top inequality that goes back at least to Grey (1994).

tion that labor earnings become a negligible fraction of the total income of extremely rich individuals (e.g. the individuals in the Forbes 400). In the model, non-entrepreneurs in these very small groups of individuals at the very top are part of these groups solely because they used to be extremely rich entrepreneurs in the recent past and not due to high labor earnings.[45] Although empirically labor earnings is a non-negligible part of the total income of conventionally studied top groups, such as the top 0.1% and 1% groups, expression (34) for entrepreneurs' share in the asymptotic top wealth group offers a useful analytical approximation for the fraction of entrepreneurs, which turns out to be quite accurate in realistic model calibrations.

Inequality dynamics The final analytical result offers important insights into the predictions of this theoretical framework regarding the transitional dynamics of inequality.

Proposition 3 (Speed of Transition). *In response to permanent and unanticipated structural shifts, the top-weighted wealth share of Es evolves according to*

$$\frac{d\varphi^E(\zeta)}{dt} = -s_t^E(\zeta)\varphi_t^E(\zeta) + \nu^{NE} + \omega w_L^\zeta \mathcal{F}_{\underset{\sim}{l}}^E(\zeta)/\mathcal{F}_t(\zeta), \tag{35}$$

with speed of transition

$$s_t^E(\zeta) = \lambda_t^E(\zeta) + \omega w_L^\zeta \mathcal{F}_{\underset{\sim}{l}}(\zeta)/\mathcal{F}_t(\zeta) - \lambda_t(\zeta), \tag{36}$$

where $\lambda^E(\zeta)$ and $\lambda(\zeta)$ are given in equations (30) and (32). For $\mu_t^E \geq 0$, the asymptotic speed of the transition in Es' top-weighted wealth share,[46]

$$s^E(\zeta) = \lim_{t\to\infty} s_t^E(\zeta) = \lambda^E(\zeta), \tag{37}$$

for $\zeta \geq 1$, is bounded above by the speed of transition of Es' aggregate wealth share:

$$\lambda^E \equiv \lambda^E(1) = \omega w_L + \nu^{NE}/m^E - \mu^E, \tag{38}$$

which is decreasing in entrepreneurial persistence and in the mean excess wealth growth rate of entrepreneurs, μ^E.

[45] Formally, the fact that E type is not an "absorbing" state at the household level (i.e. $\nu^E > 0$), so that the Pareto tail of wealth is not comprised solely of Es, gives rise to the third-degree polynomial equation (33) for the Pareto exponent. If $\nu^E = 0$, equation (33) simplifies to $\lambda^E(\zeta^*) = \nu^{NE}$, a quadratic polynomial equation in ζ^* of the type commonly found in single-type random growth models (see e.g. Gabaix (2009)).

[46] The asymptotic exponential rate of convergence is formally defined as

$$s^E(\zeta) \equiv -\lim_{t\to\infty} \frac{d\log\left(|\varphi_t^E(\zeta) - \varphi_{ss}^E(\zeta)|\right)}{dt},$$

where φ_{ss}^E denotes the (new) steady-state value of $\varphi_t^E(\zeta)$.

Expression (38) for the exponential rate of convergence of the aggregate wealth share of entrepreneurs illustrates that type persistence, as captured by the churning rate ν^{NE}, and the mean excess wealth growth rate of entrepreneurs μ^E are the key determinants of the speed of convergence following structural shifts in the economy, such as an increase in the inside equity premium. Intuitively, a lower rate of churning and higher wealth growth differential imply slower reversal of existing wealth differences between entrepreneurs and non-entrepreneurs and between richer and poorer households. Importantly, the convergence rate of the aggregate wealth share ϕ^E is an upper bound for the speed of transition of top inequality, as captured by the top-weighted wealth shares of entrepreneurs. In particular, top inequality evolves more slowly when there is large heterogeneity within entrepreneurs, as captured by the cross-sectional volatility of their realized returns, θ_Z^E.

Appendix B.2 shows that these results and intuitions readily extend to the case with bequests, although the closed-form expressions for the top-weighted inequality measures hold only as approximations around $\zeta = 1$. The intergenerational transmission of wealth in the presence of bequest motives generally amplifies the level of top inequality, all else equal.

3.4 General-Equilibrium Model Extension

The present theoretical analysis does not endogenize the level of the inside entrepreneurial equity premium, the key determinant of cross-sectional heterogeneity in returns on wealth in the model. In Appendix E, I present and analyze a general-equilibrium, endogenous-production extension of the model this section, which endogenizes the inside equity premium and shows that the key implications for wealth inequality of the partial-equilibrium model of Section 3 carry over to a setting where entrepreneurial investments are more realistically modeled as long-term, illiquid investments.

The model interprets the capital of a real-world firm as a combination of liquid (pledgeable) capital (e.g. machinery, equipment, buildings) and entrepreneurial capital. Investment in entrepreneurial capital captures investment in processes and methods tied to the skills of the individual in charge of the firm. Because the payoff of such investments is tied to the skills and effort exerted by the entrepreneur or manager, he or she must personally retain exposure to this payoff for incentive reasons. In reality, this exposure may come in the form of partial or whole equity ownership of the firm, or long-term performance-based schemes in managerial compensation.

The key long-term determinant of the inside equity premium in the model is the relative productivity of entrepreneurial capital relative to liquid capital. An increase in the relative productivity of entrepreneurial capital can generate the key predictions of the partial-equilibrium model regarding the evolution of wealth inequality.

The general-equilibrium model also makes additional predictions regarding the response of the equilibrium riskfree rate in response to an increase in the relative productivity of entrepreneurial capital via the precautionary saving channel. I study the impact of idiosyncratic entrepreneurial risk on the dynamics of the U.S. riskfree rate both empirically and analytically in the context of a related endogenous-production asset pricing model in Tsiaras (2018).

4 The Drivers of the U.S. Wealth Inequality Increase: Quantitative Model Analysis

In this section I investigate the drivers of changing U.S. wealth inequality documented in Section 2 through a calibration and quantitative analysis of the model of Section 3. In particular, I use the restrictions imposed by the structure of top inequality in the US data to infer the increase in the inside equity premium that, together with the observed increase in within-earnings heterogeneity, can account for the increase in top inequality and the relative wealth of entrepreneurs from the 1990s to the 2010s. I also examine the implications of this increase for future inequality.

Section 4.1 investigates the top-weighted inequality measures introduced in Section 3 in the SCF data. Section 4.2 describes the transition experiment at the center of the quantitative analysis of the model and Section 4.3 discusses the calibration of the labor earnings distribution in the model. Section 4.4 presents the details and conclusions from the simulated method of moments (SMM) estimation of the model. Section 4.5 discusses the intuition for and evidence consistent with the inference of high cross-sectional persistence of entrepreneurial status. Finally, Section 4.6 discusses micro-level evidence from the SCF pointing to an increase in the returns to privately-held equity as the main driver of the apparent increase in the average return differential between entrepreneurs and non-entrepreneurs.

4.1 Top-Weighted Inequality Measures in the SCF

This subsection investigates in the SCF data the top-weighted inequality measures introduced and characterized analytically in Section 3, for the empirically observable wealth concept of net worth. Matching the schedule of the top-weighted net worth share of entrepreneurs is a key goal of the model calibration, as a way to capture the relative position of entrepreneurs across the net worth distribution.

The top panel of Figure 3 plots decade averages of the schedule of top-weighted average net worth in the SCF data for different values of ζ. As is visually evident from the figure, almost all of the increase in top concentration has taken place from the 2000 to the 2010s according to this measure, with top concentration barely changing from the 1990s to 2000s. This conclusion is consistent with the evolution of top group shares, as discussed in Section

2.3.

The bottom panel of Figure 3 plots decade averages for the top-weighted net worth share of entrepreneurs in the SCF data. The figure highlights an increase in the value-weighted share of entrepreneurs throughout the net worth distribution, including at the top, from the 1990s to the 2010s, as the gap between the two schedules is roughly constant across different ζ's (in fact, the average slope is slightly higher in the 2010s). Interestingly, the increase in the intercept (at $\zeta = 1$) and the decline in the slope of the measure from the 1990s to the 2000s is consistent with the increase in the aggregate net worth share of entrepreneurs as well as the decline in their share within the top 1% group over this period (which can be seen in Tables 1 and A.1, respectively), highlighting the ability of this measure to capture the relative importance of the two groups at different parts of the distribution.

4.2 Transition Experiment

The quantitative analysis of the model centers on a parsimonious calibration exercise on transitional dynamics. Starting from the steady state of the model, which I calibrate to moments from the 1990s, I consider the impact of three permanent, concurrent structural shifts: a change in the level of the inside equity premium, a change in the cross-sectional distribution of relative permanent labor earnings (within-labor-earnings heterogeneity), and a change in the aggregate share of labor earnings in personal income (labor income share).

Starting from the old steady state, at a point in time t_0 the economy experiences an instantaneous jump in the inside equity premium of $\Delta\pi_Z$ and an instantaneous jump in the ratio of aggregate labor wealth to total wealth Δw_L. Both π_Z and w_L remain constant at their new values throughout the transition. Holding constant the ratio w_L during the transition implies that the aggregate labor share of income remains roughly constant throughout the transition at its new steady state level. At the same time t_0, while the cross-sectional distribution of net worth across households is held constant (does not jump), all households receive draws from a new distribution of relative labor earnings, retaining their relative ranking from the old distribution. Accordingly, households' labor wealth, and hence their total wealth, experiences a jump at time t_0. All other structural parameters are held constant. These shifts are not anticipated ex ante by agents.

First, I calibrate a simple specification for the labor earnings distribution using data on the labor income shares of top groups by labor income in the SCF, both in the 1990s and in the 2010s. Second, I apply the simulated method of moments (SMM) procedure to estimate several model parameters in the original (1990s) steady state of the model jointly with the long-run change in the inside equity premium $\Delta\pi_Z$ and the long-run change in the ratio of aggregate labor wealth to total wealth Δw_L, targeting wealth inequality moments from the 1990s and 2010s together with some aggregate moments.

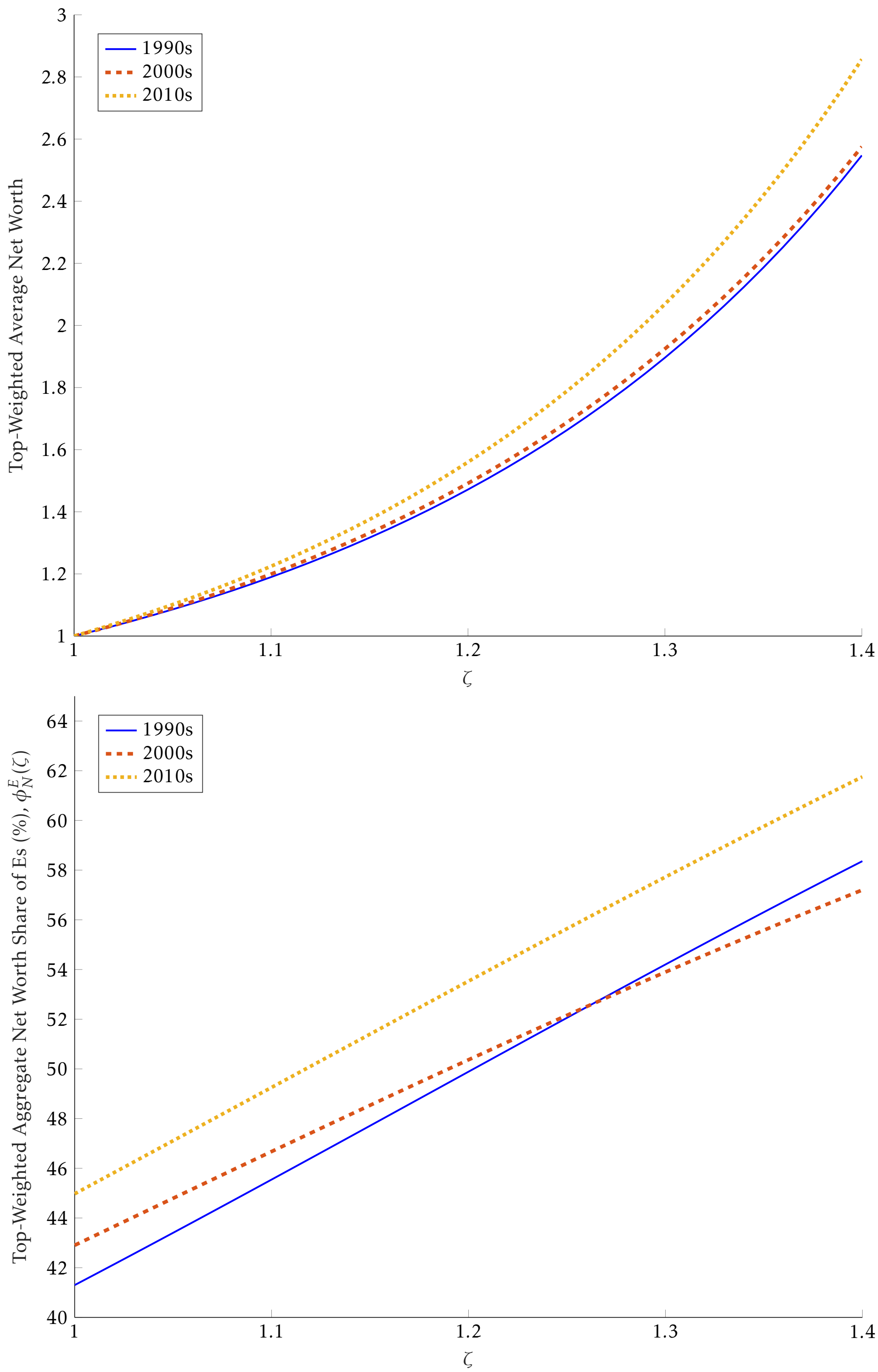

Figure 3: Top-Weighted Inequality Moments in the SCF

Notes: The top and bottom panels of this figure plot decade averages of the schedules of top-weighted average net worth and entrepreneurs' top-weighted net worth share, respectively, as a function of the exponent ζ in the SCF data. These measures (applied to net worth) are defined in equations (14) and (18), respectively. Only households with strictly positive net worth are considered when computing the measures.

4.3 Labor Earnings Calibration

I choose a simple specification for the cross-sectional distribution of permanent labor earnings across all newborn households, $f_{\underset{\sim}{l}}$, that captures well the top of the U.S. labor income distribution as it reproduces its empirically observed Pareto tail. A fraction $\mu_L = 10\%$ of newborn households receive draws from an exponential distribution for log earnings with Pareto exponent ζ_L^* and threshold $\underline{l}$, while the remaining households receive draws from a truncated normal distribution with mean m_L, standard deviation s_L, truncated at $\underline{l}$. Because this is the distribution of log earnings relative to average earnings, there are three free parameters to be calibrated.

I calibrate these parameters using information on the aggregate labor income shares of top groups by labor income in the SCF, separately for the 1990s and for the 2010s (using averages of survey waves from each decade).[47,48] First, I set ζ_L^* to the value that reproduces the observed ratio of the top 1% group share to the top 10% group share.[49] Second, I set the cutoff $\underline{l}$ to match the level of the top 10% labor income share. Although not directly targeted, the implied 5% top share is within one percentage point of the actual shares, owing to the fact that the Pareto distribution offers a remarkably good fit to the entire top end of the labor income distribution. Finally, I calibrate the remaining free parameter, which controls the truncated normal distribution, to match the labor income share of the top one-third of the population. The calibrated distributions are reported in Figure 4. The Pareto exponent ζ_L^* declines from 2.09 in the 1990s to 1.85 in the 2010s, corresponding to a significant increase in within-labor-earnings inequality.

I further allow for distinct average labor earnings for each type, E and NE, while assuming for simplicity that the distribution of earnings relative to type-specific average earnings is the same for the two types. In the SCF data, the estimate for ratio of average labor earnings across Es to average labor earnings across all households, φ_L^E/m^E, is 2.08 in the 1990s and declines slightly to 1.96 in the 2010s. These estimates use the factor decomposition of income after an adjustment for the imputed labor earnings of entrepreneurs who report no wage income in their tax returns, discussed in Section 2.2. The estimates are similar under

[47] The measure of household labor income used in the computation of the top group shares incorporates the adjustment for the imputed labor income of self-report entrepreneurs who reported no regular salaries, discussed in Section 2.2.

[48] This procedure ignores the impact of transitory labor income shocks to the cross-sectional distribution of annual labor earnings. However, the estimates from the calibration (for the evolution of the Pareto exponent, in particular) are very similar quantitatively to calibrations that allow for both transitory and persistent components in individual labor earnings (see, e.g., Hubmer, Krusell, and Smith (2016)).

[49] This computation uses the fact that, when a variable is distributed according to an exact Pareto distribution, the ratio of the top x% group share to the top y% group share, T_x/T_y, satisfies

$$\log(T_x/T_y) = \left(1 - 1/\zeta_L^*\right)\log(x/y). \tag{39}$$

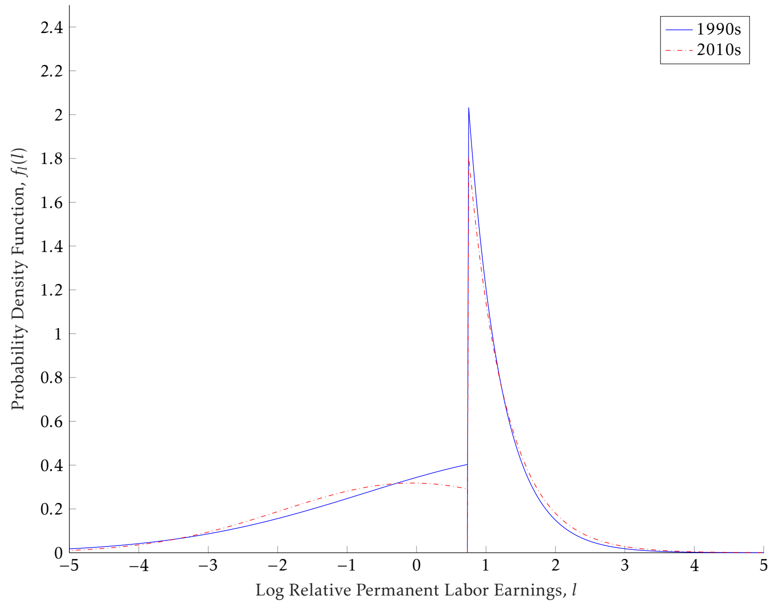

Figure 4: Calibration of Labor Earnings Distribution

Notes: This figure plots the calibrated cross-sectional distributions for relative labor earnings using SCF data from the 1990s and the 2010s. Calibrated parameters for the 1990s: $\zeta_L^* = 2.092$, $\underline{l} = 0.737$, $m_L = 2.012$, $s_L = 2.761$. Calibrated parameters for the 2010s: $\zeta_L^* = 1.846$, $\underline{l} = 0.740$, $m_L = -0.060$, $s_L = 1.889$.

the unadjusted decomposition of income (see Table 1).

4.4 SMM Model Estimation

Taking as given the calibration labor earnings distribution across households before and after the structural shift (time t_0), I estimate the model by the simulated method of moments, using simulations of one million households.

Fixed parameters I fix several parameters that are directly observable and some preference parameters, reported in the left panel of Table 2. Given the focus of the model on inequality at the top, I choose a (capital) income tax rate $\tau = 28\%$ close to the effective (and marginal) tax rates for top groups in the US (Piketty and Saez, 2007; Auerbach and Hassett, 2015) and an effective estate tax rate $\tau_D = 19\%$, following Carlitz and Friedman (2005). The assumed death rate $\omega = 4\%$ corresponds to an average working life of 25 years.[50] The real riskfree rate is set to 2.2%, the average over the years from 1989 to 1998.[51] I assume unit elasticities of

[50] Although the implied working life duration is a bit lower than conventional working life estimates of around 30 to 40 years, I choose this value because in this model the parameter ω also controls the degree of pledgeability of the future labor income stream of a household "dynasty", with lower average generation duration (higher ω) implying lower pledgeability.

[51] I compute the time series of the annual real rate using inflation expectations from the Survey of Professional Forecasters.

Parameter	Description	Value	Parameter	Description	Estimate, % (SE)
Fixed			Estimated		
τ	Income tax rate	28%	π_Z	Inside equity premium	22.2 (0.3)
τ_D	Estate tax rate	19%	ν^{NE}	Type switching rate	0.1 (0.0)
ψ	EIS	1	π_B	Price of aggregate risk	33.2 (2.4)
$\widetilde{\psi}$	EGS	1	ρ	Time discount rate	1.8 (1.3)
γ	Risk aversion coeff	4	V_D	Bequest motive	6.4 (1.1)
ω	Death rate	4.0%	w_L	Aggr labor-total wealth ratio	51.7 (3.5)
m^E	E pop fraction	11.7%	$\Delta\pi_Z$	Change in π_Z	4.6 (0.5)
r_f	Riskfree rate	2.2%	Δw_L	Change in w_L	-7.5 (0.9)

Table 2: Parameters in Baseline Model Calibration

Notes: The left panel of this table reports the model parameters directly calibrated (fixed), while the right panel reports the parameters (all in percentage-point format) estimated by the simulated method of moments (SMM). SMM standard errors for the parameter estimates using the covariance matrix of the data moments from the SCF are reported in parentheses.

intertemporal and intergenerational substitution of consumption and a coefficient of relative risk aversion equal to 4.

Targeted moments 8 model parameters are left to be calibrated: the inside equity premium, π_z; the transition rate from NE to E status, ν^{NE}; the price of aggregate risk π_B; two preference parameters (ρ and V_D); the old steady-state labor-total-wealth ratio w_L; and the changes in the inside equity premium and the labor-total-wealth ratio, $\Delta\pi_Z$ and Δw_L. I estimate these parameters via simulated method of moments to match 14 moments. Ten of these moments, overweighted in the SMM estimation, are the levels of five key net worth inequality moments in the 1990s, as well as changes in these moments from the 1990s to the 2010s. The changes in these moments are compared against the corresponding cross-sectional moments in the model simulation 19.5 years after the start of the transition.[52] The moments are the aggregate net worth share of entrepreneurs, the top-weighted net worth share of entrepreneurs evaluated a high value of $\zeta = 1.3$, and the aggregate net worth shares of the top 0.1%, 1%, and 10% groups of households by net worth. The first two moments capture crucial information on the relative position of entrepreneurs along the wealth distribution. These moment targets, estimated using SCF data, are reported in the first and third rows of Table 3, together with their corresponding bootstrap standard errors.

The four remaining targets, reported in table 4 are aggregate moments: the share of aggregate labor income in total income in the SCF data from the 1990s, the observed decline in this share from the 1990s to the 2010s (assumed to be permanent in the model), an estimate

[52] 19.5 years is the difference between the midpoint of the four SCF survey waves before 2000 (year 1993.5) and the three survey waves in the 2010s (year 2013).

	1990s		1990s → 2010s		
	Data	Model	Data	Model	Model, $\Delta\pi_Z = 0$
E share, ϕ_N^E	41.4 (0.9)	39.1	+3.8 (1.2)	+2.8	-0.6
Top-wgt E share, $\phi_N^E(1.3)$	54.2 (1.3)	52.6	+3.5 (1.7)	+3.6	-0.6
Top 1% share	32.2 (0.6)	34.6	+3.9 (0.7)	+3.4	+0.5
Top 0.1% share	12.0 (0.4)	15.4	+1.4 (0.6)	+1.8	+0.2
Top 10% share	67.7 (0.5)	67.7	+7.9 (0.6)	+3.7	+0.5

Table 3: Key Inequality Moments in the Data and in the Model

Notes: This table reports the 1990s levels and changes from the 1990s to the 2010s in key inequality measures in the data (columns 1 and 3) and in the baseline calibration of the model (columns 2 ad 4). The data estimates are from the SCF and are part of the targeted moments in the SMM estimation of the model. Numbers in parentheses denote bootstrap SCF standard errors. The last column reports the model-implied change in the inequality moments if the only structural shift is the increase in within-labor-earnings inequality, that is, assuming no change in the inside equity premium or the labor income share ($\Delta\pi_Z = \Delta w_L = 0$).

Moment	Data	Model
Labor income share, 1990s	82.0 (1.1)	79.7
Labor income share change, 1990s → 2010s	-11.4 (1.2)	-15.1
Income-net worth ratio	17.8 (0.4)	13.2
Bequests-income ratio	5.0	5.4

Table 4: Aggregate Moments in the Model Calibration

Notes: This table reports aggregate data moments targeted in the SMM estimation of the model. The remaining moment targets (net worth inequality moments) are reported in Table 3. All values reported are percentages.

of the ratio of income to net worth, and an estimate of the ratio of annual bequests to income. The estimate of the income-net worth ratio is the average value of this statistic across all SCF waves from 1989 to 2016. I use this average value because my transition experiment does not incorporate secular shifts that account for the declining trend in this ratio since the 1990s, notably the large decline in the U.S. real riskfree rate.[53,54] For the ratio of annual bequests to income I use a target of 5% that is in the range of existing estimates for annual bequests as a share of GDP, inclusive of inter-vivos transfers (Wang, 2016; Luo, 2017).

Estimated parameters The resulting estimates of the model parameters are reported in the right panel of Table 2. The original-steady-state level of the inside equity premium, estimated at 22.2%, is lower than the price of aggregate risk $\pi_B = 33.2\%$, that is, the Sharpe

[53] In the transition experiment, the riskfree rate and premium on the risky financial asset, π_B, are held constant at their old steady state levels. This assumption is made for parsimony given that these asset prices do not affect inequality directly due to households' symmetric portfolio choice (proportional wealth exposures) with respect to financial assets. As a result, the income-net worth ratio in the model remains essentially unchanged following the structural shifts.

[54] The simulated value of the income-net worth ratio in the model (at the old steady state) takes into account the adjustment in the factor decomposition of income for the imputed earnings of entrepreneurs who do not report regular wage income.

ratio on the risky financial asset to which all households have access (proxying for non-business assets, such as public equity and real estate). The corresponding average excess pre-tax returns (risk price times exposure) on the two assets are $\Pi_Z = \pi_Z \theta_Z^E = 1.7\%$ and $\Pi_B = \pi_B \theta_B = 3.8\%$, highlighting the ability of a relatively small return differential, captured by Π_Z, to generate large average wealth differences between the groups of entrepreneurs and non-entrepreneurs. The estimated entry and exit rates to and from entrepreneurship are $\nu^{NE} = 0.1\%$ and $\nu^E = 0.8\%$, respectively, corresponding to a large degree of cross-sectional persistence of entrepreneurial status.

The calibration provides a good fit for the long-run levels of top inequality, as captured by the 1990s values of the moment targets reported in the first column of Table 3, which is no small feat given the parsimonious nature of the model with only a single source of ex-ante rate-of-return heterogeneity and only two free parameters controlling this heterogeneity (π_Z and ν^{NE}).[55] Importantly, the model matches well the entire schedule of the top-weighted net worth share of entrepreneurs, including its average slope, as can be seen in Figure 5. By implication, it matches well the net worth shares of entrepreneurs within top groups; for example, the fraction of top 1% group net worth that is held by entrepreneurs within those groups is 68% in the 1990s SCF data and 63% in the original steady state of the model.

The calibration infers a 20% relative increase in the inside equity premium, from 0.22 to 0.27, corresponding to a sizeable increase in the average excess pre-tax return to entrepreneurial investments from 1.7% to 2.5%, in order to account for the observed increase in the value shares of entrepreneurs throughout the net worth distribution and the increase in top net worth inequality. The model can easily account for the magnitude of the changes in the inequality moments from the 1990s to the 2010s, with the exception of the increase in the top 10% net worth share. The estimated shifts in the return and earnings heterogeneity can together account for about half of the large increase in the top 10% share; in other words, the inequality increase is more concentrated at the very top of the distribution in the model relative to the data.

The calibrated model also generates a quantitatively realistic level for the saving rate differential between entrepreneurial and non-entrepreneurial households. The saving rates of Es and NEs, computed according to equation (12), are 29% and 17%, respectively, in the original steady state implying a saving rate differential of 13%. This is close to the range of saving rate differentials (16% to 35%) estimated in median regressions on entrepreneurial status by Gentry and Hubbard (2004) using a panel dimension in the 1982 and 1989 SCF surveys.[56] In turn, given the prevalence of entrepreneurs at the top of the income and wealth

[55] For comparison, the calibrated wealth inequality model of Benhabib, Bisin, and Luo (2015), which offers a similarly good fit for the top of the wealth distribution, features a 5-state Markov chain for the rate of return of wealth, with 10 free parameters allowing for much greater freedom in the model estimation.

[56] Gentry and Hubbard (2004) use an empirical definition of the saving rate as the change in net worth divided by an average of the (pre-tax) incomes in the two survey years, which is comparable to the definition used here,

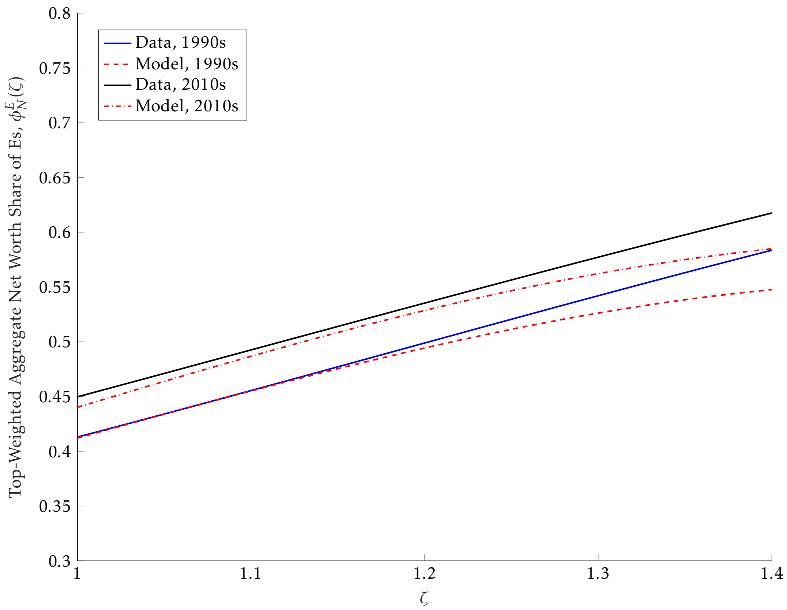

Figure 5: Top-Weighted Net Worth Share of Entrepreneurs, Data and Model

Notes: This figure plots the top-weighted net worth share of entrepreneurs as a function of the exponent ζ in the SCF data (the average schedule across the survey waves in the 1990s and in the 2010s) and in the model, both in the original steady state (1990s) and after a 19.5-year transition (2010s). This measure is defined as in equation (18). Only households with strictly positive net worth are considered when computing the measure, both in the data and in the model simulation.

distributions, saving rates are positively correlated with wealth in the model as they are in the data. The level of entrepreneurs' saving rates in the model is also close to estimates for the saving rates of households at the top of the net worth distribution.[57] In the model calibration, these large saving rate differentials are entirely driven by the superior average returns earned by entrepreneurs.[58]

The increase in the average returns to entrepreneurship, is essential for matching the increase in top wealth concentration. The fifth column of Table 3 plots the changes in inequality moments following 19.5 years of transition in a model simulation assuming that the only structural change is the increase in the cross-sectional dispersion of permanent labor earnings, that is $\Delta\pi_Z = \Delta w_L = 0$. This shift by itself can generate increases in the net worth shares of top groups that are only a fraction of the observed increases. And it is com-

equation (12). In particular, the closest model analog would be $sy_{GH} = (1-\tau)sy$. The authors also emphasize that starting entrepreneurs have higher saving rates than continuing entrepreneurs, suggesting an important role for constraints that entrepreneurs face in the external financing of new entrepreneurial ventures.

[57] For example, using 2000-2009 data, Saez and Zucman (2016) estimate "synthetic" saving rates (constructed by grouping everyone within a certain wealth fractile and calculating the ratio between changes in total wealth and total income for this group) of 35% for the top 1% wealth group, 9% for the group of individuals between the 90th and 99th percentiles and -4% for the group of individuals below the 90th percentile.

[58] The calibration assumes unit elasticities of consumption substitution, so that saving rates are given by equation (13). The differentials would be amplified if these elasticities are greater than unity, as entrepreneurs reduce their consumption-wealth ratios in order to take greater advantage of their superior returns to saving.

pletely unable to account for the notable increases in the (top-weighted) net worth shares of entrepreneurs, which decline under this transition experiment.[59]

Implications for the dynamics of inequality The sizeable inferred increase in the return differential between entrepreneurs and non-entrepreneurs is a robust feature of empirically plausible calibrations of the model. The model requires high cross-sectional persistence of entrepreneurial status. I discuss the intuition for the inference of high entrepreneurial persistence in Section 4.5. In turn, the degree of entrepreneurial persistence is the key determinant of the speed of transition of the wealth distribution following structural shifts, as highlighted by equation (38) for the speed of transition of entrepreneurs' aggregate net worth share, with high persistence implying slow transitions. In the baseline calibration, the (asymptotic) transition half-life for the aggregate wealth share of entrepreneurs is 46 years.[60] Given these slow transition dynamics, the model must assume a sizeable increase in the inside equity premium in order to match the sizeable recent shifts in the structure of top inequality that have occurred during a period of only 20 years.

The slow transition dynamics of the wealth distribution in any realistic calibration of the model, driven by the high inferred cross-sectional persistence of entrepreneurial status, also imply that recent structural shifts may have a protracted impact on inequality in the future. If the shifts persist in the future, the model predicts a long period of widening inequality in the future as the wealth distribution slowly converges to its new steady state over several decades of transition. The solid lines in Figure 6 plot the time paths of the aggregate net worth share of entrepreneurs and of the top 1% net worth group in a model simulation under the baseline calibration that assumes that the shifts in the inside equity premium and in within-labor-earnings heterogeneity are permanent and holding all else constant. The cumulative changes in the inequality moments from the 1990s to the 2030s (39 years after time $t_0 = 1993.5$) under this scenario are also reported in the second column of Table B.1. For example, the top 1% net worth share will increase by another 2.9% over the next 19.5 years from the 2010s to the 2030s, almost as much as its 3.9% increase over the 19.5 years from the 1990s to the 2010s. Even if the structural shifts are fully reversed going forward, the economy will only slowly revert to its lower 1990s levels of inequality (the original steady state of the model), over several decades of transition. The dashed lines in Figure 6 and the third column in Table B.1 depict a scenario under which the structural shifts that took place

[59]This version of the transition experiment assumes no change occurs in the aggregate share of labor income, which has recently experienced a significant decline in the data of 11% from the 1990s to the 2010s. Accounting for the latter change would further diminish the implied contribution of the increase in labor earnings inequality to the overall increase in net worth inequality.

[60]The half-life estimate is computed using the formula half-life = $\log(2)/\lambda_{ss}^E$, where λ_{ss}^E is the speed of transition λ_t^E from equation (69) evaluated at the new-steady-state level of the excess wealth growth rate of entrepreneurs, μ^E.

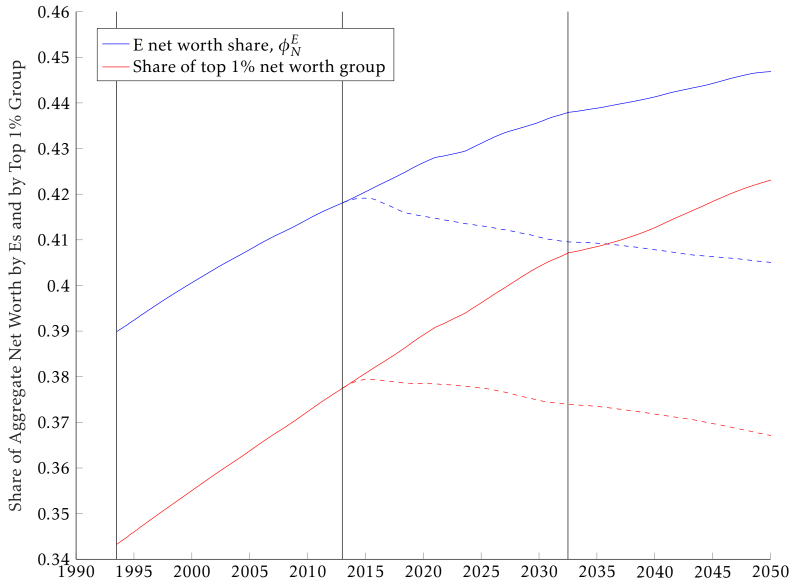

Figure 6: Evolution of Net Worth Share Held by Entrepreneurs and by the Top 1% Group in the Model Simulation

Notes: This figure plots the time paths of the aggregate net worth share of entrepreneurs and of the top 1% group by net worth in model simulations under the baseline calibration. The solid lines plot the time paths if the structural shifts are permanent, while the dashed lines plot the time paths if the structural shifts are fully reversed in the 2010s (after 19.5 years of transition following time $t_0 = 1993.5$).

at time $t_0 = 1993.5$ are fully reversed in $t_1 = 2013$ following 19.5 years of transition, so that inequality converges to its old steady state in the future. The simulation estimates show that it will take far more than 20 years for inequality to return to its lower 1990s levels.

4.5 The Robustness of High Entrepreneurial Persistence

The inference of a sizeable increase in the inside equity premium and of slow transition dynamics for the wealth distribution in the model calibration hinge critically on the high inferred cross-sectional persistence of entrepreneurial status, captured (inversely) by the type-switching rate ν^{NE} in the analytical results of Section 3.3. High entrepreneurial persistence is a robust feature of any empirically plausible calibration of the model, because the model can *jointly* match both top inequality (the top group net worth shares) and the (top-weighted) net worth share of entrepreneurs only under high persistence of entrepreneurial status across households (low ν^{NE}).

It is not immediately clear how the level of the return differential between entrepreneurs and non-entrepreneurs and the cross-sectional persistence of this differential can be disentangled using information from cross-sectional inequality moments, since both of these parameters directly affect the relative wealth of the two groups. To understand why the model identifies high entrepreneurial persistence, note from formula (29) that top-weighted average wealth, a measure capturing the degree of top wealth concentration, depends directly on the inside equity premium (through the terms μ^E-μ^{NE} and θ^E) but not on the churning rate ν^{NE}, after controlling for the schedule of entrepreneurs' top-weighted wealth share $\varphi^E(\zeta)$. Because this schedule is (approximately) observable, the degree of top concentration in the data can help identify the return differential separately from the degree of cross-sectional persistence.

If one imposes a low level of type persistence (high churning between types), the model can match the observed long-run level of top inequality or the observed long-run aggregate net worth share of the group of entrepreneurs, but not both. On one hand, a calibration that matches top inequality under low type persistence severely under-predicts the entrepreneurs' aggregate net worth share (and the entire top-weighted net worth share schedule, $\phi_N^E(\zeta)$). On the other hand, a calibration that matches the observed aggregate net worth share of entrepreneurs under low type persistence needs to assume an implausibly high level for the expected return differential between Es and NEs, implying counterfactually large levels of top inequality.

To better understand this important point, I conduct a simple comparative-statics analysis at the original (1990s) steady state of the model. Starting from the parametrization of the old steady state under the baseline calibration, I increase ν^{NE} while at the same time varying the inside premium π_Z so as to hold entrepreneurs' steady-state aggregate share of wealth ϕ^E (and of net worth, ϕ_N^E) constant. All other model parameters are held constant at their calibrated values for the 1990s. Figure 7 plots key variables as a function of the rate of entry into entrepreneurship, ν^{NE}. This entry rate varies from 0.01% to 1%, implying a corresponding variation in the exit rate from entrepreneurship, ν^E from 0.8% to 7.6%.[61] The bottom-left panel shows how the decline in type persistence translates into faster transitions, that is, a decline in the half-life implied by the asymptotic rate of convergence of the aggregate wealth share of entrepreneurs, λ^E. This half-life is 33 years in the old steady-state of the model, and it falls to under 20 years only for values of ν^{NE} greater than 0.5% (exit rates above 3.5%). Accounting for any increase in the inside equity premium relative to its 1990s level, as in the transition experiment of the baseline calibration, will imply even longer half-lifes for a given level of persistence.[62]

As can be seen in the top-right panel of Figure 7, such low levels of persistence (high churning) require implausibly high average excess returns Π_Z on entrepreneurial investments in order to hold the aggregate net worth share of entrepreneurs close its empirically observed level. In turn, these extreme return differentials imply counterfactually large levels

[61] The two churning rates are linked by the inflow-outflow balance condition (1).

[62] The relevant half-life for the transition experiment is the one computed under the new steady state dynamics and is significantly larger at 46 years, due to the fact that the convergence rate (half-life) is directly decreasing (increasing) in the excess wealth growth rate of entrepreneurs μ^E, and hence in the inside equity premium, for any given level of persistence, as can be seen in equation (38).

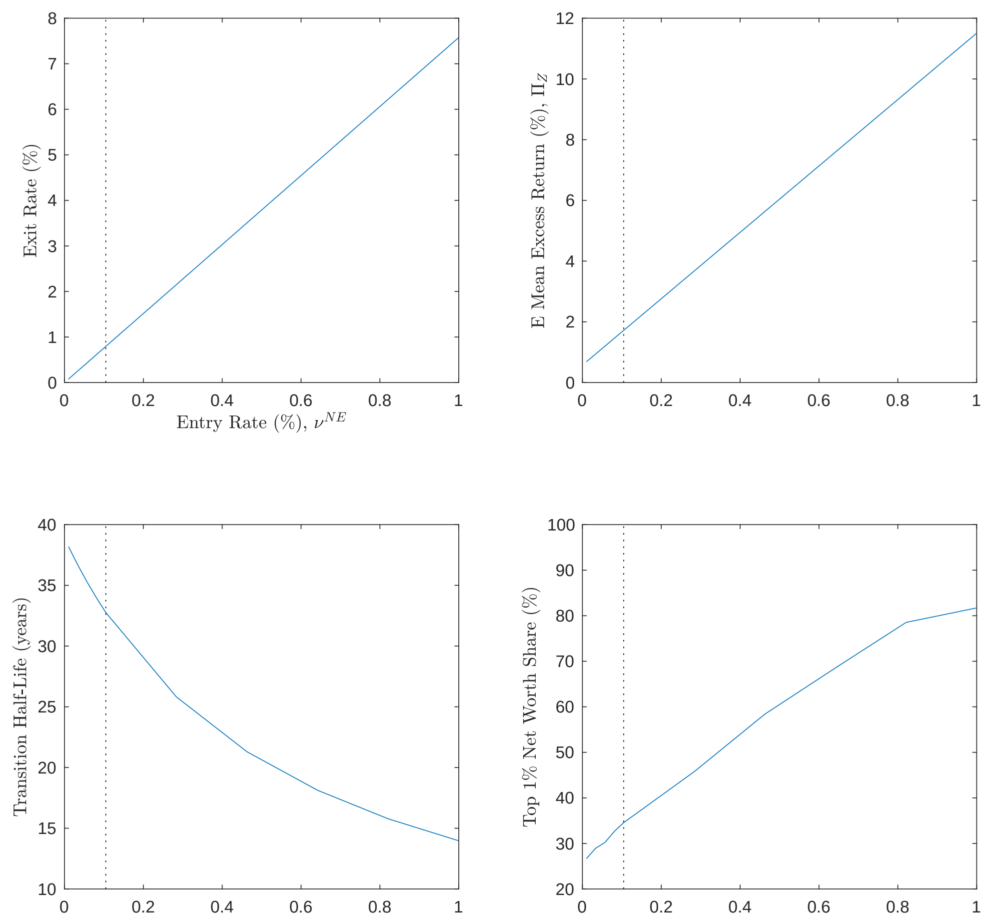

Figure 7: Comparative Statics Analysis of the Impact of Entrepreneurial Persistence

Notes: This figures plots key equilibrium objects in the steady state of the model as a function of the entry rate ν^{NE}, while also varying the inside equity premium π_Z starting from the original steady state of the baseline calibration (vertical dash line) in order to keep entrepreneurs' aggregate total wealth and net worth shares constant.

of top inequality when compared to their empirical counterparts from the 1990s, as can be seen in the bottom-right panel of Figure 7 plotting the steady-state level of the top 1% net worth share. For example, a value of $\nu^{NE} = 0.5\%$ implies a top 1% net worth share of 58%, compared to an estimate of 32% from the SCF data for the 1990s.

Multiple types A natural question that emerges from this analysis is whether the parsimonious nature of this two-type model is the reason behind its inability to generate fast transitions in response to changes in expected-rate-of-return heterogeneity while still matching quantitatively the key features of the structure of wealth inequality emphasized in this study. Would the introduction of more household types, for example in the form of heterogeneity in expected returns within the group of entrepreneurs, alter these predictions? Although I cannot provide a definitive answer to this question, the answer is negative in the case of a straightforward extension of this framework with multiple entrepreneurial types. The reason is that the rate of convergence of the aggregate wealth share of entrepreneurs depends on the average return on wealth across entrepreneurs (relative to non-entrepreneurs) but is independent of heterogeneity within the group of entrepreneurs. For the same fundamental reason, the convergence rate in equation (38) does not directly depend on the cross-sectional volatility of entrepreneurial returns θ_Z^E. Although high churning between different entrepreneurial types will likely speed up the convergence of the tail of the distribution in a model with multiple types relative to the two-type model of this paper, Proposition 3 shows that the convergence rate of the aggregate wealth share is an upper bound for the speed of convergence of the upper tail of the wealth distribution, as captured by the speed of convergence of the top-weighted wealth shares. Thus, the key prediction of slow transitional dynamics for wealth inequality survives in a multiple-type framework.

Evidence on entrepreneurial persistence Finally, existing empirical evidence lends support to the model's prediction of high persistence of entrepreneurial status ("entrepreneurs for life"). Low entry and exit rates to and from entrepreneurship have been documented empirically in the US. Using the 1984, 1989, and 1994 PSID surveys, Quadrini (2000) finds much higher rates of re-entry into entrepreneurial ventures by previous business owners relative to the general population, despite the high failure rates associated with private businesses. Moreover, recent empirical work by Fagereng et al. (2016) using panel tax data from Norway documents a high degree of persistence in the heterogeneity of rates of return on wealth across households. Although they do not directly address the persistence of entrepreneurial status, they show that their estimates of the cross-sectional persistence in return heterogeneity fall substantially when entrepreneurs are excluded from the sample.

4.6 The Drivers of the Increase in Return Heterogeneity

In this subsection, I investigate the drivers of the apparent increase in the average return differential between entrepreneurs and non-entrepreneurs. Section 4.6.1 offers micro-level evidence of an increase in the average returns to private businesses, conditional on their survival in private form, relative to returns on passive financial investments using SCF. Section 4.6.2 shows that there is no evidence of an increase in the aggregate risk taking of entrepreneurs relative to non-entrepreneurs, as captured by households' portfolio exposure in risky assets, which could account for the increase in the return differential between entrepreneurs and non-entrepreneurs. These results point to an increase in the average returns to entrepreneurial ventures as the most likely driver of the recent increase in entrepreneurial wealth.

4.6.1 Estimates of the average returns to private equity

This subsection offers an analysis of a cross-section of excess returns to entrepreneurial investments using SCF data at the household level. For each entrepreneurial household in the SCF (about 1,200 observations in the 2016 wave) I construct the long-term return to its business with the largest inflation-adjusted initial investment (cost basis) as the sum of a measure of annualized capital gains over the life of the business and an estimate of the average dividend yield of the business.[63] The construction of the capital gains component uses survey data on a household's initial investment on their business, the current market valuation of the business as estimated by the respondent, and business age. Because the SCF does not offer reliable information on the average dividend yield (distributed earnings as a share of firm value) of an individual business and to facilitate transparent comparisons over time, I use the time average (over all SCF waves) of an estimate of the aggregate dividend yield on inside private equity (8.9%) as an estimate of the dividend yield component of the return of each business. I subtract the geometric-average annual return of the S&P500 index over the life of the business to obtain an estimate of the annualized long-term return of the business in excess of the return to a liquid investment over the same period. The details of the construction and further analysis of this cross-section is presented in Appendix A.4.5.

Figure A.6 plots an estimate of the probability density function for the cross-sectional distribution of simple excess business returns from the 2016 SCF wave. The distribution displays extreme skewness and kurtosis, with a long right tail of extremely high return realizations (only partly displayed in the figure). The key limitation of this cross-sectional sample is that it only contains information on private businesses that are still in operation as

[63] The overwhelming majority of entrepreneurial households have a single business. For households with more than one business, I construct alternative measures that combine the returns to all businesses at the household level. The results from an analysis based on this alternative construction are very similar quantitatively.

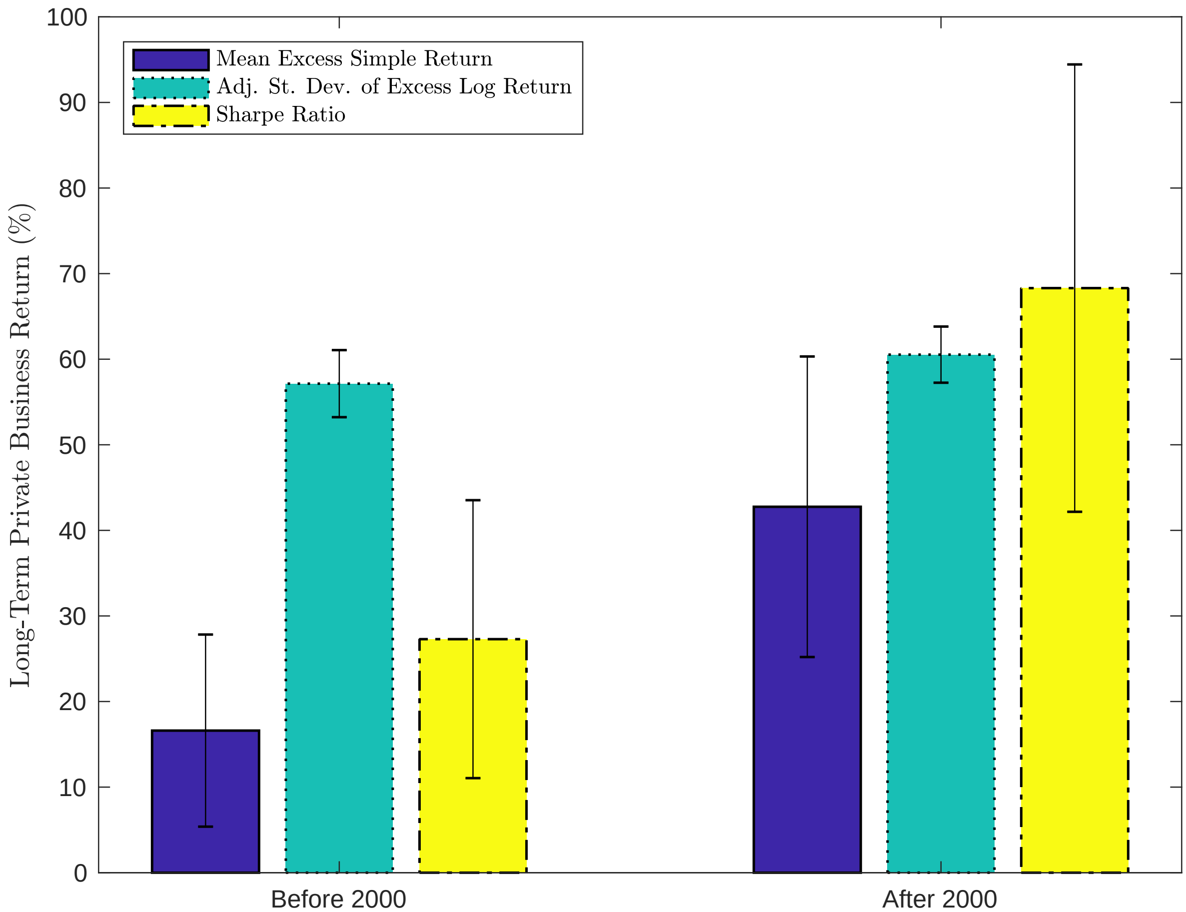

Figure 8: Cross-sectional Moments of Private Business Returns in the SCF, Conditional on Survival in Private Form

Notes: This figure plots averages across SCF waves before and after the year 2000 for the cross-sectional mean of the annualized excess simple returns and the cross-sectional standard deviation of the annualized excess log returns on private businesses, and for the ratio of these measures (Sharpe ratio). Annualized returns are adjusted by a factor of $\sqrt{T}$, where T denotes business age, when computing the standard deviation. The errorbars denote 95% confidence intervals. SCF population weights are used through the analysis.

private businesses at the time of each survey. It excludes businesses of surveyed households that have failed or have been liquidated, that have been acquired by other businesses, or that have gone public in the past. Accounting for firm failure, the most important source of risk for entrepreneurial ventures, would push down estimates of the average return and push up estimates of the volatility of returns,while accounting for firms that have been acquired or gone public would likely go in the other direction.A second limitation is that by construction the cross-sectional distribution of returns captures variation only in the capital gains component of returns but not in the dividend yield component.

Figure 8 reports the average levels of the cross-sectional mean of excess simple returns, the cross-sectional standard deviation of excess log returns across survey waves before and after the year 2000. For a comparable estimate of the volatility of returns across firms of different ages (investment horizons), annualized business returns are multiplied by a factor of $\sqrt{T}$, where T denotes business age in years, to compute an estimate of the standard deviation of one-year-ahead log returns. This is an appropriate benchmark because, in a model where one-period-ahead (excess) returns are identically and independently distributed conditional on survival, the standard deviation of the cumulative return grows with the square root of the horizon, so one would expect the term structure of the standard deviation of log returns constructed as such to be flat under that benchmark.Figure 8 also reports averages of the ratio of the cross-sectional mean over the adjusted cross-sectional standard deviation, which is an estimate of the Sharpe ratio of private business returns conditional on survival in private form.

The conditional cross-sectional mean of the excess simple returns has experienced a large increase in post-2000 surveys relative to pre-2000 survey waves, although the uncertainty in the estimates of average returns is large. In contrast, the conditional cross-sectional volatility of one-period-ahead log returns has not experienced a significant change since the 1990s. As a result, the ratio of these two cross-sectional measures (conditional Sharpe ratio) more than doubled from an average level of 27% in pre-2000 surveys to 68% in post-2000 surveys, a large and statistically significant increase of 41.0% (standard error 15.7%).[64] Although the magnitudes of these estimates are sensitive to details of the construction of returns, the qualitative conclusion of a substantial increase in conditional average returns and the conditional Sharpe ratio is robust across different constructions.

This analysis does not provide a definitive answer on the trends in the corresponding unconditional moments, after accounting for the full range of possible firm outcomes, including business failure, transition to public status, and acquisitions. For example, a possibility is that an increase in firm failure rates has occurred since the 1990s, biasing upward the reported estimates of average returns conditional on survival. However, recent empirical studies using panel data show a post-2000 decline in the (unconditional) firm-level idiosyncratic volatility of publicly traded stock returns relative to their level in the 1990s.[65] It would be surprising if the volatility of private firms followed an increasing trend since the 2000s, especially in light of the fact that it did not even share the increasing trend of public stocks

[64]Appendix Figure A.7 reports survey wave averages of the same measures for each decade (1990s, 2000s, and 2010s), and shows that the increase in average returns and the conditional Sharpe ratio is particularly pronounced during the 2000s.

[65]Brandt et al. (2010) show that the increasing trend in the firm-level idiosyncratic volatility of public stocks up until the late 1990s, emphasized by Campbell et al. (2001), was reversed by the early 2000s. In particular, the equally-weighted annualized idiosyncratic standard deviation of the simple returns to public stocks, estimated using daily data and the variance decomposition of Campbell et al. (2001) rose to 80% on average in the 1991-2000 period but came back down to around 60% in the 2000s, and remained in low pre-1990s level in the early 2010s, notwithstanding the temporary spike of the measure during the recent financial crisis. (Brandt et al., 2010; Lebedinsky and Wilmes, 2018).

during the 1980s and 1990s (Davis et al., 2006).[66]

The conclusions from the analysis above are consistent with estimates of the time variation in the aggregate returns to private equity in recent years. Kartashova (2014) repeats in an updated sample the methodology of MV2002 for estimating the premium earned by private equity over an index of publicly traded stocks using aggregate data from the SCF (aggregate dividend yield and change in the aggregate market value of private businesses). This methodology makes several adjustments at the aggregate level, including adjustments for firm births and for firm exits due to initial public offerings and mergers and acquisitions.[67] Kartashova (2014) finds that the aggregate returns to private equity in excess of the returns to public equity (the CRSP index of publicly traded stocks) were substantially higher in the 2000s relative to the 1990s. The conclusions are also consistent with those of Smith et al. (2017), who use U.S. administrative tax data linking pass-through firms to their owners and find that more than 80% of the increase from 2001 to 2014 in the income of S-corporations owned by individuals in the top 1% group by income is due to rising profitability per unit of scale (worker) rather than rising scale, strongly suggestive of an increase in the returns to these firms.

4.6.2 Aggregate risk taking by entrepreneurs and non-entrepreneurs

The model abstracts from sources of return heterogeneity other than entrepreneurship. In particular, agents in the model, which assumes homogeneous preferences across households and complete financial markets with respect to aggregate risk, choose an identical proportional wealth exposure θ_B to aggregate risk. In reality, there is large heterogeneity in the degree to which households take on aggregate risk implicitly or explicitly through their financial portfolios and nontradable assets (e.g. their labor income streams), with wealthier households being significantly more exposed to aggregate fluctuations than poorer households (see e.g. Parker and Vissing-Jorgensen (2009)). In light of the sizeable risk premia in financial assets (such as the U.S. stock market) in recent decades, this asymmetry is likely to be an important source of cross-sectional return heterogeneity. This begs the question of whether an increase in the aggregate risk exposure of entrepreneurs (i.e. private business owner-managers) relative to non-entrepreneurs since the 1990s has contributed to the apparent increase in the expected return differential between the two groups of households in

[66] Davis et al. (2006) show that the volatility of employment growth rates declined for private firms by about 50 percent between the early 1980s and the late 1990s, in contrast to the increasing trend for public firms over this period. They also estimate a ratio for the employment-weighted volatility of private firms relative to that of public firms of 1.6 in 2001.

[67] This measure of the private equity premium based on aggregate data has two important limitations. First, the return to the average single private business may be very different from the aggregate return on private equity, as pointed out by MV2002. Second, the time series for aggregate private equity returns is silent on changes in the riskiness of entrepreneurial investments.

recent years.[68]

To investigate this possibility, I examine some aspects of the portfolio allocation of entrepreneurs and non-entrepreneurs in the SCF data. Table A.5 reports the cross-sectional average share of household gross assets (net worth plus debt) invested in different types of assets across entrepreneurs and non-entrepreneurs, including within top groups by net worth. The asset categories are: inside private equity ("private equity" in the table), public equity, all equity, and all risky assets. Public equity includes all financial investments in publicly traded stocks, whether held directly, through mutual funds, through IRA accounts, or other accounts.[69] "All equity" is the sum of inside private equity, public equity, and outside (not actively managed) private equity. The measure of "all risky assets" adds housing, vehicles and "other non-financial" assets to equity.[70]

Table A.5 shows that entrepreneurs have significantly larger shares of their wealth invested in equity and risky assets in general than entrepreneurs on average, both across the population and within top groups. However, inside private equity accounts entirely for the larger total equity shares of entrepreneurs and, for top groups, there is a significant crowding-out of public equity by inside private equity in entrepreneurs' portfolios relative to those of non-entrepreneurs in a given wealth group. This crowding-out effect is consistent with the presence of a premium for inside private equity relative to public equity and need not imply differences in the risk tolerance between entrepreneurs and non-entrepreneurs. For example, the model of this paper makes this prediction, under the realistic assumption that private firms have comparable aggregate risk exposures to those of public firms.[71]

Examining the evolution of the portfolio share differentials between entrepreneurs and non-entrepreneurs, I do not find consistent evidence of an increase in the (aggregate) risk taking of entrepreneurs relative to non-entrepreneurs across SCF survey waves. In fact, the estimates of Table A.6 show that the positive difference between entrepreneurs' and non-entrepreneurs' portfolio share of equity (public plus private) appears to have declined slightly from the 1990s to the 2010s, including within top groups, although the changes are not statistically significant. There appears to be a meaningful increase in the risky asset share differential between the two groups within the top 1% group, which is completely due to housing and takes place from the 2000s to the 2010s. However, the increase is not

[68] An interesting possibility is that asymmetric aggregate risk taking can amplify the impact of an increase in the returns to entrepreneurial activity. For example, if privately-held equity has similar aggregate risk exposure to that of public equity and if there are frictions making it hard to short public equity, entrepreneurs may have to indirectly increase their aggregate risk exposure when scaling up their investments to private equity in response to an increase in the expected returns of the latter.

[69] I use the SCF Bulletin measure for household-level public equity throughout this analysis, but the conclusions are unchanged under other (narrower or broader) definitions.

[70] The remaining (non-risky) assets are mainly cash-like assets, bonds (directly and indirectly held), and retirement accounts.

[71] Recall that inside equity in the model should be interpreted as inside privately-held equity stripped of its systematic risk exposure.

statistically significant and is not reflected in the estimates for other top groups.

5 Conclusion

This paper has presented survey-based evidence pointing to an important contribution of the returns to entrepreneurs, identified empirically as owner-managers of private businesses, to the increase in U.S. wealth inequality and concentration at the top over the last three decades. Moreover, an analysis of a calibrated model of entrepreneurship and inequality shows that an increase in the average return premium to entrepreneurial investments can account for most of the recent increase in top wealth inequality and is likely to have a protracted impact on inequality in the future.

An open question for future empirical and theoretical work relates to the structural forces that have led to the apparent increase in the returns to entrepreneurial and managerial capital and whether these forces will remain relevant in the future. A conjecture is that these developments are related to the rising importance of intangible investments in the modern economy. Intangible investments, defined to include software development, R&D, and investments in process engineering, have overtaken tangible investments as a share of aggregate firm investment in most developed countries over the last three decades (Corrado, Hulten, and Sichel, 2006; Haskel and Westlake, 2018). These types of investment are by their nature less pledgeable and tied to performance of the entrepreneur or manager who undertakes them. Another possibility is that some entrepreneurs have been able to enjoy a lower effective tax rate on their business income by exploiting the differential tax treatment of pass-through legal forms of business organization in the U.S. since the 1980s.

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A Empirics Supplement

A.1 Additional Figures and Tables

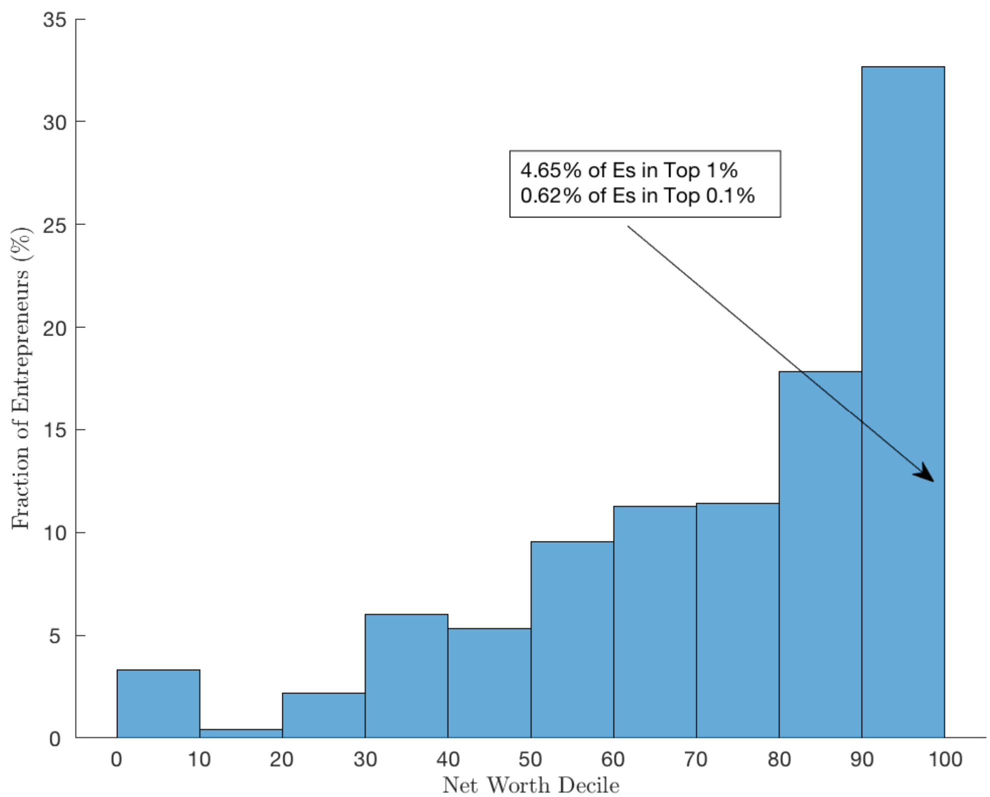

Figure A.1: Entrepreneurs Across the Net Worth Distribution

Notes: This figure plots the fraction of entrepreneurs who are in each decile of the net worth distribution across all households, using the 2016 SCF survey wave.

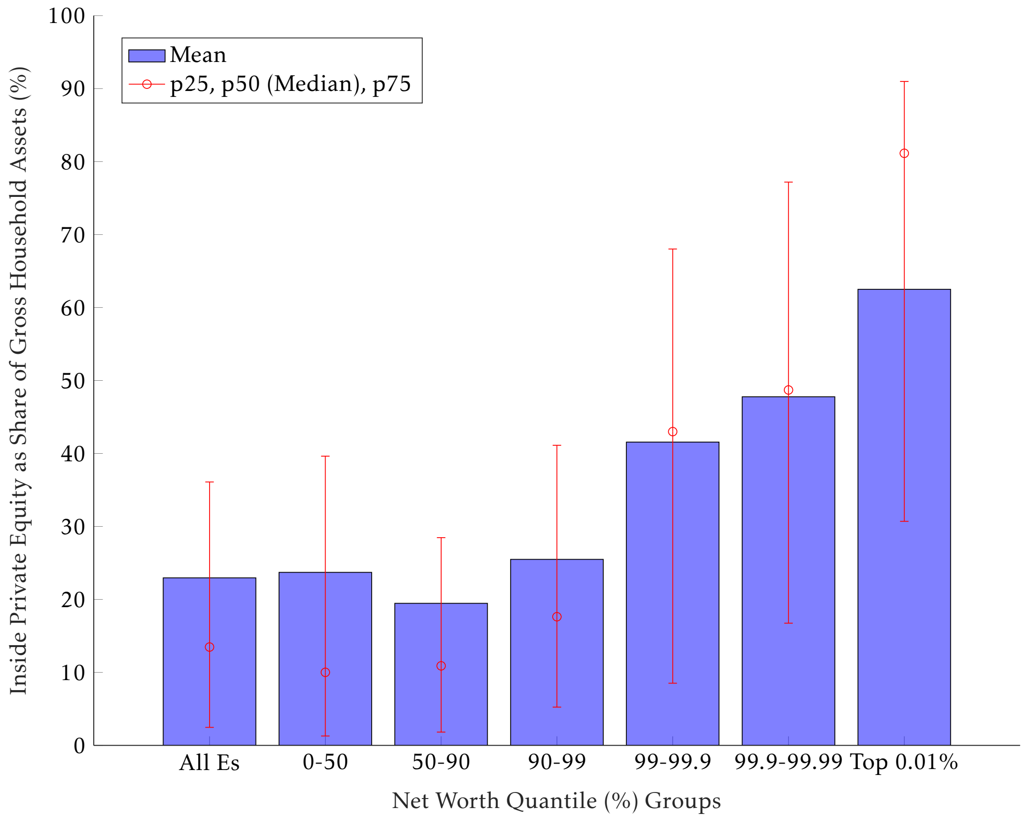

Figure A.2: Share of Inside Private Equity in Household Gross Assets Across Net Worth Quantiles

Notes: This figure displays moments (equally-weighted mean and the 25th, 50th, and 75th percentiles) of the cross-sectional distribution of the ratio of inside (i.e. actively-managed) privately-held equity over total household gross assets for different groups of entrepreneurial households. The data are from the 2016 SCF survey wave. “All Es” refers to the group of all entrepreneurs, while “0-50” refers to the group of entrepreneurs that are between the 0th and 50th percentile in terms of net worth, where the percentiles are defined across all US households. The remaining net worth quantile groups are defined similarly.

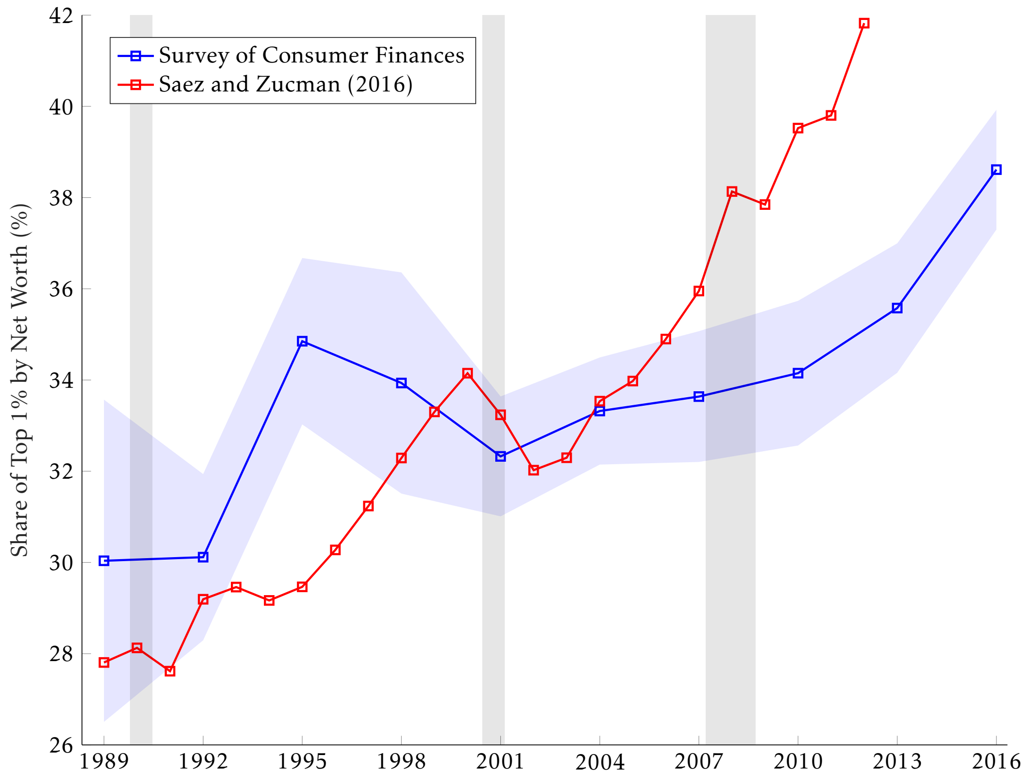

Figure A.3: Wealth and Income Top Inequality in the SCF

Notes: This figure plots the net worth share of the top 1% group by net worth over time, as measured in the SCF and also by the income-capitalization method of Saez and Zucman (2016). The colored bands represent 95% confidence intervals for the SCF estimates based on bootstrap standard errors (see Appendix A.4.3). Gray shaded areas denote NBER-designated recessions.

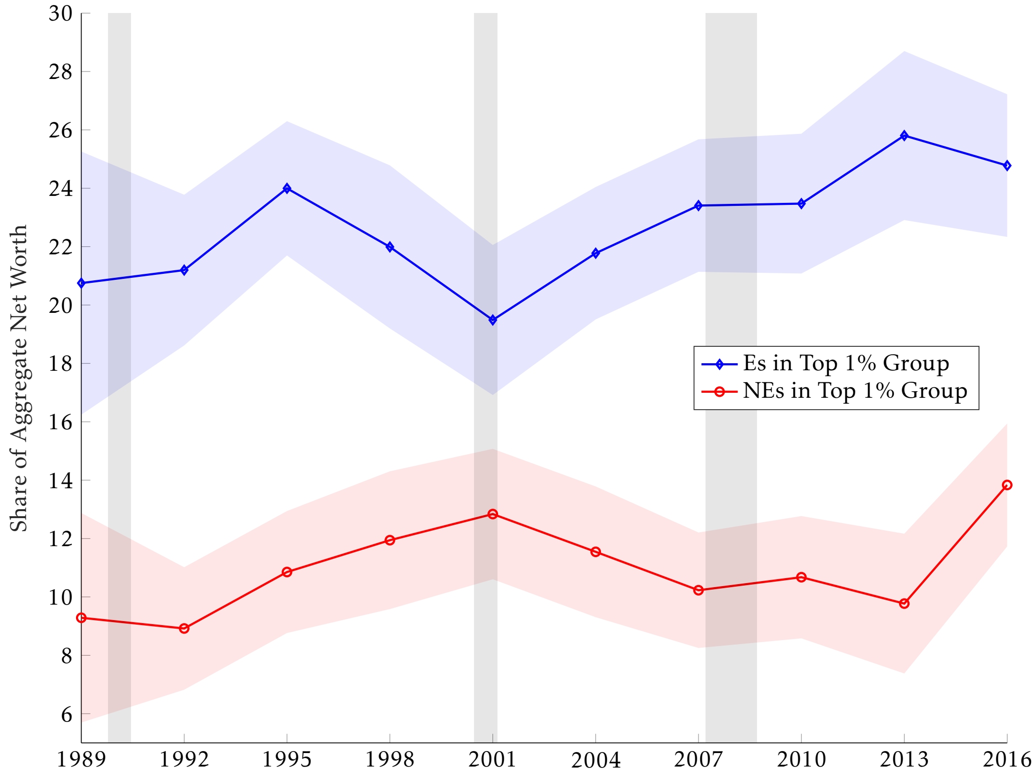

Figure A.4: Decomposition of Top 0.1% Net Worth Share by Entrepreneurial Status over Time

Notes: This figure plots the share of aggregate net worth held by the entrepreneurs (E) and non-entrepreneurs (NE) that are part of the top 0.1% group by net worth. The colored bands represent 95% confidence intervals for the SCF estimates based on bootstrap standard errors. Gray shaded areas denote NBER-designated recessions..

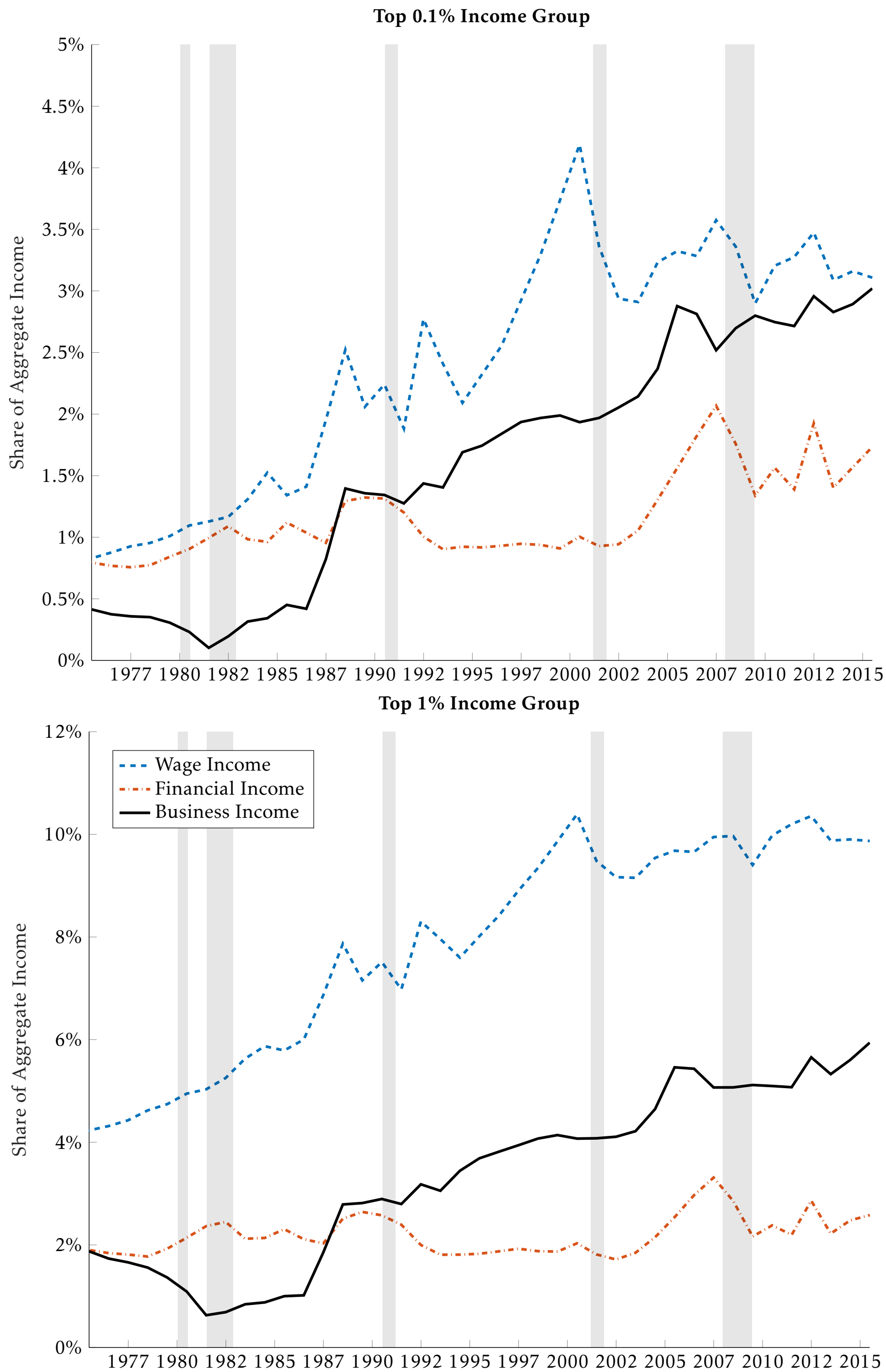

Figure A.5: Income Factor Decomposition for Top 0.1% and 1% Shares Over Time (Tax Data)
Notes: This figure plots components of the aggregate income of the top 0.1% income group (top panel) and the top 1% income group (bottom panel) since the 1970s. The data, based on IRS administrative tax data, and the classification of income based on categories (lines) in US tax returns are from an updated version of the dataset of Piketty and Saez (2003). The plotted categories do not include realized capital gains. Gray shaded areas denote NBER-designated recessions.

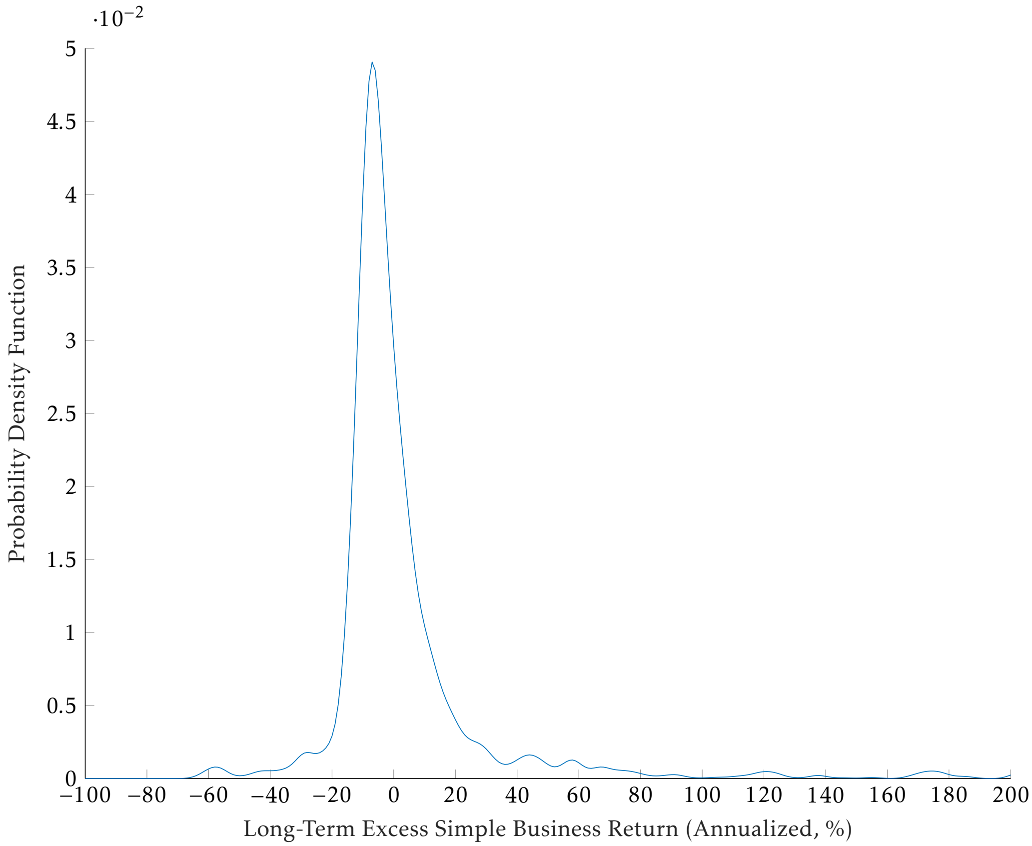

Figure A.6: Cross-Sectional Distribution of Annualized Long-Run Excess Simple Business Returns, 2016 SCF Wave

Notes: This figure plots the probability density function for annualized long-run excess simple private business returns, conditional on no firm exit (due to business failure, acquistion by another firm, or transition to a public firm), from the 2016 survey wave. The plot of the probability density estimate uses the unified dataset across all implicates and a normal kernel function (with bandwidth equal to 3).Only businesses with an inflation-adjusted cost basis of at least $5,000 and survey-year market value of at least $1,000 are considered. See Appendix A.4.5 for details on the construction of business-level returns.

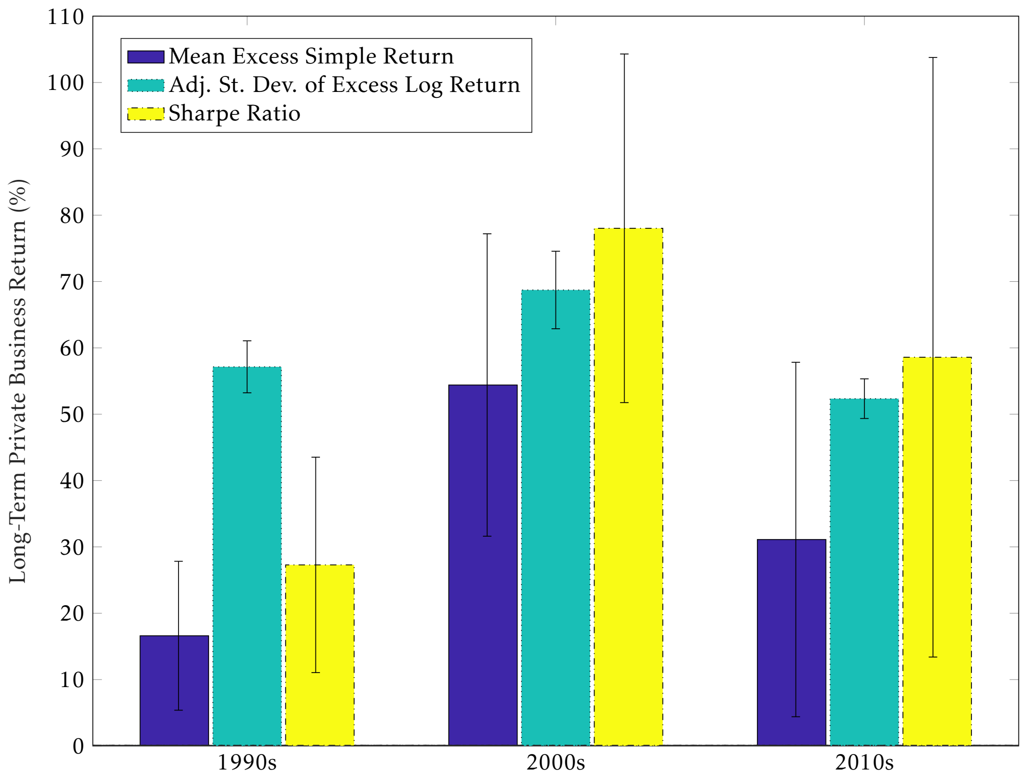

Figure A.7: Cross-sectional Moments of Excess Private Business Returns in the SCF Across Decades, Conditional on Survival in Private Form

Notes: This figure plots decade averages for the cross-sectional mean of the annualized excess simple returns and the cross-sectional standard deviation of the annualized excess log returns on private businesses in SCF, and for the ratio of these measures (Sharpe ratio). Annualized returns are adjusted by a factor of $\sqrt{T}$, where T denotes business age, when computing the standard deviation. The errorbars denote 95% confidence intervals.

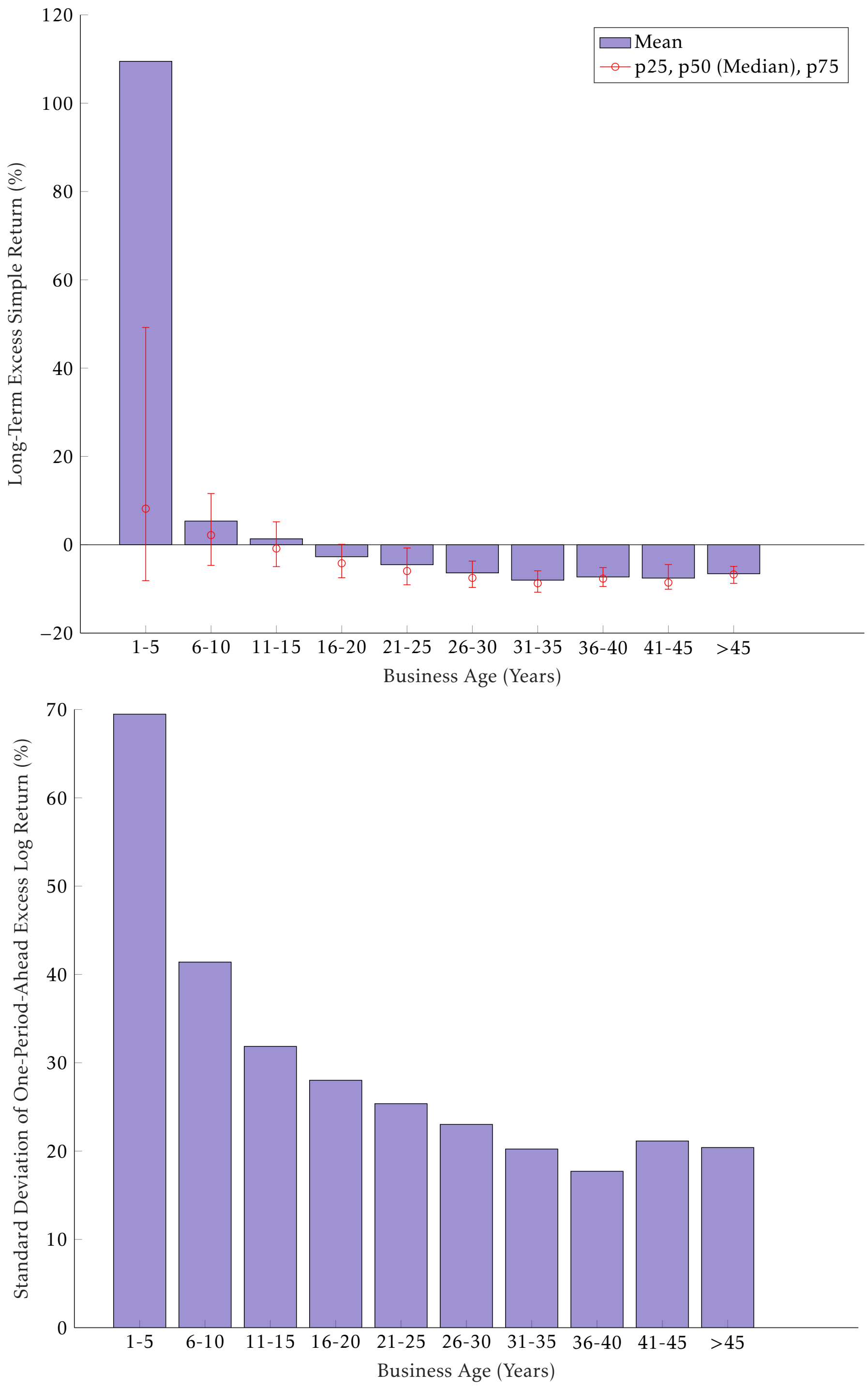

Figure A.8: Mean and Standard Deviation of Long-Run Excess Private Business Return Against Business Age, Conditional on Survival

Notes: This figure plots cross-sectional moments of annualized long-run excess private business returns for different business age groups at the time of the survey. The top panel plots the equally-weighted mean and the 25th, 50th, and 75th percentiles of excess simple returns, and the bottom panel plots the standard deviation of excess log returns. The reported moments are averages across the three SCF survey waves in the 2010s. Annualized returns are adjusted by a factor of $\sqrt{T}$, where T denotes business age, when computing the standard deviation. See Appendix A.4.5 for details on the construction of the long-run entrepreneurial business returns at the household level.

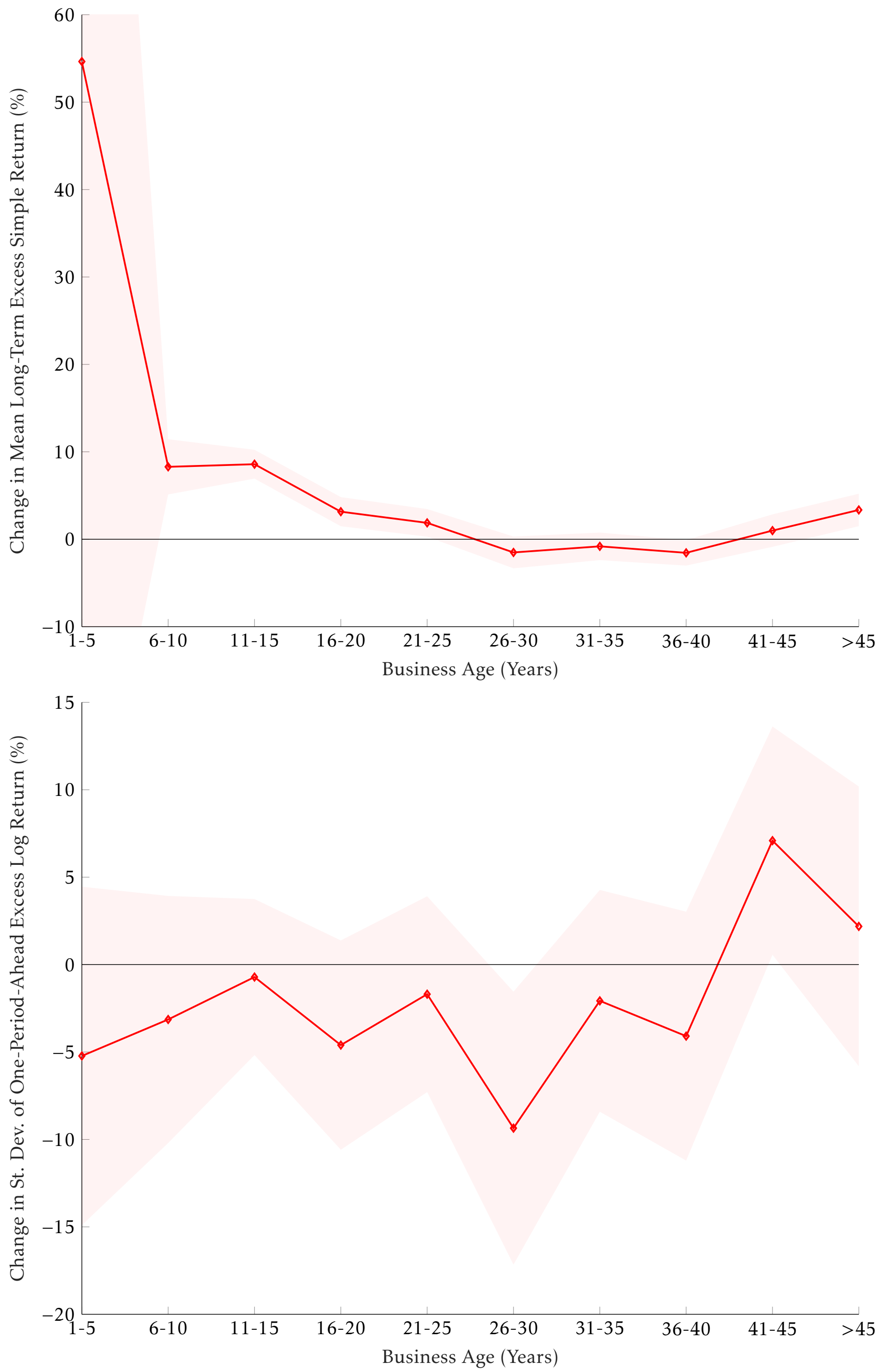

Figure A.9: Change in Mean and Standard Deviation of Long-Run Excess Private Business Return Against Business Age, Conditional on Survival (1990s to 2010s)
Notes: This figure plots the change from the 1990s to the 2010s in cross-sectional moments of annualized long-run excess private business returns for different business age groups at the time of the survey. The top panel plots the change in the cross-sectional mean of excess simple returns and the bottom panel plots the standard deviation of excess log returns. Annualized returns are adjusted by a factor of $\sqrt{T}$, where T denotes business age, when computing the standard deviation.

	Net Worth			Income		
	Top 1% Share	By Es	By NEs	Top 1% Share	By Es	By NEs
Level,	32.23	21.98	10.25	14.97	9.25	5.72
90s	(0.60)	(0.80)	(0.66)	(0.64)	(0.69)	(0.42)
Change,	0.86	-0.43	1.29	4.54	1.24	3.30
90s → 00s	(0.71)	(1.06)	(0.92)	(0.94)	(0.85)	(0.89)
Change,	3.02	3.13	-0.11	0.61	1.21	-0.60
00s → 10s	(0.56)	(1.03)	(0.90)	(0.89)	(0.74)	(0.98)
Change,	3.88	2.70	1.18	5.15	2.45	2.70
90s → 10s	(0.73)	(1.10)	(0.92)	(0.85)	(0.88)	(0.73)

Table A.1: Decomposition of the Increase in the Top 1% Net Worth and Income Shares By Group

Notes: This table reports the decomposition of the top 1% share (in percentage point format) and its increase by group, entrepreneurs (E) and non-entrepreneurs (NE), for net worth and income from the 1990s to the 2010s. The top 1% group is defined using household rankings with respect to net worth and income, respectively."Share by Es" refers to the share of aggregate net worth or income held by the entrepreneurs that are part of the top 1% group. For all rows, the components of each decomposition sum to the total inequality level or change. SCF standard errors are reported in parentheses.

	Net Worth			Income		
	Top 0.1% Share	By Es	By NEs	Top 1% Share	By Es	By NEs
Level,	12.02	8.79	3.24	6.03	4.17	1.86
90s	(0.41)	(0.48)	(0.35)	(0.58)	(0.57)	(0.28)
Change,	-0.47	-0.55	0.08	1.40	-0.47	1.87
90s → 00s	(0.50)	(0.61)	(0.49)	(0.78)	(0.67)	(0.67)
Change,	1.91	2.15	-0.24	0.23	1.09	-0.85
00s → 10s	(0.51)	(0.63)	(0.47)	(0.69)	(0.56)	(0.78)
Change,	1.44	1.60	-0.16	1.64	0.62	1.02
90s → 10s	(0.59)	(0.70)	(0.48)	(0.74)	(0.72)	(0.57)

Table A.2: Decomposition of the Increase in the Top 0.1% Net worth and Income Shares By Group

Notes: This table reports the decomposition of the top 0.1% share (in percentage point format) and its increase by group, entrepreneurs (E) and non-entrepreneurs (NE), for net worth and income from the 1990s to the 2010s. The top 0.1% group is defined using household rankings with respect to net worth and income, respectively."Share by Es" refers to the share of aggregate net worth or income held by the entrepreneurs that are part of the top 0.1% group. For all rows, the components of each decomposition sum to the total inequality level or change. SCF standard errors are reported in parentheses.

	T	$\widetilde{T}^E$	$\widetilde{T}^{NE}$	ϕ^E	T^E_{in}	T^{NE}_{in}
Net Worth	36.11	24.69	11.43	45.21	54.61	20.84
Non-Bus	30.30	16.44	13.85	33.67	48.82	20.84
Bus	63.72	63.72	0.00	100.00	63.72	
Income	20.12	11.70	8.42	27.43	42.55	11.62
Labor	13.55	5.30	8.26	22.45	23.62	10.67
Capital	35.63	27.33	8.30	40.36	67.68	13.93
Population	1.00	0.63	0.38	11.43	5.53	0.43

Table A.3: Top 1% Share Decomposition by Group

Notes: This table reports the decomposition of the value share of the top 1% by group, entrepreneurs (E) and non-entrepreneurs (NE), for several key variables, where the top 1% group is defined with respect to total net worth for rows 1-3 and 7 (net worth and its components, and population) and with respect to total income for rows 4-6 (income and its components). Business net worth refers to inside (actively-managed) private equity and non-business net worth refers to all other components of net worth. The decompositions for labor and capital income are reported after an adjustment for the unpaid labor of self-employed entrepreneurs, detailed in Appendix A.4.4. All estimates are averaged over the three latest survey waves (2010, 2013, 2016) and are reported in percentage point format.

	Top 1% Income Share							
	Total	Labor			Capital			Other
			by Es	by NEs		by Es	by NEs	
Level,	14.97	6.24	3.57	2.67	8.55	5.56	2.99	0.19
90s	(0.64)	(0.34)	(0.29)	(0.30)	(0.66)	(0.64)	(0.31)	(0.05)
Change,	4.54	2.44	0.42	2.02	2.19	0.89	1.29	-0.08
90s $\rightarrow$ 00s	(0.94)	(0.57)	(0.45)	(0.52)	(0.93)	(0.73)	(0.70)	(0.07)
Change,	0.61	0.36	-0.47	0.83	-0.21	1.60	-1.81	0.46
00s $\rightarrow$ 10s	(0.89)	(0.67)	(0.45)	(0.67)	(0.82)	(0.57)	(0.69)	(0.24)
Change,	5.15	2.80	-0.04	2.84	1.97	2.49	-0.52	0.38
90s $\rightarrow$ 10s	(0.85)	(0.60)	(0.41)	(0.59)	(0.83)	(0.78)	(0.42)	(0.24)

Table A.4: Decomposition of the Increase in the Top 1% Income Share by Factor and Group

Notes: This table reports the decomposition of the top 1% share and its increase by group, entrepreneurs (E) and non-entrepreneurs (NE), for both labor and capital income from the 1990s to the 2010s. Household-level estimates of labor and capital income are adjusted for the unpaid labor if self-employed entrepreneurs, as discussed in the text and detailed in Appendix A.4.4. The top 1% group is defined using household rankings with respect to total income.

	Entrepreneurs				Non-Entrepreneurs	
	Equity			Risky Assets	All Equity	Risky Assets
	Private	Public	All			
All	23.2	10.2	34.0	83.6	9.4	75.5
	(0.4)	(0.3)	(0.5)	(0.3)	(0.1)	(0.2)
Top 1%	38.3	17.7	59.5	84.3	42.7	68.5
	(1.8)	(1.2)	(1.5)	(1.0)	(2.0)	(1.9)
Top 0.1%	47.4	16.6	69.5	85.9	48.8	74.6
	(2.7)	(1.6)	(2.2)	(1.2)	(3.9)	(2.9)
Top 0.01%	53.6	18.3	75.2	86.2	70.6	82.8
	(4.8)	(3.5)	(3.2)	(2.9)	(6.2)	(4.6)

Table A.5: The Portfolio Allocation of Entrepreneurs and Non-Entrepreneurs, 2010s

Notes: This table reports the cross-sectional average of the fraction of household gross assets held in different types of risky assets for different groups of households. See the text (Section 4.6.2) for details on the asset categories. Top groups are defined using household rankings with respect to household net worth. All estimates are averaged over the three latest survey waves (2010, 2013, 2016) and are reported in percentage point format. SCF standard errors are reported in parentheses.

	Equity		Risky Assets	
	1990s	1990s → 2010s	1990s	1990s → 2010s
All	25.6	-1.1	6.9	1.3
	(0.6)	(0.7)	(0.6)	(0.7)
Top 1%	18.3	-1.5	11.3	4.6
	(3.2)	(4.0)	(2.5)	(3.3)
Top 0.1%	23.7	-2.9	12.4	-1.1
	(4.8)	(6.6)	(5.7)	(6.5)
Top 0.01%	22.0	-17.5	13.2	-9.8
	(7.3)	(10.2)	(7.8)	(9.6)

Table A.6: Change in Portfolio Share Differentials Between Entrepreneurs and Non-Entrepreneurs

Notes: This table reports the level in the 1990s and the change from the 1990s to the 2000s in the difference between entrepreneurs' and non-entrepreneurs' cross-sectional average of the fraction of household gross assets held in equity ("All Equity" in Table A.5) and in risky assets, for different groups of households by net worth. All estimates are reported in percentage point format. SCF standard errors are reported in parentheses.

A.2 Group Decomposition of the Top 1% Group for Several Variables

Table A.3 shows a decomposition of the top 1% share for several variables by entrepreneurial status:

$$\begin{aligned} T &= \widetilde{T}^{E} + \widetilde{T}^{NE} \\ &= \phi^{E} T_{\text{in}}^{E} + (1-\phi^{E}) T_{\text{in}}^{NE}, \end{aligned} \tag{40}$$

where T is the aggregate value share of the top 1% group, $\widetilde{T}^{E}$ ($\widetilde{T}^{NE}$) is the aggregate value share of the entrepreneurs (non-entrepreneurs) that belong to the top 1% group, ϕ^{E} is the aggregate value share of entrepreneurs (reported in Table 1), and T_{in}^{E} (T_{in}^{NE}) is the aggregate value held by entrepreneurs (non-entrepreneurs) in the top 1% group expressed as a share of the aggregate value of all entrepreneurs (non-entrepreneurs). The latter measure can be interpreted as a measure of concentration at the top *within* each subpopulation of households.[72]

The group decomposition for the 2010s, reported in Table A.3, offers several insights on top inequality. First, net worth and capital income are more concentrated at the top relative to labor income (and thus total income); the top 1% net worth (income) groups own 36% of aggregate net worth (capital income), while the top 1% income group owns only 14% of aggregate labor income. Inside private equity in particular is extremely concentrated at top with the entrepreneurs in the top 1% group by net worth holding 64% of aggregate inside private equity. (By definition, only entrepreneurs hold inside private equity.) Second, entrepreneurs within the top 1% groups hold a larger fraction of aggregate net worth and capital income relative to non-entrepreneurs (25% for Es relative to 11% for NEs), while non-entrepreneurs hold a larger fraction of labor income (8% for NEs relative to 5% for Es). Third, the degree of within-group top inequality as proxied by the measures T_{in}^{E} and T_{in}^{NE} is larger for the group of entrepreneurs for all variables. The results are quantitatively very similar for the unadjusted factor decomposition of income (see the discussion of Section and Appendix A.4.4).

A.3 Group and Factor Decomposition of Household Income

The trends in top income inequality are more complex relative to those for net worth inequality discussed in Section 2.3 due to the importance of labor income. An appropriately capitalized version of the total current and future labor income stream of households is a key component of the total economic wealth of households that is missing from the conventional measure of net worth (net assets). Table A.4 decomposes the top 1% income share by factor

[72] For all variables, the top 1% group is constructed using the rankings of households with respect to *total* net worth or income, not the rankings with respect to the corresponding wealth or income components.

and, additionally, by group within each factor. It shows that labor and capital income each account for roughly half (60% for labor and 40% for capital, excluding the "other income" category) of the increase in top income inequality since the 1990s.[73]

Further, Table A.4 shows that the increase in the labor income of the top 1% income group mainly occurred during the 1990s and almost exclusively through non-entrepreneurs. In contrast, the increase in capital income is exclusively due to entrepreneurs and has mostly taken place since 2000. Another important development is the collapse of the capital income of non-entrepreneurs following the Great Recession of 2007-2009, which resulted in an overall decline of the total income of non-entrepreneurs from the 2000s to 2010s (see the last column of Table A.1) and a slight decline in the total capital income of the top 1% group (see the fifth column of Table A.4). The capital income of entrepreneurs in the top group in fact fell by the same amount (2.3 percentage points of aggregate total income from the 2007 survey to the 2010 survey), but the decline was not enough to reverse the sustained increase in the capital income of this subgroup over all other three-year time periods from 2001 to 2013.

An analysis of US tax data also suggests that an important shift in the composition of top income inequality and its increase occurred around the early 2000s, as emphasized by Guvenen and Kaplan (2017). Figure A.5 plots the components of the income of the top 1% and 0.1% groups by income over time, using the classification of pre-tax income into wage income (wages, salaries, and bonuses, including exercised employee stock options), financial income (interest income, dividends, and rents) and business income (referred to as "entrepreneurial income" in Piketty and Saez (2003)), the latter category defined to include the profits of partnerships, sole proprietorships, and type-S (pass-through) corporations, and royalties. The secular trends based on tax data are qualitatively consistent with the results of the analysis based on the SCF. Figure A.5 shows that the growth in the wage income for the top groups has stopped or even reversed since 2000 (certainly so for the top 0.1% group), while business income has experienced an unabated and sustained increase since the 1980s up until the end of the data series in 2015, with a significant uptick in the early 2000s. Income classified as financial income also experienced a similar uptick in the 2000s, but that was reversed in large part during the Great Recession of 2007-2009, consistent with the decline in the capital incomes of both entrepreneurs and non-entrepreneurs in the SCF data discussed above.

[73]Table A.4 shows that the contributions of labor and capital income to overall income inequality are of comparable magnitudes, both in levels and in changes over time. This is perhaps surprising both for the level decomposition, given the much larger share of labor income in total income (70% in the 2010s), as well as the change decomposition, given the decline in the labor share. The explanation for the former observation is that capital income is in general much more unequally distributed than labor income, and the reason for the latter is that the decline in the labor share over the last 30 years was accompanied by a large increase in within-labor-income inequality.

Taking stock, the present analysis based on SCF data suggests a compositional change in top income inequality and its increasing trend, from within-labor-income inequality in the 1990s to within-capital-income inequality since the early 2000s, a change confirmed by existing analyses based on tax data. Moreover, the group decomposition of top income and net worth inequality strongly suggests that the increase in capital income and net worth inequality at the top is primarily due to entrepreneurs and thus due to increasing returns to actively-managed forms of capital investments, as opposed to "passive" financial investments, to which all households have access regardless of entrepreneurial status.

A.4 Additional Details on SCF Data Analysis

A.4.1 Sample Selection Design

The SCF employs a dual-frame sample selection design incorporating an area-probability sample, which provides good information on financial variables that are broadly distributed in the population, and a special list sample selected from a sample of recent tax records. The list sample oversamples households that are more likely to be wealthy, thus providing superior information on financial variables that are highly correlated with wealth, such as business assets, ownership of stocks, and real estate investments. As with other surveys, response rates for the SCF have fallen in recent years, a problem affecting the wealthy oversample in particular. Changes in the survey design have been implemented recently to address this issue (Bricker, Henriques, and Moore, 2017).

A.4.2 Bulletin Measures of Household Assets and Income

The SCF Bulletin measure of household gross assets is the sum of all financial assets owned by the households, including directly-held stocks and bonds, transaction accounts, mutual funds, life insurance, annuities, trusts, and quasi-liquid retirement (IRA and Keogh) accounts, and of the household's non-financial assets, which include real estate, vehicles, and actively and non-actively managed private business equity (including net loans from the household to its business). The net worth measure subtracts from the gross assets measure a measure of the value of the household's total debt, including mortgages, collateralized and non-collateralized loans, and credit card debt.

The net worth measure, constructed using all available information in the survey, misses two important components of US household wealth relative to the economically preferable concept of total non-labor wealth that includes all assets, net of liabilities, over which a family has a legal claim that can be used to finance its present and future consumption: defined-benefit (DB) pension wealth (which has grown in importance in recent years) and the wealth of the members of the Forbes 400 list of wealthiest individuals, which are by design excluded from the survey. Bricker et al. (2016) argue that accounting for these missing components via macro-level imputations leaves the usual measures of wealth concentration at the top of the distribution largely unchanged, with the addition of DB pension wealth pushing them down and that of the Forbes 400 wealth pushing them up.

SCF total income largely corresponds to the IRS measure of income, but also contains certain non-taxable sources of income.

A.4.3 Statistical Inference in the SCF

The SCF employs a multiple imputation technique to deal with the issue of missing or incomplete information in households' responses. In particular, five distinct observations called implicates are provided in the data for each household surveyed. Any missing values for the household are replaced with five values generated to simulate the conditional sampling distribution of the missing values.

To properly analyze the data, one must use repeated-imputation inference (RII) techniques (Rubin, 1987; Montalto and Sung, 1996). RII requires that separate analyses be performed on each of the five data sets corresponding to the five implicates. Specifically, the best point estimate for a statistic is constructed as the average of the five separate point estimates computed in each of the five data sets. An estimate of the total variance of the overall point estimate is given by the formula:

$$\sigma^2 = \sigma^2_{\text{within}} + \left(1 + \frac{1}{m}\right)\sigma^2_{\text{between}}, \tag{41}$$

where σ^2_{within} is an estimate of within-imputation variance, or an estimate of the sampling variance of the point estimate within an implicate dataset, and $\sigma^2_{\text{between}}$ is between-imputation variance defined as the sample variance of the set of the five point estimates. $1+1/m$, where $m = 5$ is the number of implicates, is an adjustment factor for using a finite number of implicates.

To obtain good estimates of within-imputation variance, capturing uncertainty due to sampling variability and taking into account the survey sample design, the SCF group at the Fed makes available a set of 999 bootstrap replicate weights for the first implicate of each raw observation. These weights are produced using the same sample selection techniques used to choose the actual survey sample.[74] The sampling variance of the point estimates across these replicates is then an estimate of σ^2_{within}.

Confidence intervals at the $100 \cdot \alpha$ significance level are constructed as

$$x \pm t_{\nu}(\alpha/2)\,\sigma, \tag{42}$$

[74] These techniques are not fully disclosed publicly, which is why the SCF group directly provides these replicate weights.

where x denotes the point estimates and $t_\nu(\alpha/2)$ is the $100 \cdot \alpha/2$ percentile of the student t distribution with $\nu = (m-1)(1+1/r_m)^2$ degrees of freedom, where

$$r_m = \frac{(1+1/m)\,\sigma^2_{\text{between}}}{\sigma^2_{\text{within}}} \tag{43}$$

is the relative increase in variance due to non-response.[75]

To compute statistics involving estimates from different survey waves, such as decade averages or relative changes over time, I use the standard Gaussian rules for the propagation of uncertainty, which assume that sampling errors are uncorrelated across survey waves.

A.4.4 Imputation of Labor Income for Self-Employed Entrepreneurs in the SCF

The starting point for the factor decomposition of pre-tax income at the household level are the reported components of pre-tax household income through a series of survey questions that closely follow key lines in the IRS personal tax return, Form 1040. In particular, my pre-adjustment measure of labor income is reported wage and salary income (which includes bonuses). My pre-adjustment capital income consists of the following categories: sole proprietorship and farm income; rental real estate, royalties, partnerships, S corporations, trusts etc.; realized net capital gains; tax-exempt and taxable interest income; dividend income, ordinary and qualified; Social Security, pensions, annuities etc. A third category of "other income" includes the remaining categories, mainly transfer income: unemployment and worker's compensation; child support, alimony; welfare transfers (TANF, SNAP tec.); any other sources.

I perform a regression-based imputation of the labor income of self-employed entrepreneurs who report that they are not paid regular wages. The method follows Moskowitz and Vissing-Jørgensen (2002) and is similar in spirit to the methodology of the Bureau of Labor Statistics for calculating its estimate of the labor share. It makes use of the work hours in a year reported for all households in the SCF, including the unpaid entrepreneurs, and a predicted estimate of the hourly wage of the unpaid entrepreneurs based on a regression of the log hourly wage of employed households (separately for the household head and partner) on demographic characteristics (age, education, and gender).

My post-adjustment measure of labor (capital) income is wage (capital) income plus (minus) the imputed labor income of self-employed entrepreneurs reporting zero wage income.

My estimate of the labor share (pre- or post-adjustment) is aggregate labor income divided by the sum of aggregate labor income and aggregate capital income, that is, I ignore the "other income" category.

[75] For most statistics in the SCF the degrees of freedom are large enough that the corresponding t distribution essentially coincides with the standard normal.

A.4.5 Construction and Further Analysis of Long-Term Private Business Returns in the SCF

For each household, I consider its primary business with the largest inflation-adjusted cost basis. I construct the annualized long-term return to this business as the sum of a measure of annualized capital gains over the life of the business and an estimate for the average dividend yield of the business.

For the latter return component, I use the time average (across survey waves) of the aggregate dividend yield of inside private equity, which is 8.9%. The aggregate dividend of inside private equity for a given survey year is constructed as the aggregate net income (profit) of all private businesses, adjusted for the share of each business owned by a managing household, for corporate taxation,[76] for retained earnings (a uniform 30% earnings retention rate for all businesses), and for the imputed labor income of self-employed entrepreneurs who do not report regular wages (see Appendix A.4.4).[77]

To construct a measure of the capital gains component of the return I take the reported cost basis as the value of the original investment and the reported current market value of the business (multiplied the share owned by the household) as the current price. I then compute the geometric-average annualized capital gains rate taking into account the age of the business. Businesses with an inflation-adjusted cost basis of less than $5,000 or survey-year market value of less than $1,000, in 2016-equivalent US dollars, are excluded from the sample throughout the analysis in order to avoid data outliers due to artificially large capital gains or extremely negative log returns (for businesses with near-zero survey-year market value), respectively.

Appendix Figures A.8 and A.9 display levels and changes in conditional cross-sectional moments for different age groups of firms. Figure A.8 displays the term (firm age) structure of estimate moments of one-year-ahead private business returns for 5-year age groups, while Figure A.9 plots changes in these moments from their average levels in the 1990s to their average levels in the 2010s. The top panel of Figure A.9 shows that the increase in average returns is mainly driven by the youngest firms (up to 20 years old), confirming that the increase is a relatively recent phenomenon.[78] The bottom panel of Figure A.9 shows that the change in the estimated standard deviation from the 1990s to the 2010s is not statistically significant at the 5% level for almost all age groups and the estimate of the change is negative

[76] I assume a uniform 30% corporate tax rate if the business is a non-S corporation. I impute the legal classification of non-primary businesses (in the SCF the legal form is reported only for the first two or, prior to 2010, the first three businesses) based on the legal classification of the primary businesses. If the legal classification of the primary businesses is "mixed", that is, if at least one business is pass-through and at least another one is a non-S corporation, I assume all remaining businesses are non-S corporations.

[77] All income data in the survey are in fact for the year preceding the survey year.

[78] On average across the three survey waves in the 2010s, about 29% of firms in the sample are of age 1-5 years, 19% are of age 6-10, 15% are of age 11-15, 10% are of age 16-20, 8% are of age 21-25, and the remaining 19% are older than 25 years.

for most age groups. Similar conclusions follow when looking at other indicators of the riskiness of the return distribution, such as higher cross-sectional moments.

B Model Supplement

This appendix contains supplemental material on the model of Sections 3 and 4. Section B.1 contains additional figures and tables related to the analysis of Sections 3 and 4. Section B.2 contains additional exposition, details, and results on the model of Section 3. The proofs of the lemmas and propositions of Sections 3 and B.2 are located in a separate appendix, D.

B.1 Additional Figures and Tables

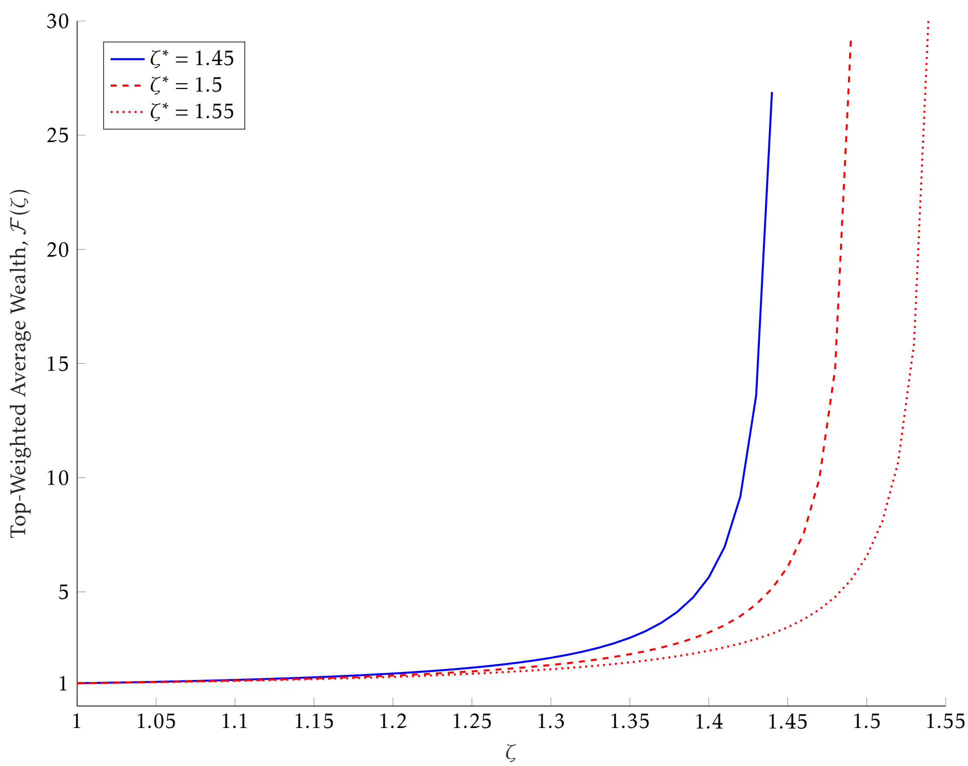

Figure B.1: Top-Weighted Average Wealth When Wealth Follows An Exact Pareto Distribution

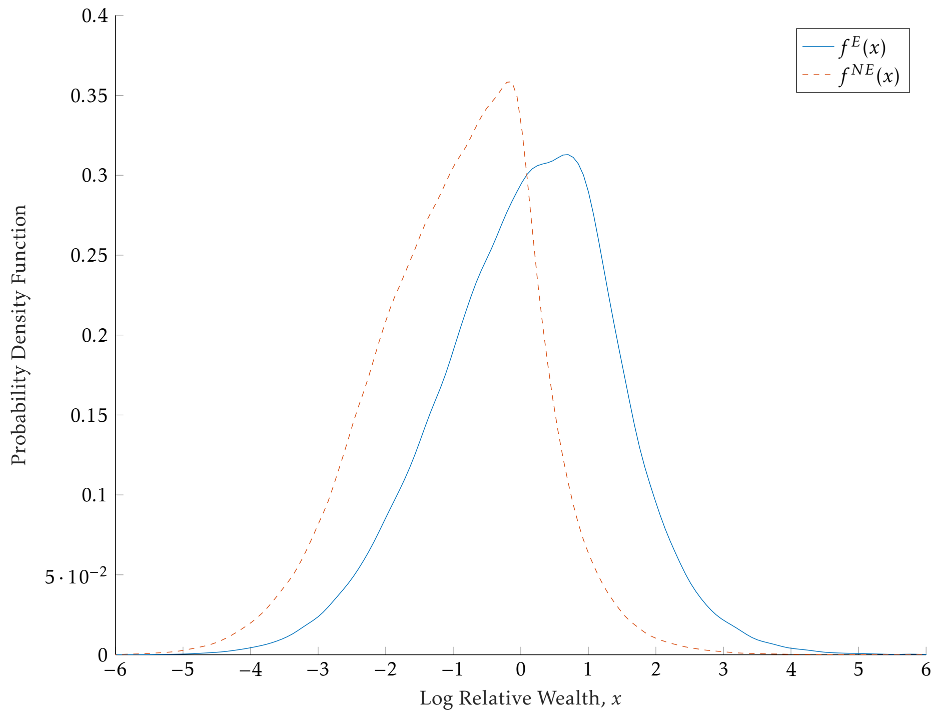

Figure B.2: The Cross-sectional Distribution of Log Relative Wealth in the Model

Notes: This figure plots the scaled cross-sectional distributions (probability density functions) for the log relative wealth of Es and NEs in the original steady state of the model, calibrated to fit inequality moments from the 1990s (see Section 4).

	1990s → 2010s	1990s → 2030s no reversal	1990s → 2030s reversal
E share, ϕ_N^E	+2.8	+4.7	+1.7
Top-wgt E share, $\phi_N^E(1.3)$	+3.6	+6.4	+1.8
Top 1% share	+3.4	+6.3	+2.8
Top 0.1% share	+1.8	+3.7	+1.5
Top 10% share	+3.7	+5.8	+2.3

Table B.1: Future Change in Inequality Under the Baseline Model Calibration

Notes: This table reports the cumulative change in the top inequality moments targeted in the calibration from the 1990s to the 2030s (39 years of transition) under two scenarios: fully permanent structural shifts (column 2); and full reversal of the shifts at time $t_1 = 2013$.

B.2 Supplement for Section 3

B.2.1 Optimal Agent Policies

Proposition B.1 (Optimal Policies)**.** *Let $U^s = (\exp(v^s)W)^{1-\gamma}/(1-\gamma)$ denote the equilibrium lifetime utility of an agent of type $s \in \{E, NE\}$ with wealth W.*

Then the optimal consumption and bequest (annuity) policies are characterized by consumption-wealth ratios $c^s \equiv C_{it}/W_{it}$ and bequeathed-to-surviving-wealth ratios $w_D^s \equiv W_{Dit}/W_{it} = (1-\tau_D)(1-\theta_D^s)$ given by: ,

$$c^s = (\rho+\omega)^{\psi}\exp\left((1-\psi)v^s\right), \tag{44}$$

$$w_D^s = \left(\frac{1-\tau_D}{1-\tau}V_D\right)^{\widetilde{\psi}}\exp\left((1-\widetilde{\psi})v^s\right). \tag{45}$$

The optimal wealth exposure to the (after-tax) liquid financial asset return is the same for all agents,

$$(1-\tau)\theta_B = \frac{\pi_B}{\gamma}, \tag{46}$$

and the optimal wealth exposure of entrepreneurs to the (after-tax) inside equity return is

$$\theta^E = (1-\tau)\theta_Z^E = \frac{\pi_Z}{\gamma}. \tag{47}$$

B.2.2 Wealth Dynamics

This section states the results for the general case with bequests, $V_D > 0$ and thus $w_D(s) > 0$ for $s \in \{E, NE\}$.

The aggregate wealth growth rate g_t and the average wealth growth rate across surviving households $\widetilde{g}_t$ satisfy:

$$\widetilde{g}_t = \varphi_t^E \mu_W^E + (1-\varphi_t^E)\mu_W^{NE} \tag{48}$$

$$g_t = \widetilde{g}_t - \omega\left(1 - w_L - w_{Dt}\right), \tag{49}$$

where $w_{Dt} \equiv \varphi_t^E w_D^E + (1-\varphi_t^E) w_D^{NE}$.

The log relative wealth of surviving E and NE households evolves according to

$$dw_{it}^E = \left(\widetilde{\mu}_t^E - \left(\theta^E\right)^2/2\right)dt + \theta^E dZ_{it} \tag{50}$$

$$dw_{it}^{NE} = \widetilde{\mu}_t^{NE} dt, \tag{51}$$

where

$$\widetilde{\mu}_t^E = \mu_W^E - g_t = \mu_t^E + \omega\left(1 - w_L - w_{Dt}\right) \tag{52}$$

$$\widetilde{\mu}_t^{NE} = \mu_W^{NE} - g_t = \mu_t^{NE} + \omega\left(1 - w_L - w_{Dt}\right). \tag{53}$$

As in the main text, $\mu_t^E = \mu_W^E - \widetilde{g}_t$ and $\mu_t^{NE} = \mu_W^{NE} - \widetilde{g}_t$ are the mean excess wealth growth rates of surviving Es and NEs, respectively, relative to the average growth rate of all surviving households.

The log relative wealth of a newborn household i of type s_i with permanent log relative labor earnings draw $\underset{\sim}{l}_i$ whose parent household j has log relative wealth w_j at the time of its death, is given by

$$\underset{\sim}{w}_{it} = h(w_j, \underset{\sim}{l}_i, s_i) \equiv \log\Big(w_D(s_i)\exp(w_{jt}) + w_L \exp(\underset{\sim}{l}_i)\Big). \tag{54}$$

Standard arguments imply that the dynamics of the cross-sectional distributions of log relative wealth across E and NE households, $f^E(w)$ and $f^{NE}(w)$, satisfy the following *Forward Kolmogorov equations*:

$$\frac{df^E(w)}{dt} = -(\widetilde{\mu}_t^E - \frac{1}{2}(\theta^E)^2)f^{E\prime}(w) + \frac{1}{2}(\theta^E)^2 f^{E\prime\prime}(w) + \nu^{NE} f^{NE}(w) - \Big(\nu^E + \omega\Big) f^E(w) + \omega B^E\{f^E\}(w) \tag{55}$$

$$\frac{df^{NE}(w)}{dt} = -(\widetilde{\mu}_t^{NE}) f^{NE\prime}(w) + \nu^E f^E(w) - (\nu^{NE} + \omega) f^{NE}(w) + \omega B^{NE}\{f^{NE}\}(w), \tag{56}$$

where the functional operator B^s for $s \in \{E, NE\}$ is defined as

$$B^s\{f^s\}(w) = \begin{cases} f_{\underset{\sim}{l}}^s\left(w - \log(w_L)\right) & \text{if } w_D(s) = 0 \\ \int_{l < w - \log(w_L)} \frac{\exp(w - \widetilde{h}(w,l,s))}{m^s w_D(s)} f^s\Big(\widetilde{h}(w,l,s)\Big) f_{\underset{\sim}{l}}^s(l)\,dl & \text{if } w_D(s) > 0. \end{cases} \tag{57}$$

Here, $m^{NE} = 1 - m^E$, and function $\widetilde{h}(w,l,s)$ is the inverse function of $h(w,l,s)$ (with respect to the argument w),

$$\widetilde{h}(w,l,s) \equiv \log\left(\exp(w) - w_L \exp(l)\right) - \log(w_D(s)). \tag{58}$$

The Forward Kolmogorov equations for the equilibrium labor earnings distributions are:

$$\frac{df_l^E(l)}{dt} = \nu^{NE} f_l^{NE}(l) - (\nu^E + \omega) f_l^E(l) + \omega f_{\underset{\sim}{l}}^E(l) \tag{59}$$

$$\frac{df_l^{NE}(l)}{dt} = \nu^E f_l^E(l) - (\nu^{NE} + \omega) f_l^{NE}(l) + \omega f_{\underset{\sim}{l}}^{NE}(l). \tag{60}$$

For notational simplicity, I omit the dependence of the cross-sectional distributions on time in equations (55)–(57) and (59)–(60), as I also do in equations (90)–(91) describing the law of motion for top-weighted average wealth.

Lemma B.1 (Labor Earnings Inequality in Steady State)**.** *In steady state, the equilibrium distribution of log relative labor earnings across all surviving households, $f_l(l)$, coincides with that of log relative labor earnings across newborn households, $f_{\underset{\sim}{l}}(l)$.*

The steady-state top-weighted labor earnings share of Es is given by

$$\varphi_L^E(\zeta) = m^E \frac{\omega \varphi_{\underset{\sim}{L}}^E(\zeta)/m^E + \nu^{NE}/m^E}{\omega + \nu^{NE}/m^E}, \tag{61}$$

where $\varphi_{\underset{\sim}{L}}^E(\zeta) \equiv \int \exp(\zeta l) f_{\underset{\sim}{l}}^E(l)dl / \int \exp(\zeta l) f_{\underset{\sim}{l}}(l)dl$ is the top-weighted labor earnings share of newborn Es relative to all newborn households.

Although distributions $f_l(l)$ and $f_{\underset{\sim}{l}}(l)$ coincide in steady state, this is not true in general for the type-specific distributions, $f_l^s(l) \neq f_{\underset{\sim}{l}}^s(l)$ for $s \in \{E, NE\}$, because of churning between the two types.

Proposition B.2 (Top-Weighted Moments)**.** *Consider the general case with bequests. In steady state, the top-weighted aggregate wealth share of entrepreneurs $\varphi^E(\zeta)$ and top-weighted average wealth $\mathcal{F}(\zeta)$ satisfy*

$$\varphi^E(\zeta) \approx m^E \frac{w_L^\zeta(\mathcal{F}_l(\zeta)/\mathcal{F}(\zeta))(\omega + \nu^{NE}/m^E)\varphi_L^E(\zeta)/m^E + \left(1 - w_L^\zeta(\mathcal{F}_l(\zeta)/\mathcal{F}(\zeta))\right)\nu^{NE}/m^E}{\lambda^E(\zeta)} \tag{62}$$

and

$$\mathcal{F}(\zeta) \approx \frac{\omega w_L^\zeta}{\lambda(\zeta)} \mathcal{F}_l(\zeta), \tag{63}$$

where

$$\lambda^E(\zeta) \equiv \omega\left(1 - \zeta(1 - w_L - w_{Dt}) - (w_D^E)^\zeta\right) + \nu^{NE}/m^E - \mu^E\zeta - (\theta^E)^2\zeta(\zeta - 1)/2 \tag{64}$$

$$\lambda^{NE}(\zeta) \equiv \omega\left(1 - \zeta(1 - w_L - w_{Dt}) - (w_D^{NE})^\zeta\right) + \nu^{NE}/m^E - \mu^{NE}\zeta, \tag{65}$$

and

$$\begin{aligned}\lambda(\zeta) &\equiv \lambda^E(\zeta)\varphi^E(\zeta) + \lambda^{NE}(\zeta)(1 - \varphi^E(\zeta)) - \nu^{NE}/m^E \\ &= \omega\left(1 - \zeta(1 - w_L - w_{Dt}) - \varphi^E(\zeta)(w_D^E)^\zeta - (1 - \varphi^E(\zeta))(w_D^{NE})^\zeta\right) \\ &\quad - \mu^E \frac{\varphi^E(\zeta) - \varphi^E}{1 - \varphi^E}\zeta - \varphi^E(\zeta)\left(\theta^E\right)^2 \zeta(\zeta - 1)/2.\end{aligned} \tag{66}$$

Equations (62) *and* (63) *hold exactly for* $\zeta = 1$ *and approximately so for* ζ *close to 1.*

Proposition B.3 (Pareto Tail in Steady State)**.** *Consider the general case with bequests. The results of Proposition 2 still hold, but with functions* $\lambda^E(\zeta)$ *and* $\lambda^{NE}(\zeta)$ *defined as in equations* (64) *and* (65).

Proposition B.4 (Speed of Transition)**.** *Consider the general case with bequests. In response to permanent and unanticipated structural shifts, the top-weighted wealth share of Es evolves approximately as*

$$\frac{d\varphi^E(\zeta)}{dt} \approx -s_t^E(\zeta)\varphi_t^E(\zeta) + \nu^{NE} + \omega w_L^\zeta \mathcal{F}_{\underset{\sim}{l}}^E(\zeta)/\mathcal{F}_t(\zeta), \tag{67}$$

with approximate speed of transition

$$s_t^E(\zeta) = \lambda_t^E(\zeta) + \omega w_L^\zeta \mathcal{F}_{\underset{\sim}{l}}(\zeta)/\mathcal{F}_t(\zeta) - \lambda_t(\zeta), \tag{68}$$

where $\lambda^E(\zeta)$ *and* $\lambda(\zeta)$ *are given in equations* (64) *and* (66).

The convergence rate for Es' aggregate wealth share is:

$$\lambda_t^E \equiv \lambda_t^E(1) = \omega\Big(w_L - (1-\varphi_t^E)(w_D^E - w_D^{NE})\Big) + \nu^{NE}/m^E - \mu_t^E, \tag{69}$$

which is decreasing in entrepreneurial persistence and in the mean excess wealth growth rate of entrepreneurs, μ_t^E.

C Epstein-Zin Preferences with Bequests: Characterization and Derivation

This appendix introduces an extension to the continuous-time version of Epstein-Zin (EZ) preferences (the Kreps-Porteus case of the stochastic differential utility class of Duffie and Epstein (1992)) that allows for utility from bequests in settings where agents are subject to random times of death. Section C.1 introduces the specification, Section C.2 formally derives it as the continuous-time limit of a natural extension of discrete-time EZ preferences. The optimal policies implied by this specification for the setting of the model of Section 3 are given in Proposition B.1.

C.1 Preference Specification

Households die stochastically at a Poisson rate ω. They derive utility from their own consumption stream as well as from the level of wealth bequeathed to their offspring.

The lifetime utility U_{it} of household i over its consumption in case of survival $[C_{i\tau}]_{\tau\geq t}$

and its bequeathed wealth in case of death $[W_{Di\tau}]_{\tau \geq t}$ is defined recursively through

$$U_{it} = E_t\left[\int_t^{\infty} f\left(C_{i\tau}, U_{Di\tau}, U_{i\tau}\right) d\tau + \lim_{\tau\to\infty} U_{i\tau}\right], \tag{70}$$

where felicity function f is given by (under γ, ψ, $\widetilde{\psi} \neq 1$)

$$f\left(C, U_D, U\right) = \frac{1}{1-1/\psi}\Bigg[(\rho+\omega)C^{1-1/\psi}\left((1-\gamma)U\right)^{1-\vartheta} + \omega\frac{\vartheta}{\widetilde{\vartheta}}\left((1-\gamma)U_D\right)^{\widetilde{\vartheta}}\left((1-\gamma)U\right)^{1-\widetilde{\vartheta}} \\ -\left(\rho+\omega\left(1+V_D\frac{\vartheta}{\widetilde{\vartheta}}\right)\right)(1-\gamma)U\Bigg]. \tag{71}$$

Here, the utility from bequeathed wealth and the composite parameters are defined, respectively, as

$$U_{Dt} \equiv V_D^{1/\widetilde{\vartheta}}\frac{W_{Dt}^{1-\gamma}}{1-\gamma}, \tag{72}$$

and

$$\vartheta \equiv \frac{1-1/\psi}{1-\gamma}, \tag{73}$$

$$\widetilde{\vartheta} \equiv \frac{1-1/\widetilde{\psi}}{1-\gamma}. \tag{74}$$

Parameters ρ, γ, and ψ denote the rate of pure time preference, the coefficient of relative risk aversion, and the elasticity of intertemporal substitution (EIS), respectively, as in the standard EZ preference specification. Specification (70)– (74) introduces two additional exogenous parameters: the marginal value from bequeathed wealth $V_D \geq 0$ (with $V_D = 0$ corresponding to no bequest motive), and the elasticity of *intergenerational* substitution of consumption $\widetilde{\psi}$.

In equation (70), consumption $C_{i\tau}$ denotes the household's consumption flow at time τ conditional on survival up to time τ, while $U_{Di\tau}$ is the level of wealth bequeathed to the agent's offspring conditional on death taking place exactly at time τ.[79]

Section C.2 shows that specification (70)– (74) is the continuous-time limit of a continuum of discrete-time optimization settings, indexed by period duration $\Delta > 0$, in which the agent's utility $\widehat{U}_t(\Delta)$ is defined recursively through

$$\widehat{U}_t(\Delta) = \left\{(1-\beta(\Delta))\,C_t^{1-1/\psi} + \beta(\Delta)\mathbb{E}_t\left[\left((1-\pi_D(\Delta)V_D)\,\widehat{U}_{t+\Delta}(\Delta)^{1-1/\widetilde{\psi}} + \pi_D(\Delta)V_D\,W_{D,t+\Delta}^{1-1/\widetilde{\psi}}\right)^{\frac{1-\gamma}{1-1/\widetilde{\psi}}}\right]^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}, \tag{75}$$

[79]Formally, the expectation in (70) is taken with respect to all events other than the household's random time of death.

where

$$\beta(\Delta) \equiv \exp(-(\rho+\omega)\Delta) \tag{76}$$

$$\pi_D(\Delta) \equiv 1-\exp(-\omega\Delta). \tag{77}$$

The utility of specification (70)– (74) corresponds to the limit

$$U_t = \lim_{\Delta\to 0} \frac{\widehat{U}_t(\Delta)^{1-\gamma}}{1-\gamma}. \tag{78}$$

C.1.1 Special Cases

When $\psi = 1$, $\widetilde{\psi} \neq 1$, the felicity function is given by

$$\begin{aligned} f(C,U_D,U) =& (\rho+\omega)\log(C)(1-\gamma)U + \frac{\omega V_D}{1-1/\widetilde{\psi}} W_D^{1-1/\widetilde{\psi}}((1-\gamma)U)^{1-\widetilde{\vartheta}} \\ & -\left(\frac{\rho+\omega}{1-\gamma}\log((1-\gamma)U) + \frac{\omega V_D}{1-1/\widetilde{\psi}}\right)(1-\gamma)U. \end{aligned} \tag{79}$$

When $\psi \neq 1$, $\widetilde{\psi} = 1$ and $V_D > 0$,

$$\begin{aligned} f(C,U_D,U) =& \frac{\rho+\omega}{1-1/\psi} C^{1-1/\psi}((1-\gamma)U)^{1-\vartheta} + \omega V_D \log(W_D)(1-\gamma)U \\ & -\left(\frac{\rho+\omega}{1-1/\psi} + \omega V_D \log((1-\gamma)U)U\right). \end{aligned} \tag{80}$$

When $\psi = 1$, $\widetilde{\psi} = 1$, and $V_D > 0$,

$$\begin{aligned} f(C,U_D,U) =& (\rho+\omega)\log(C)(1-\gamma)U + \omega V_D \log(W_D)(1-\gamma)U \\ & -(\rho+\omega(1+V_D))\log((1-\gamma)U)U. \end{aligned} \tag{81}$$

Finally, the simple case $\gamma = 1/\psi = 1/\widetilde{\psi}$ corresponds to time-separable power utility with bequests:

$$U_t = \int_t^\infty \exp\left(-(\rho+\omega)(s-t)\right)\left[\frac{C_s^{1-\gamma}}{1-\gamma} + \omega V_D \frac{W_{Ds}^{1-\gamma}}{1-\gamma}\right]. \tag{82}$$

C.1.2 Transversality Condition

A transversality condition must hold in any equilibrium to ensure that the agent's lifetime utility and policy functions are well-defined (finite):[80]

$$\lim_{\tau\to\infty} \exp(-(\rho+\omega)(\tau-t))\mathbb{E}_t\left[\mathcal{U}_{i\tau}\right] = 0, \tag{83}$$

where $\mathcal{U}_{it}$ is the household's utility under the optimal consumption stream (the value function), $\mathcal{U}_{it} = \max U_{it}$.

C.2 Derivation

Proposition C.1. *Assume that the continuum of discrete-time optimization settings given by equations* (75)–(77) *and indexed by period duration* $\Delta > 0$ *satisfies the following condition:*

$$\lim_{\Delta \searrow 0} \frac{\mathbb{E}_t[\widehat{U}_{t+\Delta}(\Delta)^{1-\gamma}] - \widehat{U}_t(\Delta)^{1-\gamma}}{\Delta}$$

exists and is finite.

Then,

$$\lim_{\Delta\to 0} \frac{\widehat{U}_t(\Delta)^{1-\gamma}}{1-\gamma} = U_t, \tag{84}$$

where U_t *satisfies the recursive definition of equations* (70)– (74).

Proof. To Be Added. □

[80] The limit in (70) is zero only under the stronger transversality condition $\lim_{\tau\to\infty}\mathbb{E}_t\left[\mathcal{U}_{i\tau}\middle|t_{iD}=\infty\right]=0$. See the appendix to Duffie and Epstein (1992) written by Duffie, Epstein, and Skiadas on this point. However, a weaker transversality condition of the form of (83) suffices for the validity of the equilibrium.

D Proofs of the Analytical Results of Sections 3 and Appendix B.2

Proof of Proposition B.1. Applying Ito's Lemma on the conjecture for the value function $U = (\exp(v^s)W)^{1-\gamma}/(1-\gamma)$ when the agent is of type $s \in \{E, NE\}$, we have

$$\frac{dU}{(1-\gamma)U} = \frac{dW}{W} - \frac{1}{2}\gamma(1-\tau)^2\left(\theta_B^2 + \theta_Z^2\right)dt + \frac{W^{1-\gamma}}{\gamma}\left[\exp((1-\gamma)v^{-s}) - \exp((1-\gamma)v^s)\right]dF_t^s, \quad (85)$$

where $-s$ refers to the other type and F_t^s is a Poisson counting process governing the transition from type s to type $-s$.

The Hamilton-Jacobi-Bellman (HJB) equation is

$$0 = \max f(C, U_D, U) + \mathbb{E}_{t^-}[dU], \quad (86)$$

where the felicity function f is given by (71). Dividing (86) by $(1-\gamma)U$ and rearranging, we obtain the HJB for type s as

$$\begin{aligned}
\frac{\rho + \omega(1 + V_D\frac{\tilde{\vartheta}}{\vartheta})}{1 - 1/\psi} = \max_{cw,\theta_D,\theta_B,\theta_Z} & \frac{\rho+\omega}{1-1/\psi} cw^{1-1/\psi}\exp(-(1-1/\psi)v^s) \\
&+ \frac{\omega}{1-1/\widetilde{\psi}} V_D\left[(1-\tau_D)(1-\theta_D)\right]^{1-1/\widetilde{\psi}}\exp(-(1-1/\widetilde{\psi})v^s) \\
&+ (1-\tau)\left(r_f + \omega\theta_D + \pi_B\theta_B + \mathbb{1}_{s=E}\pi_Z\theta_Z\right) - cw \\
&- \frac{1}{2}\gamma(1-\tau)^2(\theta_B^2 + \theta_Z^2) + \frac{\nu^s}{1-\gamma}\left[\exp((1-\gamma)(v^{-s} - v^s)) - 1\right] \quad (87)
\end{aligned}$$

The first-order condition (FOC) with respect to the consumption-wealth ratio cw is

$$(\rho+\omega)cw^{-1/\psi}\exp(-(1-1/\psi)v^s) = 1, \quad (88)$$

which yields (44).

Using $w_D = (1-\tau_D)(1-\theta_D)$, the FOC with respect to the fraction of wealth invested in the annuity asset θ_D can be written as

$$\omega V_D(1-\tau_D)w_D^{-1/\widetilde{\psi}}\exp(-(1-1/\widetilde{\psi})v^s) = (1-\tau)\omega, \quad (89)$$

which yields (45).

Optimal policies (46) and (47) follow immediately as the FOCs with respect to θ_B and θ_Z, respectively. □

The proofs of the propositions regarding the long-run level and dynamics of inequality use the following lemma:

Lemma D.1 (Law of Motion for Top-Weighted Average Wealth). *Top-weighted average wealth across E and NE households, $\mathcal{F}^E(\zeta)$ and $\mathcal{F}^{NE}(\zeta)$, evolves over time according to*

$$\frac{d\mathcal{F}^E(\zeta)}{dt} = -\left(\widetilde{\lambda}_t^E(\zeta) - \nu^{NE}\right)\mathcal{F}^E(\zeta) + \nu^{NE}\mathcal{F}^{NE}(\zeta) + \omega\mathcal{B}^E(\zeta) \tag{90}$$

$$\frac{d\mathcal{F}^{NE}(\zeta)}{dt} = -\left(\widetilde{\lambda}_t^{NE}(\zeta) - \nu^{E}\right)\mathcal{F}^{NE}(\zeta) + \nu^{E}\mathcal{F}^{E}(\zeta) + \omega\mathcal{B}^{NE}(\zeta), \tag{91}$$

where

$$\widetilde{\lambda}_t^E(\zeta) = \omega + \nu^{NE}/m^E - \widetilde{\mu}_t^E\zeta - (\theta^E)^2\zeta(\zeta-1)/2 \tag{92}$$

$$\widetilde{\lambda}_t^{NE}(\zeta) = \omega + \nu^{NE}/m^E - \widetilde{\mu}_t^{NE}\zeta, \tag{93}$$

and

$$\mathcal{B}^s(\zeta) = \mathcal{B}^s\{f^s\}(\zeta) \equiv \begin{cases} w_L^{\zeta}\mathcal{F}_{\underset{\sim}{l}}^s(\zeta) & \text{if } w_D(s) = 0 \\ \frac{1}{m^s}\int_l\int_w \exp(\zeta h(w,l,s)) f^s(w) f_{\underset{\sim}{l}}^s(l)\,dw\,dl & \text{if } w_D(s) > 0, \end{cases} \tag{94}$$

for $s \in \{E, NE\}$.

Proof of Lemma D.1. The proof uses the following assumption on the equilibrium probability density functions $f_t^E(w)$ and $f_t^{NE}(w)$:[81] Let $v(w)$ be an arbitrary bounded and twice differentiable function; then the integrated conjuncts vanish,

$$\left[\left(\left(\widetilde{\mu}^E - \frac{1}{2}(\theta^E)^2\right)f^E(w) - \frac{1}{2}(\theta^E)^2 f^{E\prime}(w)\right)v(w) + \frac{1}{2}(\theta^E)^2 f^E(w)v'(w)\right]_{-\infty}^{+\infty} = 0 \tag{95}$$

$$\left[f^{NE}(w)v(w)\right]_{-\infty}^{+\infty} = 0. \tag{96}$$

Multiplying the FK equations (55) and (56) by $\exp(\zeta w)$ and integration from $-\infty$ to $+\infty$,

[81] This integrated conjunct assumption is in fact necessary for the Forward Kolmogorov equations to hold, as it ensures that the backward and forward Kolmogorov operators are formal adjoint operators. See any text on stochastic processes, e.g. Hanson (2007). It essentially requires that the density functions decline to zero fast enough as $w \to +\infty$ and as $w \to -\infty$ so that the cross-sectional distributions are convergent.

we get

$$\frac{d\mathcal{F}^E(\zeta)}{dt} = -\left(\widetilde{\mu}_t^E - \frac{1}{2}(\theta^E)^2\right)\int_{-\infty}^{+\infty} \exp(\zeta w) f^{E\prime}(w)dw + \frac{1}{2}(\theta^E)^2 \int_{-\infty}^{+\infty} \exp(\zeta w) f^{E\prime\prime}(w)dw$$
$$+ \nu^{NE}\mathcal{F}^{NE}(\zeta) - (\nu^E + \omega)\mathcal{F}^E(\zeta) + \omega \int \exp(\zeta w) B^E\{f^E(w)\}dw \tag{97}$$

$$\frac{d\mathcal{F}^{NE}(\zeta)}{dt} = -\widetilde{\mu}_t^{NE} \int_{-\infty}^{+\infty} \exp(\zeta w) f^{NE\prime}(w)dw + \nu^E \mathcal{F}^E(\zeta) - (\nu^{NE} + \omega)\mathcal{F}^{NE}(\zeta)$$
$$+ \omega \int \exp(\zeta w) B^{NE}\{f^{NE}(w)\}dw \tag{98}$$

Using integration parts and condition (95) with $v(w) = \exp(\zeta w)$, the first two terms on the right-hand side (RHS) of (97) equal

$$\left(\widetilde{\mu}_t^E - \frac{1}{2}(\theta^E)^2\right)\zeta\mathcal{F}^E(\zeta) + \frac{1}{2}(\theta^E)^2\zeta^2\mathcal{F}^E(\zeta). \tag{99}$$

Similarly, using integration parts and condition (96), the first term on the RHS of (98) equals

$$\widetilde{\mu}_t^{NE}\zeta\mathcal{F}^{NE}(\zeta). \tag{100}$$

In the case of no bequests $w_D(s) = 0$ for $s \in \{E, NE\}$,

$$\begin{aligned}
\mathcal{B}^s\{f^s\}(\zeta) \equiv& \int \exp(\zeta w)^s_{\underset{\sim}{l}}(w - \log(w_L))dw \\
=& w_L^\zeta \int \exp(\zeta l) f_{\underset{\sim}{l}}(l)dl \\
=& w_L^\zeta \mathcal{F}_{\underset{\sim}{l}}(\zeta).
\end{aligned} \tag{101}$$

With bequests, $w_D(s) > 0$,

$$\begin{aligned}
\mathcal{B}^s\{f^s\}(\zeta) \equiv& \frac{1}{m^s}\int\int_{l<w-\log(w_L)} \exp(\zeta w)\frac{\partial\widetilde{h}(w,l,s)}{\partial w} f^s(\widetilde{h}(w,l,s)) f^s_{\underset{\sim}{l}}(l)dldw \\
=& \frac{1}{m^s}\int\int \exp(\zeta h(\widehat{w}, l, s)) f^s(\widehat{w}) f^s_{\underset{\sim}{l}}(l)dld\widehat{w},
\end{aligned} \tag{102}$$

where the first line changes the order of integration, and the second line uses a change of variables $(l, w) \to (l, \widehat{w} = \widetilde{h}(w, l, s))$.

Collecting the results above and using $\nu^E + \nu^{NE} = \nu^{NE}/m^E$, which follows from the inflow-outflow balance condition, (1), we obtain equations (90) and (91).

□

Proof of Lemma 1. Equations (23) and (25) follow by evaluating equations (103) and (61) at

$\zeta = 1$ and using the fact that $\varphi^E_{\underset{\sim}{L}}(1)/m^E = \underset{\sim}{a}^E$.

Equation (24) follows from the definition of net worth $N_{it} = W_{it} - W_{Lit}$ and the linearity of the expectation operator. □

Proof of Proposition 1. Under no bequests, $\widetilde{\lambda}^E(\zeta)$ and $\widetilde{\lambda}^{NE}(\zeta)$, given in equations (92) and (93), are equal to $\lambda^E(\zeta)$ and $\lambda^{NE}(\zeta)$, given in equations (30) and (31), respectively.

Using the fact that, in steady state, $d\mathcal{F}^E(\zeta)/dt = 0$, and dividing both sides of equation (90) by $\mathcal{F}(\zeta)$, we get

$$\varphi^E(\zeta) = \frac{\omega w_L^\zeta \mathcal{F}^E_{\underset{\sim}{l}}(\zeta)/\mathcal{F}(\zeta) + \nu^{NE}}{\lambda^E(\zeta)}. \tag{103}$$

Equations (103) and (61) imply equation (28).

Adding equations (90) and (91), we obtain the law of motion of top-weighted average wealth,

$$\frac{d\mathcal{F}(\zeta)}{dt} = -\lambda_t(\zeta)\mathcal{F}(\zeta) + \omega w_L^\zeta \mathcal{F}_{\underset{\sim}{l}}(\zeta). \tag{104}$$

Using the fact from Lemma B.1 that, in steady state, $\mathcal{F}_{\underset{\sim}{l}}(\zeta) = \mathcal{F}_l(\zeta)$ and imposing $d\mathcal{F}(\zeta)/dt = 0$, we obtain equation (29) for the steady-state level of top-weighted average wealth.

The positive dependence of the steady-state levels of both top-weighted inequality measures on μ^E and, for $\zeta > 1$, on θ^E is obvious. For the impact of the level of type persistence, inversely related to the churning rate ν^{NE}, note that

$$\frac{\partial \varphi^E(\zeta)}{\partial \nu^{NE}} = \frac{1}{\lambda^E(\zeta)}\left[-\left(\frac{\varphi^E(\zeta)}{m^E} - 1\right) + w_L^\zeta \frac{\mathcal{F}_l(\zeta)}{\mathcal{F}(\zeta)}\left(\frac{\varphi_L^E(\zeta)}{m^E} - 1\right)\right] \tag{105}$$

Given that $w_L^\zeta \mathcal{F}_l(\zeta)/\mathcal{F}(\zeta) = \lambda(\zeta)/\omega < 1$ from equation (29), a sufficient condition for $\partial\varphi^E(\zeta)/\partial\nu^{NE} < 0$ is $\phi^E(\zeta) > \phi_L^E(\zeta)$, which is the empirically relevant case.

□

Proof of Proposition 2. I offer two proofs for the proposition characterizing the Pareto tail of wealth.

The first proof proceeds by substituting the conjecture $f^E(w) \to c^E \exp(-\zeta^* w)$ and $f^{NE}(w) \to c^{NE}\exp(-\zeta^* w)$ as $w \to \infty$ into the system of FK equations (55) and (56). We use the result that, when the Pareto tail of wealth is thicker than that of labor earnings, $\zeta_L^*(s) > \zeta^*$, for $s \in \{E, NE\}$, where $f^s_{\underset{\sim}{l}}(l) \to c^s_L \exp(-\zeta^*_L(s) l)$,[82]

$$\lim_{w\to\infty} \frac{f^s_{\underset{\sim}{l}}(w - \log(w_L))}{f^s(w)} = \lim_{w\to\infty} \frac{c^s_L w_L^{\zeta^*_L(s)}}{c^s_L} \exp(-(\zeta^*_L(s) - \zeta^*)w) = 0. \tag{106}$$

[82]The Pareto tail exponent of the distribution of equilibrium relative earnings, $f_l(l)$, is $\min\{\zeta^*_L(E), \zeta^*_L(NE)\}$.

Note that this result also implies that wealth and net worth have the same Pareto tail exponent. Substituting the conjectures into equations (55) and (56), dividing by $\exp(\zeta^* w)$, and using (106), we get a system of two equations in two unknowns, the ratio c^E/c^{NE} and ζ^*:

$$c^E\left(\lambda^E(\zeta^*) - \nu^{NE}\right) = c^{NE}\nu^{NE} \tag{107}$$

$$c^{NE}\left(\lambda^{NE}(\zeta^*) - \nu^E\right) = c^E\nu^E. \tag{108}$$

This system yields expressions (33) and (34).

A second method of proof uses a "Tauberian" result from Mimica (2016), also included as Proposition 7 in the Appendix of Gabaix et al. (2016). From equation (29), a number $-\widehat{\zeta} \geq 0$ that satisfies $\lambda(\widehat{\zeta}) = 0$ constitutes a negative abscissa of convergence for the Laplace transform of $f(w)$, that is, $\mathcal{F}(\zeta)$ converges for $0 < \zeta < \widetilde{\zeta}$, diverges for $\zeta > \widetilde{\zeta}$, and has a singularity at $\widetilde{\zeta}$. The Tauberian result then implies that this number $\widetilde{\zeta}$ is the Pareto tail exponent of $f(w)$. Finally, note that equation (33) defining ζ^* is equivalent to $\lambda(\zeta^*) = 0$. Therefore, $\zeta^* = \widetilde{\zeta}$ is the Pareto tail exponent of wealth.

□

Proof of Proposition 3. Using the laws of motion (90), (91), and (104) for $\mathcal{F}^E(\zeta)$, $\mathcal{F}^{NE}(\zeta)$, and $\mathcal{F}(\zeta)$, respectively, we have

$$\begin{aligned}\dot{\varphi}^E(\zeta) &= \frac{\dot{\mathcal{F}}^E(\zeta)}{\mathcal{F}(\zeta)} - \varphi^E(\zeta)\frac{\dot{\mathcal{F}}(\zeta)}{\mathcal{F}(\zeta)} \\ &= -\left[\lambda_t^E(\zeta) + \omega w_L^\zeta \frac{\mathcal{F}_{\underset{\sim}{l}}(\zeta)}{\mathcal{F}(\zeta)} - \lambda_t(\zeta)\right]\varphi^E(\zeta) + \nu^{NE} + \omega w_L^\zeta \frac{\mathcal{F}^E_{\underset{\sim}{l}}(\zeta)}{\mathcal{F}(\zeta)}\end{aligned} \tag{109}$$

Using equation (104) and the fact that, as $t \to \infty$, $d\mathcal{F}(\zeta)/dt \to 0$, we obtain

$$s^E(\zeta) = \lim_{t\to\infty} s_t^E(\zeta) = \lambda^E(\zeta). \tag{110}$$

Finally, using $w_L < 1$ and $\mu^E > 0$, it follows that $\lambda^E(\zeta)$ is a decreasing function of ζ, so that $\lambda^E(1)$ is an upper bound for the asymptotic speed of transition.

Also note that an analytical upper bound for the speed of transition of $\varphi_t^E(\zeta)$ at any time t is given by $\lambda^E(1) + \dot{\mathcal{F}}_t(\zeta)/\mathcal{F}_t(\zeta)$.

□

Proof of Lemma B.1. Adding the FK equations for relative labor earnings, equations (59)–(60), and using the fact that $f_l(l) = f_l^E(l) + f_l^{NE}(l)$ and $f_{\underset{\sim}{l}}(l) = f_{\underset{\sim}{l}}^E(l) + f_{\underset{\sim}{l}}^{NE}(l)$ yields

$$\frac{df_l(l)}{dt} = \omega\left(f_{\underset{\sim}{l}}(l) - f_l(l)\right). \tag{111}$$

In steady state, the cross-sectional distribution is time-invariant, $df_l(l)/dt = 0$, for all $l \in \mathbb{R}$, so $f_l(l) = f_{\utilde{l}}(l)$. This also implies $\mathcal{F}_l(l) = \mathcal{F}_{\utilde{l}}(l)$.

Multiplying equation (59) by $\exp(\zeta l)$ and integrating over l, we obtain

$$\frac{d\mathcal{F}_l^E(\zeta)}{dt} = \nu^{NE}\mathcal{F}_l^{NE}(\zeta) - (\nu^E + \omega)\mathcal{F}_l^E(\zeta) + \omega\mathcal{F}_{\utilde{l}}^E(\zeta). \tag{112}$$

Imposing time invariance of the earnings distribution in steady state, dividing both sides by $\mathcal{F}_l(l)$ and using $\mathcal{F}_l^{NE}(\zeta)/\mathcal{F}_l(\zeta) = 1 - \varphi_L^E(\zeta)$, we get

$$0 = \nu^{NE}\left(1 - \varphi_L^E(\zeta)\right) - (\nu^E + \omega)\varphi_L^E(\zeta) + \omega\varphi_{\utilde{L}}^E(\zeta). \tag{113}$$

Solving for $\varphi_L^E(\zeta)$, we have

$$\varphi_L^E(\zeta) = \frac{\omega\varphi_{\utilde{L}}^E(\zeta) + \nu^{NE}}{\omega + \nu^E + \nu^{NE}} = \frac{\omega\varphi_{\utilde{L}}^E(\zeta) + \nu^{NE}}{\omega + \nu^{NE}/m^E}, \tag{114}$$

where the second equality follows from the inflow-outflow condition, (1). □

Proof of Propositions B.2 and B.4. Consider a first-order approximation of $\exp(\zeta h(w,l,s))$ as a function of ζ around $\zeta = 1$:

$$\begin{aligned}\exp(\zeta h(w,l,s)) &= [w_D(s)\exp(w) + w_L\exp(l)]^\zeta \\ &\approx \left(w_D^s\right)^\zeta \exp(\zeta w) + w_L^\zeta \exp(\zeta l).\end{aligned} \tag{115}$$

Using this approximation in the case of bequests, we obtain

$$\begin{aligned}\mathcal{B}^s\{f^s\}(\zeta) &= \frac{1}{m^s}\int\int \exp(\zeta h(w,l,s)) f^s(w) f_{\utilde{l}}^s(l)\,dl\,dw \\ &\approx \left(w_D^s\right)^\zeta \mathcal{F}^s(w) + w_L^\zeta \mathcal{F}_{\utilde{l}}^s(l)\end{aligned} \tag{116}$$

It follows that the type-specific top-weighted average wealth measures approximately follow the laws of motion:

$$\frac{d\mathcal{F}^E(\zeta)}{dt} \approx -\left(\widetilde{\lambda}^E(\zeta) - \omega\left(w_D^E\right)^\zeta - \nu^{NE}\right)\mathcal{F}^E(\zeta) + \nu^{NE}\mathcal{F}^{NE}(\zeta) + \omega w_L^\zeta \mathcal{F}_{\utilde{l}}^E(\zeta) \tag{117}$$

$$\frac{d\mathcal{F}^{NE}(\zeta)}{dt} \approx -\left(\widetilde{\lambda}^{NE}(\zeta) - \omega\left(w_D^{NE}\right)^\zeta - \nu^E\right)\mathcal{F}^{NE}(\zeta) + \nu^E\mathcal{F}^E(\zeta) + \omega w_L^\zeta \mathcal{F}_{\utilde{l}}^{NE}(\zeta), \tag{118}$$

These laws of motion have the same form as those for the case of no bequests, but with

$$\lambda^E(\zeta) = \widetilde{\lambda}^E(\zeta) - \omega\left(w_D^E\right)^\zeta \tag{119}$$

$$\lambda^{NE}(\zeta) = \widetilde{\lambda}^{NE}(\zeta) - \omega\left(w_D^{NE}\right)^\zeta. \tag{120}$$

Equations (62)–(63) and (67)–(69) then follow by repeating the steps in the proofs of Propositions 1 and 3.

□

Proof of Proposition B.3. Performing a change of variables,

$$\begin{aligned}\mathcal{B}^s\{f^s\}(\zeta) &= \int_{l<w-\log(w_L)} \frac{\exp(w-\widetilde{h}(w,l,s))}{m^s w_D(s)} f^s\left(\widetilde{h}(w,l,s)\right) f_{\underset{\sim}{l}}^s(l)dl \\ &= -\frac{1}{m^s}\int_{-\infty}^{+\infty} \frac{\exp(w)}{\exp(w)-w_D(s)\exp(\widetilde{h})} f^s(\widetilde{h}) f_{\underset{\sim}{l}}^s\left(\log(\exp(w)-w_D(s)\exp(\widetilde{h}))-\log(w_L))\right)d\widetilde{h}.\end{aligned} \tag{121}$$

Dividing this by $f^s(w)$ and taking the limit as $w\to\infty$,

$$\begin{aligned}\lim_{w\to\infty}\frac{\mathcal{B}^s\{f^s\}(\zeta)}{f^s(w)} &= -\frac{1}{m^s}\int_{-\infty}^{+\infty} f^s(\widetilde{h})\left[\lim_{w\to\infty}\frac{\exp(w)}{\exp(w)-w_D(s)\exp(\widetilde{h})}\cdot\right. \\ &\qquad\qquad \left.\frac{f_{\underset{\sim}{l}}^s\left(\log(\exp(w)-w_D(s)\exp(\widetilde{h}))-\log(w_L)\right)}{f^s(w)}\right]d\widetilde{h} \\ &= -\frac{1}{m^s}\int_{-\infty}^{+\infty} f^s(\widetilde{h})\left[\lim_{w\to\infty} w_L^{\zeta_L^*(s)}\left[1-w_D(s)\exp(\widetilde{h}-w)\right]^{-(1+\zeta_L^*(s))}\exp(-(\zeta_L^*(s)-\zeta^*)w)\right]d\widetilde{h} \\ &= 0\end{aligned} \tag{122}$$

where the first line follows by changing the order of the limit and the integration, the second line uses the fact that

$$\lim_{l\to\infty} f_{\underset{\sim}{l}}^s(l) = \exp(-\zeta_L^*(s)l) \tag{123}$$

and the third line uses the fact that wealth has a thicker tail than earnings $\zeta^* < \min\{\zeta_L^*(E), \zeta_L^*(NE)\}$.

□

E A General-Equilibrium Model of Entrepreneurial Capital

This section introduces and analyzes a general-equilibrium (GE), endogenous-production extension of the model of Section 3. Sections E.1–E.3 introduce the different components of the model. Section E.4 characterizes the inside equity premium in terms of fundamentals. Section E.5 briefly addresses the equilibrium determinants of the riskfree rate.

To ease the reader's introduction to the key features of the model, I delegate many technical details, including generalizations of propositions and additional results, to Online Appendix F. The proofs of all formal results are located in Online Appendix G.

E.1 Setting

The overlapping-generations and labor earnings structure and household preferences are as in the model of Section 3. As I only analyze the GE model qualitatively, for expositional simplicity I assume no taxes, $\tau = \tau_D = 0$, and no bequests, $V_D = 0$.

Production of the final good requires as inputs labor and two types of capital, liquid capital K and entrepreneurial capital K^E. A stock of liquid capital can be operated by any agent and traded frictionlessly in financial markets. In contrast, entrepreneurial capital is tied to an entrepreneur and subject to a financial (skin-in-the-game) friction: only he can operate his stock of entrepreneurial capital and expand it via investment; its future income stream cannot be pledged to outsiders. The rest of this subsection presents the details of the setup.

E.1.1 Production

There are three types of firms: entrepreneurial firms, liquid firms, and final-good producers.

Intermediate goods An entrepreneurial firm, operated by an entrepreneurial household, holds and accumulates entrepreneurial capital, which produces a good that serves as an intermediate input to final good production. Denoting the quantity of the capital stock of firm i by $\mathcal{K}^E_{it}$, the firm's cumulative flow of the entrepreneurial good S^E_{it} follows[83]

$$dS^E_{it} = K^E_{it} dJ^E_{it}, \tag{124}$$

where

$$K^E_{it} \equiv A^E_{it} \mathcal{K}^E_{it} \tag{125}$$

[83] Unless otherwise indicated, all equations describing the evolution of *household- or firm-level* variables in stochastic differential equation form are conditional on the survival of the household or firm over the time increment.

denotes the *quality-adjusted* stock of firm i's capital, accounting for firm-specific productivity A_{it}^E. The dynamics of firm-level productivity are discussed in Section E.1.3.

J_{it}^E is a scaled Poisson process,[84]

$$dJ_{it}^E = \frac{1}{\lambda^E} d\Upsilon_{it}^E, \tag{126}$$

where Υ_{it}^E is a Poisson counting process with intensity λ^E. Note that $\mathbb{E}_{t^-}[dJ_{it}^E] = dt$. J_{it}^E captures a firm-specific production risk with a purely transitory effect on the firm's income stream.Applying a law of large numbers across the continuum of entrepreneurial firms, the aggregate quantity of the entrepreneurial good produced at time t equals $\mathbb{E}_t^*[dS_{it}^E] = K_t^E dt$ where $\mathbb{E}_t^*[\cdot]$ is the cross-sectional expectation operator and K_t^E is the aggregate (quality-adjusted) stock of entrepreneurial capital.[85]

The entrepreneurial good is traded on a Walrasian market at price p_t^E in units of the final good (the numeraire). The cumulative business income flow $\mathcal{Y}_{it}^E$ to entrepreneur i thus follows $d\mathcal{Y}_{it}^E = p_t^E dS_{it}^E$ and the corresponding aggregate entrepreneurial income flow is simply $p_t^E K_t^E dt$.

The production function of liquid firms, which produce a different intermediate input, is defined symmetrically. The cumulative flow S_{it} of the intermediate good of liquid firm i with capital stock $\mathcal{K}_{it}$ follows $dS_{it} = K_{it} dJ_{it}$ where $K_{it} \equiv A_{it}\mathcal{K}_{it}$ is the quality-adjusted stock of firm i's capital and J_{it} again captures a source of idiosyncratic income risk with $\mathbb{E}_{t^-}[dJ_{it}] = dt$. The intermediate good produced by liquid capital is traded at a unit price p_t.

Final-good producers Final-good producer j combines $\mathcal{G}_{jt}^E$ and $\mathcal{G}_{jt}$ units of intermediate inputs from entrepreneurial and liquid capital, respectively, with $\mathcal{L}_{jt}$ units of labor to produce Y_{jt} units of the final good according to the production function

$$Y_{jt} = \left(\mathcal{G}_{jt}^c\right)^{\varsigma} \left(A_t^L \mathcal{L}_{jt}\right)^{1-\varsigma}, \tag{127}$$

where A_t^L denotes the productivity of labor, and the composite intermediate input from capital is defined as

$$\mathcal{G}_{jt}^c \equiv \left[\Xi\left(\mathcal{G}_{jt}^E\right)^{1-\frac{1}{\varepsilon}} + (1-\Xi)\left(\mathcal{G}_{jt}\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}} \tag{128}$$

[84] More generally, J_{it}^E can be any Lévy subordinator process, that is, a stochastic process with non-negative increments that are stationary and identically and independently distributed (iid) over time, which is also iid across firms and satisfies $\mathbb{E}_{t^-}[dJ_{it}^E] = dt$.

[85] All exogenous sources of idiosyncratic risk in the model, such as the process J_{it}^E, are distributed independently across households and independently from each other and from the aggregate source of risk B_t. See Appendix F.2 on the application of the law of large numbers.

where $\varepsilon > 1$ is the elasticity of intratemporal substitution between entrepreneurial and liquid capital intermediate goods, and $\Xi \in (0,1)$ controls the productivity of entrepreneurial capital relative to liquid capital.

Factor income shares The equilibrium policies of final-good producers (see appendix F.3) imply the following distribution of aggregate income among the three factors of production:

$$p_t^E K_t^E = \xi_t \varsigma Y_t \tag{129}$$

$$p_t K_t = (1 - \xi_t)\, \varsigma Y_t \tag{130}$$

$$L_t = p_t^L A_t^L = (1 - \varsigma)\, Y_t, \tag{131}$$

where Y_t is aggregate output and p_t^L is the wage (i.e. aggregate earnings L_t) normalized by the level of aggregate productivity. (Equation (131) assumes a unit aggregate labor supply.) Thus, parameter ς equals the capital income share, and

$$\xi_t = \xi\left(\eta_t ; \Xi, \varepsilon\right) \equiv \left[1 + \frac{1-\Xi}{\Xi} \eta_t^{-\left(1-\frac{1}{\varepsilon}\right)}\right]^{-1} = \frac{\Xi (K_t^E)^{1-1/\varepsilon}}{\Xi (K_t^E)^{1-1/\varepsilon} + (1-\Xi)(K_t)^{1-1/\varepsilon}}, \tag{132}$$

is the share of entrepreneurial income in total capital income. Here,

$$\eta_t \equiv \frac{K_t^E}{K_t} \tag{133}$$

is the entrepreneurial capital ratio, that is, the ratio of the two aggregate (productivity-adjusted) stocks of capital. Under the assumption that entrepreneurial and liquid capital are substitutes in production in the Pareto-Edgeworth sense, $\varepsilon > 1$, the share of entrepreneurial capital is increasing in the entrepreneurial capital ratio. In particular, an increase in the relative productivity of entrepreneurial capital leads to a higher capital ratio and hence to a higher share of capital income accruing to entrepreneurial firms.

Labor productivity I assume that labor productivity is proportional to the composite aggregate capital stock, possibly due to knowledge spillovers from capital accumulation as in Romer's (1986) endogenous growth model with capital externalities:

$$A_t^L \equiv b^{1/(1-\varsigma)} K_t^c, \tag{134}$$

where

$$K_t^c \equiv \left[\Xi \left(K_t^E\right)^{1-\frac{1}{\varepsilon}} + (1-\Xi)\left(K_t\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}} \tag{135}$$

and $b > 0$ is a constant. An implication of this assumption, and its *raison d'être*, is that equilibrium aggregate output is proportional to the composite capital stock,

$$Y_t = bK_t^c. \tag{136}$$

E.1.2 Households

Every household is characterized by its type (E or NE), the level of its liquid (i.e. non-business) wealth $\widetilde{W}_{it}$ and, in the case of entrepreneurial households, its stock of entrepreneurial capital K_{it}^E.

E.1.3 Entrepreneurial firms

The cumulative entrepreneurial income flow and the idiosyncratic component of the productivity of entrepreneurial firm (household) i evolve according to

$$d\mathcal{Y}_{it}^E = p_t^E K_{it}^E dJ_{it}^E \tag{137}$$

$$\frac{dA_{it}^E}{A_{it^-}^E} = \sigma dB_t + dJ_{Pit}^E - dF_{it}^E - d\overline{F}_{it}^E, \tag{138}$$

where the diffusion B_t captured aggregate, total factor productivity shocks, process J_{it}^E, introduced in section E.1.1, captures purely transient income shocks, process J_{Pit}^E captures permanent income shocks, and F_{it}^E and $\overline{F}_{it}^E$ are Poisson counting process with intensities ν^E and $\overline{\nu}^E$, capturing business failure shocks as discussed below.

I assume that transient and permanent income shocks are driven by the same idiosyncratic underlying shocks,

$$dJ_{Pit}^E = s_{Kt} dJ_{it}^E, \tag{139}$$

where s_{Kt} may depend on the aggregate state of the economy.

The inclusion of both transient and permanent income shocks and the specification of their relationship in (139) are made for tractability reasons, as I discuss in Section E.3.

Failure risk Entrepreneurial firms of surviving households fail at a rate $\nu^E + \overline{\nu}^E$ losing all of their capital (their capital becomes completely unproductive, hence worthless), a risk captured by the counting processes F_{it}^E and $\overline{F}_{it}^E$. If failure is driven by an F_{it}^E shock, $dF_{it}^E = 1$, the household operating the firm becomes a non-entrepreneur. If failure is driven by an $\overline{F}_{it}^E$ shock, $d\overline{F}_{it}^E = 1$, the household receives an idea (blueprint) for a new entrepreneurial venture immediately upon failure of its existing firm, retaining its status as an entrepreneurial household. This structure serves to disentangle the persistence of entrepreneurial status, governed by the churning rate ν^E as in the model of Section 3, from the business failure rate, governed by $\nu^E + \overline{\nu}^E$.

If the household dies, its stock of entrepreneurial capital is similarly lost.[86]

Capital investment Entrepreneurial firms accumulate capital via an investment technology subject to convex adjustment costs, as in the q-theory of Hayashi (1982). In particular, the quantity of the capital stock of an entrepreneurial firm i evolves as

$$\frac{d\mathcal{K}^E_{it}}{dt} = \left(\chi(\iota^E_{it}) - \delta^E\right)\mathcal{K}^E_{it}, \tag{140}$$

where $\chi(\cdot)$ is a concave function, $\iota^E_{it} \equiv I^E_{it}/K^E_{it}$ is the ratio of the firm's investment expenditure flow I^E_{it} to its *productivity-adjusted* capital stock K^E_{it}, and δ^E is the rate of depreciation of entrepreneurial capital.

Combining equations (138) and (140), a household's quality-adjusted capital stock $K^E_{it} \equiv A^E_{it}\mathcal{K}^E_{it}$ evolves according to

$$\frac{dK^E_{it}}{K^E_{it^-}} = \left(\chi(\iota^E_{it}) - \delta^E\right)dt + \sigma dB_t + dJ^E_{Pit} - dF^E_{it} - d\overline{F}^E_{it}. \tag{141}$$

E.1.4 Entrepreneurial firm creation

At every point in time a mass $\nu^{NE}dt$ of non-entrepreneurial households become entrepreneurs and establish new entrepreneurial firms, an event that occurs according to the household-specific Poisson counting process G^{NE}_{it} capturing the arrival of an idea (blueprint) for a new entrepreneurial venture. Additionally, a mass $\overline{\nu}^E dt$ of entrepreneurial households (whose businesses have just failed) also establish new entrepreneurial firms.

Founders of new entrepreneurial firms decide on their initial investment in the firm, $\underset{\sim}{I}^E_{it}$.[87] (A tilde below a variable indicates that the variable is associated with entrepreneurial firm creation.) The process of new firm creation is subject to convex costs at the aggregate level, capturing diminishing returns to the implementability of new ideas and methods. In particular, the initial (productivity-adjusted) capital stock of an entrepreneurial firm is given by

$$\underset{\sim}{K}^E_{it} = \underset{\sim}{\chi}_t\left(\frac{\underset{\sim}{I}^E_t}{K^E_t}\right)\underset{\sim}{I}^E_{it}, \tag{142}$$

[86] An alternative assumption with the same qualitative implications is that, upon the household's death, its stock of entrepreneurial capital is converted into liquid capital. As I abstract from bequests and do not calibrate the model in this section, I choose the simpler assumption that the household's stock of entrepreneurial capital is fully destroyed upon death.

[87] Capital investment expenditures at the time of firm creation are discrete (lumpy) at the household level, in contrast to the smooth flow of capital investment expenditures by existing entrepreneurial firms.

where $\underset{\sim}{\chi}_t(\cdot)$ is a concave function and

$$\underset{\sim}{I}_t^E dt \equiv \int_{i \in \underset{\sim}{\mathcal{I}}^E_{[t,t+dt)}} \underset{\sim}{I}_{it}^E di \tag{143}$$

is aggregate investment by founders of new entrepreneurial ventures.[88,89]

E.1.5 Liquid firms

Liquid firms face no financial frictions. As discussed in the next subsection, ownership claims to liquid firms are pooled together, so that all idiosyncratic risks to their income and productivity are fully diversified away. Hence, the structure of their idiosyncratic risks is irrelevant for the determination of aggregate equilibrium prices and quantities. Therefore, without loss of generality for the present purposes, I assume a time-invariant set of liquid firms I^K and that firm-level productivity is only affected by aggregate risk (TFP shocks):

$$\frac{dA_{it}}{A_{it}} = \sigma dB_t, \tag{144}$$

for all $i \in I^K$.

Liquid firms accumulate capital via the same investment technology as entrepreneurial firms,

$$\frac{d\mathcal{K}_{it}}{dt} = (\chi(\iota_{it}) - \delta)\mathcal{K}_{it}, \tag{145}$$

where $\iota_{it} \equiv I_{it}/K_{it}$ and δ is the rate of depreciation of liquid capital.

Combining equations (145) and , the productivity-adjusted stock of liquid capital $K_{it} \equiv A_{it}\mathcal{K}_{it}$ evolves as

$$\frac{dK_{it}}{K_{it}} = (\chi(\iota_{it}) - \delta)\, dt + \sigma dB_t. \tag{146}$$

E.1.6 Financial markets

Financial markets are formally complete with respect to all risks, aggregate as well as idiosyncratic. However, entrepreneurs are not able to issue claims against the future income streams of their entrepreneurial firms for moral hazard (skin-in-the-game) reasons.[90] More-

[88]Function $\underset{\sim}{\chi}(\underset{\sim}{\iota}^E)$ is defined on $\underset{\sim}{\iota}^E \geq 0$, satisfies $\underset{\sim}{\chi}(0) = 0$ and $\underset{\sim}{\chi}'(0) = \infty$, and may be a function of the aggregate state vector Σ_t defined in Section E.3.

[89]$\underset{\sim}{\mathcal{I}}^E_{[t,t+dt)}$ denotes the set of households who become entrepreneurs over the infinitesimal time interval $[t, t + dt)$.

[90]The framework can easily be extended (without loss of tractability) to allow for partial outside financing of entrepreneurial capital investments, by assuming that entrepreneurs must at all times retain ownership of a minimum fraction $\Phi \in (0, 1]$ of the future income stream of its entrepreneurial firm.

over, the arrival of an idea (blueprint) for a new entrepreneurial firm is not contractible ex-ante. Appendix F.5 discusses related microfoundations for the assumed form of the financial friction.

The market value of a liquid firm can be written as $Q_{it}K_{it}$, where Q_{it} equals the market price of the claim to a unit of firm i's capital stock.[91] By absence of arbitrage, liquid capital price Q_{it} equals the present discounted value of the future stream of net profits to the firm normalized by its current capital stock:

$$Q_{it} = \max_{[\iota_{i\tau}]_{\tau=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} \frac{M_\tau}{M_t} (p_\tau - \iota_{i\tau}) \frac{K_{i\tau}}{K_{it}} d\tau \right], \tag{147}$$

where M_t denotes the pricing kernel for financial assets.[92] Liquid firms choose their investment policy to maximize market value as in the neoclassical theory of the firm.

Taking into account all types of traded assets, aggregate liquid (non-business) wealth in the economy is the sum of aggregate financial wealth and aggregate capitalized labor wealth:

$$\widetilde{W_t} = Q_t K_t + W_{Lt} = Q_t K_t + Q_t^L A_t^L, \tag{149}$$

where

$$Q_t^L = \mathbb{E}_t \left[\int_t^{\infty} \frac{M_\tau}{M_t} \exp(-\omega(\tau - t)) p_\tau^L \frac{A_\tau^L}{A_t^L} d\tau \right] \tag{150}$$

denotes aggregate labor wealth normalized by the level of labor productivity.

E.2 Equilibrium

In this subsection I discuss the optimization problem of households and market clearing for final output. The formal statement of equilibrium is given in Definition 1 in the appendix.

Households solve the following maximization problem for all $t \in [t_{i0}, t_{iD})$, where t_{i0} is household i's time of birth and t_{iD} its random time of death:

$$\mathcal{U}_{it} \equiv \max_{C_i, \Theta_i, I_i^E, \underset{\sim}{I}_i^E} U_{it}, \tag{151}$$

[91] In this model, average or Tobin's q equals marginal q in q-theory terms due to the proportional specification of investment adjustment costs in (145).

[92] The (unique) pricing kernel in this economy follows

$$\frac{dM_t}{M_t} = -r_{ft} dt - \pi_{Bt} dB_t, \tag{148}$$

where r_{ft} and π_{Bt} denote the riskfree rate and the price of aggregate risk, respectively. To the extent that they can be traded, idiosyncratic risks have a zero market price because they are fully diversified away by risk pooling.

subject to

$$\widetilde{W}_{it'} \geq 0 \qquad \forall t' \in [t, t_{iD}), \tag{152}$$

where the utility function U_{it} is given by (70) and the households' liquid or tradable wealth at time t is

$$\widetilde{W}_{it} = \widetilde{W}_{it_{i0}} + \underbrace{\int_{t_{i0}}^{t} \widetilde{W}_{i\tau} dR_{i\tau}^{\Theta}}_{\text{financial \& labor income}} + \underbrace{\mathcal{Y}_{it}^{E} - \int_{t_{i0}}^{t} I_{i\tau}^{E} d\tau - \sum_{\underset{\sim}{t}} I_{i\underset{\sim}{t}}^{E}}_{\text{entrepreneurial income}} - \int_{t_{i0}}^{t} C_{i\tau} d\tau, \tag{153}$$

and $\widetilde{W}_{it_{i0}} = Q_{t_{i0}}^{L} A_{t_{i0}}^{L}$.[93] Θ_i summarizes the household's financial portfolio policy, which affects the returns to its liquid portfolio dR_{it}^{Θ}, and $\underset{\sim}{t}$ denotes a stopping time associated with the creation of a new entrepreneurial firm.

The solvency constraint takes the form of (152) because I have defined initial wealth $\widetilde{W}_{it_{i0}}$ to include the present discounted value of all pledgeable sources of future wealth, in particular future labor income. In other words, the constraint still allows for real-world borrowing. However, liquid wealth does not account for households' future business income, which is non-pledgeable.[94]

Non-entrepreneurs Non-entrepreneurs face a simple consumption and financial portfolio choice problem as in standard financial portfolio choice theory, and their (non-business) wealth $\widetilde{W}_{it}$ evolves according to equations (3)– (5).

Due to the scale independence of preferences, non-entrepreneurs' value function can be written as

$$\mathcal{U}_{it}^{NE} = \mathcal{U}_t(\widetilde{W}_{it}^{NE}) = \frac{1}{1-\gamma}\left(\widetilde{W}_{it}^{NE} \exp\left(V_t^{NE}\right)\right)^{1-\gamma}, \tag{154}$$

where the normalized value function V_t^{NE} is the same across non-entrepreneurs, since they face the same investment opportunities going forward. For notational simplicity, I omit the dependence of V^{NE} on the aggregate state of the economy, as I also do below for V^E.

Entrepreneurs The problem of an entrepreneur is more complicated because of his additional decision over the capital investment policy of his entrepreneurial firm. A key

[93] If i is part of the initial cohort, $t_{i0} = 0$, its initial wealth is as described in Section F.6.

[94] This constraint never binds strictly in the model. The reason is that if it did, that is if entrepreneurs at any point chose to hold only illiquid wealth (E capital) and zero liquid wealth, they would be left with no wealth at all and be forced to choose zero consumption in the event that a business failure shock occurred in the next instant, an event with strictly positive probability. Under Epstein-Zin preferences, zero consumption at any point implies infinite disutility so it cannot be part of the optimal consumption policy.

household-level state variable for entrepreneurs is their liquidity ratio

$$\kappa_{it} \equiv \frac{K_{it}^E}{\widetilde{W}_{it}^E}, \tag{155}$$

that is, the ratio of their entrepreneurial (productivity-adjusted) capital stock to their liquid wealth. Using this definition, an entrepreneur's liquid wealth follows

$$\frac{d\widetilde{W}_{it}^E}{\widetilde{W}_{it}^E} = \left(r_{ft} + \omega - c\widetilde{w}_{it}^E\right)dt + \theta_{Bit}\left(\pi_{Bt}dt + dB_t\right) + \kappa_{it}\left(p_t^E dJ_{it}^E - \iota_{it}^E dt\right) \tag{156}$$

where, as in Section 3, $c\widetilde{w}_{it}^E \equiv C_{it}^E/\widetilde{W}_{it}^E$, and θ_{Bit} captures the entrepreneur's financial portfolio choice. The law of motion (156) shows that the effective proportional exposure of liquid wealth to income risk,

$$s_{\widetilde{W}it} \equiv p_t^E \kappa_{it}, \tag{157}$$

is endogenously increasing in the price of the entrepreneurial good p_t^E and in the entrepreneur's liquidity ratio κ_{it}. Forces such as an increase in aggregate entrepreneurial productivity that increase the equilibrium price of the entrepreneurial good and lead entrepreneurs to invest more of their wealth into entrepreneurial capital will lead to an increase in the quantity of idiosyncratic income risk borne by entrepreneurs.

The scale independence of preferences again implies a simplified form for the value function:

$$\mathcal{U}_{it}^E = \mathcal{U}_t(\widetilde{W}_{it}^E, K_{it}^E) = \frac{1}{1-\gamma}\left(\widetilde{W}_{it}^E \exp\left(V_t^E(\kappa_{it})\right)\right)^{1-\gamma}, \tag{158}$$

where now the normalized value function $V_t^E(\kappa_{it})$ depends not only on the aggregate state of the economy but also on the entrepreneur's allocation of his wealth between liquid and illiquid forms, captured by the liquidity ratio.

Founders of new entrepreneurial firms When a household receives an idea for a new business venture, it faces a one-off choice over the initial capital investment $\underset{\sim}{I}_{it}^E$ in its new entrepreneurial firm. It solves

$$\max_{\underset{\sim}{I}_{it}^E, \widetilde{W}_{it} \geq 0} \mathcal{U}_t\left(\widetilde{W}_{it}^E, \underset{\sim}{K}_{it}^E\right), \tag{159}$$

subject to the budget constraint

$$\underset{\sim}{I}_{it}^E + \widetilde{W}_{it}^E \leq \widetilde{W}_{it^-}, \tag{160}$$

where the initial capital stock $\underset{\sim}{K}_{it}^{E}$ is given by (142) as a function of $\underset{\sim}{I}_{it}^{E}$, and $\widetilde{W}_{it^-}$ ($\widetilde{W}_{it}^{E}$) is the household's level of liquid wealth right before (after) it establishes the entrepreneurial firm.

The dynamic programming formulation of the optimization problems for each household type and their optimality conditions are presented in appendix F.7.

Final good market clearing Final output is used for consumption and capital investment:

$$Y_t = C_t^E + C_t^{NE} + I_t^E + \underset{\sim}{I}_t^E + I_t, \tag{161}$$

where C_t^E and C_t^{NE} are the aggregate consumption levels of the groups of entrepreneurs and non-entrepreneurs, I_t^E and $\underset{\sim}{I}_t^E$ are aggregate investment expenditures by existing and new entrepreneurial firms, respectively, and I_t is aggregate investment by liquid firms.

E.3 Aggregation

The aggregate state of the economy Σ is essentially described by the following variables: the aggregate capital ratio $\eta \equiv K^E/K$, the aggregate liquid wealth share of entrepreneurs $\widetilde{\varphi}^E \equiv \widetilde{W}^E/\widetilde{W}$, and the joint cross-sectional distribution of relative liquid wealth and liquidity ratios across households, $f(\widetilde{w}, \kappa, s)$, where $s \in \{E, NE\}$. As the in the model of Section 3, the aggregate state is unaffected by fully symmetric (total factor productivity) shocks driven by Brownian motion B_t, as all agents choose the same proportional exposures to aggregate risk.

In general, the model features a non-degenerate cross-sectional distribution of liquidity ratios across entrepreneurs. Heterogeneity in liquidity ratios translates into heterogeneity across entrepreneurs in the exposure of total household wealth to inside entrepreneurial equity, which is realistic (see Figure A.2). However, for tractability purposes, I make a set of ad-hoc functional form assumptions implying that all entrepreneurs optimally choose the same liquidity ratio, which coincides with the aggregate liquidity ratio, $\kappa_t \equiv K_t^E/\widetilde{W}_t^E$. These assumptions result in a parsimonious two-type model, directly comparable to the two-type model of Section 3. In particular, the cross-sectional distribution of relative wealth and liquidity ratios is not needed as part of the aggregate state vector Σ, resulting in a finite-dimensional aggregate state space for the model.[95]

Proposition E.1 (Tractable Aggregation Assumptions, TAA). *Assume that, at all times,*

1. *the exposure of entrepreneurial capital productivity to idiosyncratic income shocks satisfies*

$$s_{Kt} = p_t^E \kappa_t, \tag{162}$$

[95] This implies in particular that, starting from the model's steady state, the transition paths of aggregates in response to structural shifts are independent of the cross-sectional distributions.

2. *function* $\underset{\sim}{\chi}(\cdot)$ *in equation* (142), *controlling the efficiency of investment in new entrepreneurial firms, satisfies*

$$\underset{\sim}{\chi}_t\left(\frac{\underset{\sim}{I}_t^E}{K_t^E}\right) = \frac{1 - V_\kappa^E(\kappa_t)\kappa_t}{V_\kappa^E(\kappa_t)}, \tag{163}$$

where κ_t *is the aggregate liquidity ratio of entrepreneurs.*

Then, the economy converges in the long-run to a degenerate cross-sectional distribution for liquidity ratios, equal to κ_t *for all entrepreneurs.*

E.4 The Valuation of Inside Equity

Proposition E.2 (Shadow Prices)**.** *Assume the Tractable Aggregation Assumptions of Proposition E.1 hold, and* $\kappa_{it} = \kappa_t$ *for all* $i \in E$.

Define the shadow capital price of inside equity

$$Q_t^E \equiv \frac{V_\kappa^E(\kappa_t)}{1 - V_\kappa^E(\kappa_t)\kappa_t}, \tag{164}$$

total household wealth

$$W_{it} \equiv \widetilde{W}_{it} + \underbrace{Q_t^E K_{it}^E}_{\text{inside equity}}, \tag{165}$$

and the shadow risk prices

$$\pi_{Jt} \equiv 1 - \left(1 + \frac{p_t^E \kappa_t}{\lambda^E}\right)^{-\gamma} > 0 \tag{166}$$

$$\pi_{Ft} \equiv 1 - \left(1 + Q_t^E \kappa_t\right)\exp\left((1-\gamma)\left(V_t^{NE} - V_t^E(\kappa_t)\right)\right) < 0 \tag{167}$$

$$\pi_{\overline{F}t} \equiv 1 - \left(1 + Q_t^E \kappa_t\right)^\gamma < 0. \tag{168}$$

Then, the equilibrium consumption and portfolio policies of agents coincide with those in a setting (the dual economy) without illiquid investments in entrepreneurial capital but where each entrepreneur i perceives his wealth to be given by (165) *and can frictionlessly trade on the sources of risk* J_{it}^E, F_{it}^E, *and* $\overline{F}_{it}^E$ *at risk prices* π_J, π_F, *and* $\pi_{\overline{F}}$, *respectively.*

The shadow capital price of inside equity Q^E does *not* satisfy a pricing equation analogous to equation (147) for the liquid capital price Q; it implies a higher discount rate on the future cash flows of the entrepreneurial firm because undiversifiable idiosyncratic firm-level risk is priced in equilibrium.[96]

[96] One can define the price of *outside* entrepreneurial equity, as the normalized present discounted value of the future stream of net profits of the entrepreneurial firm, discounted using the pricing kernel from financial

The proportional exposures of total entrepreneurial household wealth, given by equation (165), to firm-level risks J_{it}^E, F_{it}^E, and $\overline{F}_{it}^E$ are given by:

$$\theta_{Jt}^E = p_t^E \kappa_t > 0 \tag{170}$$

$$\theta_{Ft}^E = \theta_{\overline{F}t}^E = -\frac{Q^E(\kappa_t)\kappa_t}{1 + Q^E(\kappa_t)\kappa_t} < 0. \tag{171}$$

To relate the expected returns on inside equity in this setting, which features jump risks at the firm level, to the expected returns on inside equity in the diffusion setting of the partial-equilibrium model of Section 3, it is useful to define the process Z_{it} through

$$dZ_{it} = \frac{1}{\theta_t^E}\left(\theta_{Jt}^E\left(dJ_{it}^E - dt\right) + \theta_{F_t}^E\left(dF_{it}^E - \nu^E dt\right) + \theta_{\overline{F}t}^E\left(d\overline{F}_{it}^E - \overline{\nu}^E dt\right)\right), \tag{172}$$

where

$$\theta_t^E \equiv \sqrt{\left(\theta_{Jt}^E\right)^2/\lambda^E + \left(\theta_{Ft}^E\right)^2\nu^E + \left(\theta_{\overline{F}t}^E\right)^2\overline{\nu}^E} \tag{173}$$

is a measure of the overall proportional exposure of total household wealth to firm-level risks. By the properties of Poisson processes, the increment of the Z_{it} process has zero mean and variance equal to dt, just like the Brownian motion assumed in the setting of Section 3.

Proposition E.3 (The Inside Equity Premium in Equilibrium)**.** *The excess return earned by the average entrepreneur on entrepreneurial investments is*

$$\Pi_{Zt} = \pi_{Jt}\theta_{Jt}^E + \nu^E\pi_{Ft}^E\theta_{Ft}^E + \overline{\nu}^E\pi_{\overline{F}t}^E\theta_{\overline{F}t}^E. \tag{174}$$

The inside equity premium (Sharpe ratio) can be defined as

$$\pi_{Zt} \equiv \frac{\Pi_{Zt}}{\theta_t^E}, \tag{175}$$

and is increasing in the relative productivity of entrepreneurial capital, Ξ, all else equal.

Wealth inequality Once expressed using the formulation of Proposition E.2 in terms of shadow prices, the dynamics of log relative total household wealth $w_{it} \equiv \log(W_{it}/W_t)$ satisfy a set of Forward Kolmogorov equations, given in Appendix F.8, which are very similar

markets (which assigns a zero price to idiosyncratic risk):

$$\widetilde{Q}_{it}^E = \mathbb{E}_t\left[\int_t^\infty \frac{M_\tau}{M_t}\left(p_\tau^E - \iota_{i\tau}^E\right)\frac{K_{i\tau}^E}{K_{it}^E}d\tau\right], \tag{169}$$

Then, in equilibrium the shadow price of inside equity is always lower than the price of outside equity, $Q_{it}^E < \widetilde{Q}_{it}^E$. (There is a zero gross supply of outside equity in this setting.)

qualitatively to the Forward Kolmogorov equations (55)–(56) in the partial-equilibrium setting of Section 3. Hence, the key implications of the partial-equilibrium model regarding the impact of entrepreneurship dynamics on the evolution of inequality also apply in this setting.

E.5 Precautionary Savings and the Riskfree Rate

Proposition E.4 (The Riskfree Rate in Equilibrium)**.** *In the case of a unit elasticity of intertemporal substitution, $\psi = 1$, the riskfree rate is given by*

$$r_{ft} = \rho + g_t + \omega(1 - w_{Lt}) - (\gamma\sigma^2 + \varphi_t^E \Pi_{Zt}). \tag{176}$$

where g_t is the aggregate wealth growth rate, $w_{Lt} \equiv W_{Lt}/W_t$, and φ^E is the aggregate (total) wealth share of entrepreneurs.

In steady state, the riskfree rate is decreasing in the relative productivity of entrepreneurial capital, Ξ, all else equal.

The expression for the riskfree rate is similar to the standard expression in a representative agent economy. The riskfree rate is increasing in the time discount rate ρ, increasing in the aggregate wealth growth rate (with an adjustment of $\omega(1-w_L)$ for the wedge between the average wealth growth rate of surviving households and the aggregate wealth growth rate), and decreasing in precautionary saving demand, an effect captured by the last two terms in equation (176). An increase in the inside equity premium, which may be due to an increase in the relative productivity of entrepreneurial capital Ξ, raises the quantity of idiosyncratic risk borne by entrepreneurs on average in equilibrium, since entrepreneurs increase their wealth exposure to inside equity in order to take advantage of its higher return. In turn, this makes their future consumption stream riskier, increasing their demand for precautionary savings. As a result, the riskfree rate as well as discount rates on all liquid financial assets decline in equilibrium.

Thus, besides its effects on the evolution of US wealth inequality in recent years, an increase in the inside equity premium may also help to account for part of the long-term decline in the US real riskfree rate since the 1980s.