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Improving the Conceptual Understanding of Kinematics Through Graphical Analysis

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Abstract

In this study, we use graphical analysis to develop a learner-centered approach to understanding kinematics. The learner-centered approach offers three advantages: it is consistent with pedagogy that has been shown to be effective for learning, it can be naturally integrated with real-time data collection using motion detectors or video analysis, and it provides a mechanism for developing insight into both physics and calculus. Students discover fundamental principles through a series of laboratory modules. The learning process is integrated into a conceptual framework through a variety of activities and application projects.

Introduction

Graphical analysis is an approach to learning kinematics that uses slope and area relationships among motion graphs to solve for unknown variables. Because this is essentially a graphical approach to finding derivatives and integrals, an understanding of graphical analysis is both useful to students learning calculus and broadly applicable to many other subjects. Although mentioned in many textbooks, graphical analysis is typically presented as an optional alternative to the use of constant acceleration equations for solving kinematics problems. While students may learn to solve problems more quickly through the application of constant acceleration equations, we feel that their understanding of motion—particularly the general case in which acceleration may vary with time—does not match the richer learning experience offered by graphical analysis. A graphical analysis approach allows students to *visualize* motion while working more directly with fundamental principles. Graphical analysis also takes greater advantage of advances in laboratory technology, including real-time data collection using motion detectors (an ideal tool for measuring, viewing and manipulating motion graphs for motion with constant or time-varying acceleration) and video analysis.

To produce the most effective learning, we have developed our kinematics curriculum based upon learner-centered principles. In this paper we will present our approach using the framework of the National Research Council (NRC) findings on effective learning. In a study with strong implications for teaching, the NRC has recently reported the following points as key to successful learning.[1]

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they won't change or they may learn for the test and revert to preconceptions.

2. To develop competence in an area, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and applications.
3. A metacognitive approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

Anyone who has taught physics will agree that students enter the class with many preconceptions. For example, students have tremendous difficulty with situations in which instantaneous velocity is zero while acceleration is not zero, such as a ball at the top of its flight[2]. Referring to Key Point #2, the NRC cites several studies that illustrate the differences in how experts and novices organize their understanding. For example, when asked to verbally explain their approach to solving physics problems, experts mentioned major principles and laws while competent beginners described the equations that they would use and how they would manipulate them. The NRC also points out that organizing information into a conceptual framework allows students to apply what was learned in new situations and to learn related information more quickly.

We feel that teaching graphical analysis using a learner-centered pedagogy offers three advantages for the study of kinematics: it is consistent with the NRC key findings, it can be naturally integrated with real-time data collection using motion detectors or video analysis, and it provides a mechanism for developing insight into both physics and calculus. Although we use graphical analysis throughout our study of dynamics and other subjects in physics, in this paper we will present only our work on one-dimensional kinematics. The curriculum implementation that we will describe took place in an 11th-grade physics class in the Brunswick School, Greenwich, CT (a college-preparatory private day school). Our major learning goals for kinematics were the following:

- Given a description of one-dimensional motion, students will be able to draw the position, velocity and acceleration versus time graphs.
- Given a position, velocity or acceleration time graph, students will be able draw the other two graphs and solve for all values.
- Students will be able to solve word problems using graphical analysis.
- Students will understand the terms used to describe motion (such as distance, displacement, speed, instantaneous and average values, etc.) and be able to calculate them from motion graphs.
- Through directed laboratory investigations students will "discover" the slope/area relationships among motion graphs.
- Students will derive the equations of motion for constant acceleration using graphical analysis and understand their application.
- Students will see that all motion in Newtonian mechanics can be understood through graphical analysis. All one-dimensional kinematics problems (and later the components of motion in multiple dimensions) will be treated in fundamentally the same way.
- Students will learn that their problem-solving potential is limited only by their ability to calculate the areas and slopes of graphs—and thus they will see the need for calculus.
- Students will build this understanding within a conceptual framework for dynamics and all physics.

We have found that at the completion of the unit, most students achieved most of these goals. In fact most students attained a sound understanding of phenomena such as the kinematics of a bouncing ball. (Note: later in the course they revisited the causes of the bouncing ball motion through an F=ma, conservation of energy[3] and impulse-momentum approach.)

Engaging Initial Student Understanding

As already noted in the NRC's Key Point #1, effective learning may not take place if the preconceptions that students bring to the classroom are not engaged. We have designed a sequence of laboratories and in-class activities designed to expose and address student preconceptions—both predictable and unpredictable. What are the fundamental components of each laboratory?

- Making individual predictions based upon reason
- Discussing and reconciling differences among group members
- Making measurements for required experimental procedures as well as student-designed laboratory extensions
- Discussing and reconciling the differences between measured and predicted results

To illustrate these components we will describe a laboratory in which frictionless carts are used to generate smooth graphs from which students can make inferences about their motion.

<u>Example laboratory: Accelerating Up and Down an Incline</u>

To introduce the concept of acceleration, we have developed a laboratory in which students study the motion graphs for a cart that:

- moves up an incline after it is pushed and released,
- moves down an incline when released from rest and
- combines the two motions by moving up an incline, slowing down, and then returning to its starting point.

We feel that this experiment is particularly important because it provides an effective means for both examining the nature of acceleration and allowing students to see a slow-motion version of free fall that can be referred to later.

Students begin by making predictions of the motion graphs in a pre-lab homework assignment. Although making incorrect predictions is part of the learning process, students must show that their predictions are based upon reason (correct or incorrect). When they come to class, students break into groups of three and begin the laboratory by discussing and debating the differences in their predictions with the goal of reaching a single prediction for the entire group. This often leads to intense discussion and debate as each member defends his or her reasoning. During this time and throughout the laboratory, the teacher visits each group to help focus the discussion and to gather information that will direct the post-lab discussion.

Following the discussion period the students begin the experiment. By this time student interest has increased because they now have a vested interest in addressing the controversies that have arisen in their group discussions. Although we give general guidelines for the experiment, the students are left with many decisions that make each group's result unique. A typical set of student motion graphs is shown in Fig. 1. The measurements that students make are guided by a series of questions designed to focus attention on important issues, promote group discussion and ensure that differences and similarities between predicted and measured motion are noted (see Fig. 2).

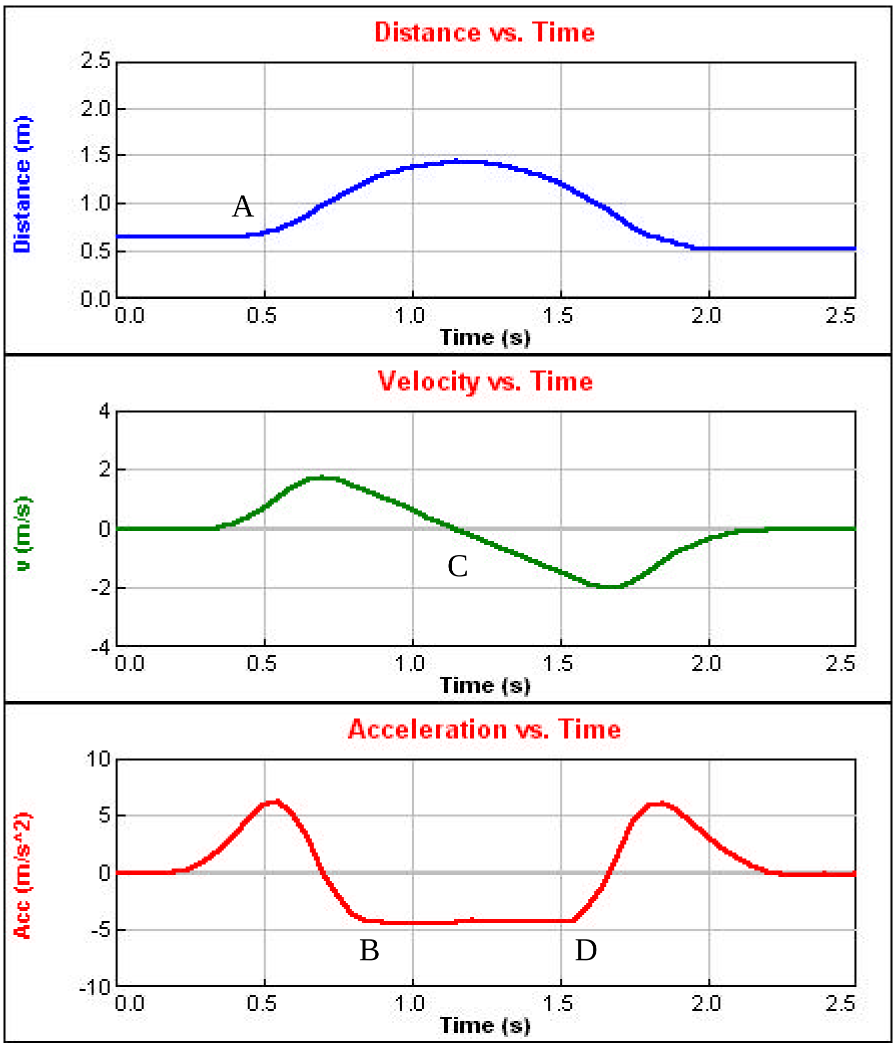

Figure 1. Typical graphs of position (distance from the detector), velocity, and acceleration vs. time created with Logger Pro[4] for a cart going up and down a ramp. The motion detector is at the bottom of the ramp. Point "A" on the position-time graph is when the push started, point "B" on the acceleration-time graph is when their hands left the cart, point "C" on the velocity-time graph is when the cart reached the turning point, and point "D" on the acceleration-time graph is when they stopped the cart.

Cart released from rest at the top of the ramp

- How do the position graphs differ from the position graphs for steady (constant velocity) motion?
- What feature of the velocity graphs indicates that the cart was speeding up? How does the velocity vary in time as the cart speeds up? Does it increase at a steady rate or in some other way?
- During the time that the cart is speeding up, is the acceleration positive or negative? How does speeding up while moving toward the detector result in this sign of acceleration? Explain by looking at how the velocity is changing. What would the sign of the acceleration be if the cart were speeding up while moving away from the detector? Check your prediction by moving the motion detector to the top of the track and letting the cart go from the top.
- How does the acceleration vary in time as the cart speeds up? Is this what you expect based upon the velocity graph? Explain.
- How is the magnitude (size) of the acceleration represented on a velocity-time graph? How is it represented on an acceleration-time graph?

Cart pushed up the incline and released

(Note: Students are required to sketch the graph in their notebooks and the label their graphs with "A" when the push started, "B" when their hands left the cart, "C" when the cart reached the turning point and "D" when they stopped the cart.)

- Did the cart "stop" at point C? How much time did it spend at zero velocity before it started back toward the detector?
- According to your acceleration graph, what is the acceleration at the instant the cart reaches its turning point? Is it any different from the acceleration during the rest of the motion?
- Explain the observed value and sign of the acceleration at the turning point in terms of velocities.
- Explain the nature of the graphs during your push (A to B) and when you stopped the cart (D).

Figure 2. Sample Questions for the "Cart on an Incline" Laboratory

The laboratory concludes with a class discussion of key concepts. Because the instructor has observed and talked with groups during the laboratory, this class discussion can be directed to focus on the students' own experiences. For the "Cart on an Incline" laboratory, some issues for discussion include:

- Acceleration is approximately constant and a function only of the ramp angle.
- The sign of acceleration has a carefully defined meaning.
- The definition of the coordinate system is important. (Placing the motion detector at the top or bottom of the ramp results in a different acceleration sign for the same motion.)
- The magnitude of acceleration is not arbitrary and should be part of the prediction.

This is also a time to extend the learning experience beyond the laboratory setup. Through this process students see that physics applies not only to the ideal case but to all cases. It is also a chance to motivate the study of future topics. For example, by questioning what will happen to the motion graphs as the ramp angle approaches a horizontal or vertical inclination, the discussion moves naturally into free fall. Or, stated in a different way, how is the motion of a ball thrown up in the air and caught again the same as—and how is it different from—the motion of the cart on the ramp?

<u>Example activity: Walking and Talking the Graphs</u>

The classroom activities are designed to reinforce and build upon the laboratory experiences. One of the most important is for students to draw a graph on the board and have other students walk the motion while explaining their reasoning. Common misconceptions—such as: positive acceleration means that speed is increasing, a direction reversal means a change in the sign of acceleration, negative velocity means that motion must be taking place on the negative side of the origin, and so on—are all exposed and addressed in this lively activity.

Figure 3 shows a typical graph of a complexity that most students will be able to handle by the end of the kinematics unit. Note that we do not indicate whether this is a position, velocity or acceleration versus time graph: the students will walk all three possibilities. This is particularly valuable for helping students see how the same graph features imply different motion, depending on which variable is being graphed. For example, in Figure 3 the graph begins with a horizontal line. By walking this same graph shape three times, students will practice initially standing still on the positive side of the origin (position-time graph), walking in the positive direction at a constant rate (velocity-time graph), and walking with increasingly positive velocity (acceleration-time graph). Interesting details, such as the importance of initial conditions and the physical meaning of discontinuous slopes, also come out in this exercise and are discussed later in this paper.

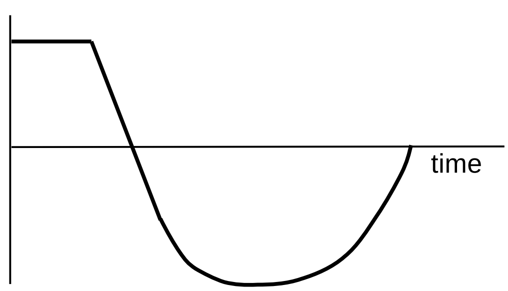

Figure 3. Typical student graph that students must walk three times while discussing their motion—once as a position graph, once as a velocity graph and once as an acceleration graph.

Building a deep understanding within the context of a conceptual framework

In response to the NRC's Key Point #2, it should be noted that the study of dynamics and its applications is traditionally divided into about ten textbook chapters. Thus it is not surprising that students tend to concentrate on learning techniques to solve problems within each chapter and often don't see how it all fits together. In our approach we teach all of dynamics within a simple conceptual framework (Fig. 4). Students learn that the role of kinematics in this framework is the description of motion.

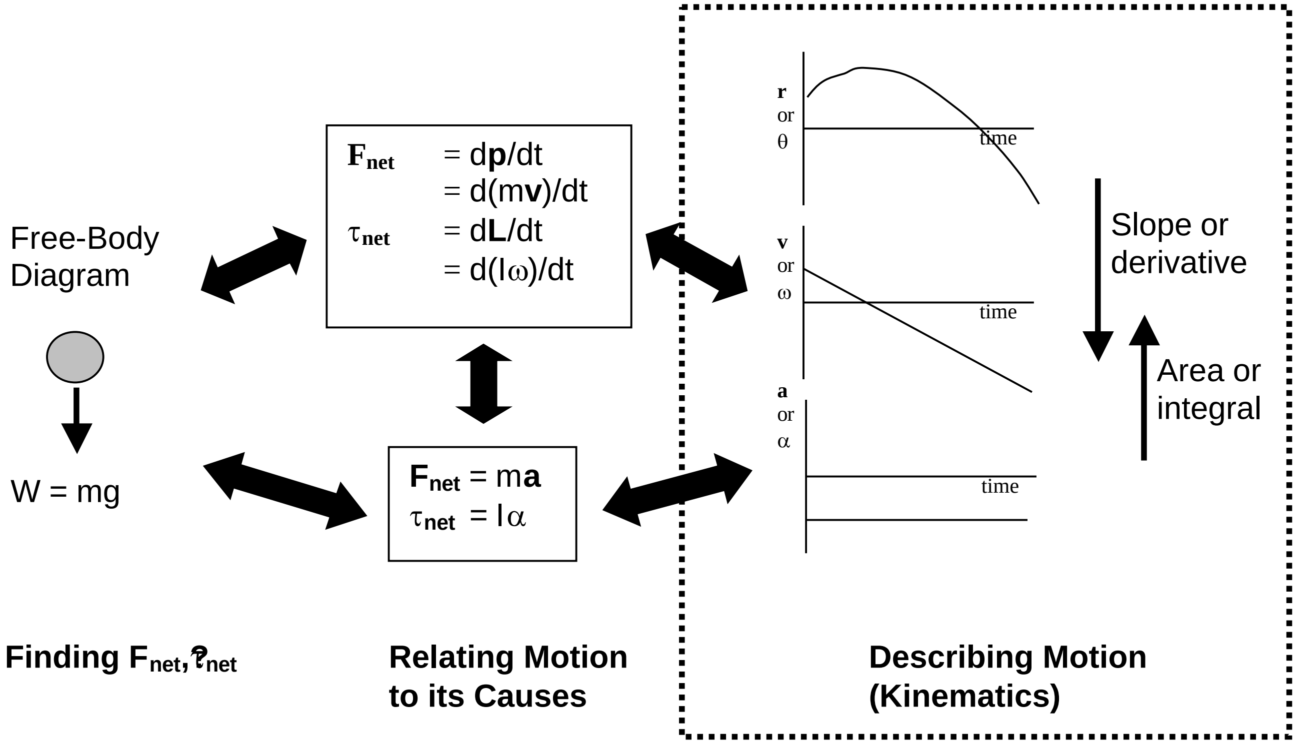

Figure 4. Understanding kinematics within the conceptual framework of dynamics

From Figure 4, the two fundamental ideas that students will build upon are that (1) motion can be described through position, velocity and acceleration versus time graphs—and so it is important to thoroughly understand how the features of each graph can be related to real-world motion—and (2) the graphs are related through their slopes and areas. At the heart of each laboratory, activity or problem solution are the following relationships that students discover through directed laboratory activity.

- The slope of position (x) versus time (t) = velocity (v) at that time.
- The slope of velocity (v) versus time (t) = acceleration (a) at that time.
- The area under velocity (v) versus time (t) = change in position (Δx) for that time period.
- The area under acceleration (a) versus time (t) = change in velocity (Δv) for that time period.

We will now examine how students build the content knowledge and skills for kinematics upon this framework.

Constant Acceleration Motion

Our approach for studying constant acceleration motion differs in a fundamental way from the more traditional approach that uses constant acceleration equations. Compared to graphical analysis, the role of constant acceleration equations in a dynamics framework is less easily

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grasped because these equations are only a special case of general motion. Students consider general motion only during the derivation of the equations, and their *application* does not require any understanding of their *origin*. However, in our approach to graphical analysis students essentially derive the constant acceleration equations each time they solve a problem. A simple one-dimensional word problem will illustrate the thought process that our students learn to follow when solving all kinematics problems.

> *A car is moving at 25 m/s when the rider applies the brakes, giving the car a constant deceleration. During braking the car travels 80 meters while reducing its speed to 15 m/s. Find the acceleration of the car and the time required for braking.*

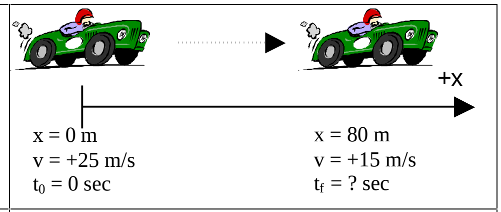
The solution to this problem is illustrated in Figure 5. Of particular importance to this solution procedure is that the students see the motion graphically illustrated every time they solve a problem. This reinforces both their ability to relate the graphical representation to the actual motion and their understanding of the connections among the graphs. They also solve each problem beginning with fundamental principles (i.e. motion graphs and slope/derivative and area/integral relationships) that are in the conceptual framework shown in Figure 4. In this manner free fall and later multi-dimensional and rotational kinematics are approached the same way. Even the derivation of the constant acceleration equations is approached this way as a homework question for practicing graphical analysis.

Non-constant acceleration

Graphical analysis provides students with a tool that can be used to analyze all motion. Constant acceleration problems and non-constant acceleration problems are solved using the same procedure illustrated in Figure 5. In the graphical analysis approach, the only difference that occurs for non-constant acceleration is that students must use an approximate method for estimating the slopes and areas of curves. Motion detectors with software such as MacMotion and Logger *Pro*[4] are designed to make these calculations easy. Video-analysis software such as VideoPoint[5] provides another effective tool for analyzing non-constant acceleration motion. An example is the motion of the cart on the ramp discussed earlier in this paper. While the cart is pushed or caught it has a varying acceleration (see Figure 1). Walking humans and falling coffee filters are two other examples that students study in the laboratory. Later, when we introduce forces as the cause of motion and explore the dynamics conceptual framework more fully, students will be introduced to a similar graphical approach in which the forces are often time-varying.

Graphical analysis as a step toward calculus-based physics

Students soon discover on their own that most time-varying acceleration problems can be solved only by approximate methods using graphical analysis. At this point students have already developed an understanding of many calculus concepts, making the transition to a calculus approach a natural one as students learn to replace slopes and areas with derivatives and integrals. Although the most important and obvious calculus preparation takes place through the practice of graphing the slopes and area of functions, other useful concepts are also learned to support the study of calculus.

<table>
<tr>
<td><u>Step 1</u>
Write down a coordinate system for solving the problem.</td>
<td>
</td>
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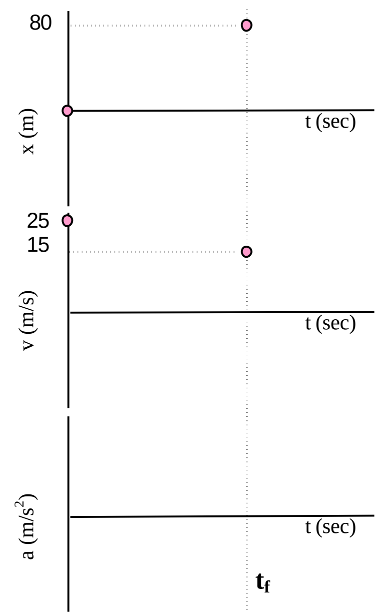
<tr>
<td><u>Step 2</u>
Draw axes for position, velocity, and acceleration versus time graphs (one set for each dimension of the problem). Place on these axes the information given in the problem statement.

As in any solution procedure, students must learn to interpret the meaning of words such as from rest, stop, drop, etc. in terms of graphical information. In this case we know the initial position and velocity and the final position and velocity for our coordinate system. There may also be implicit information in some problems. For example, students know from laboratories that the constant acceleration approximation is more appropriate in some free-fall situations than others.</td>
<td>
</td>
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<tr>
<td><u>Step 3</u>
Draw in the functions for each graph.

At this point students must use their understanding of the slope/area relationships among the graphs as well as their physical meaning. Because it is given that acceleration is constant and the change in velocity is negative, the acceleration graph must be a negative constant (a_0). Because acceleration is constant, the slope of velocity must be constant and therefore v is a straight line (because slope of v = a). Because v decreases in a straight line from +25 to +15, the position graph must be a parabola that increases in value with a slope that decreases from +25 m/s to +15 m/s (because slope x = v).

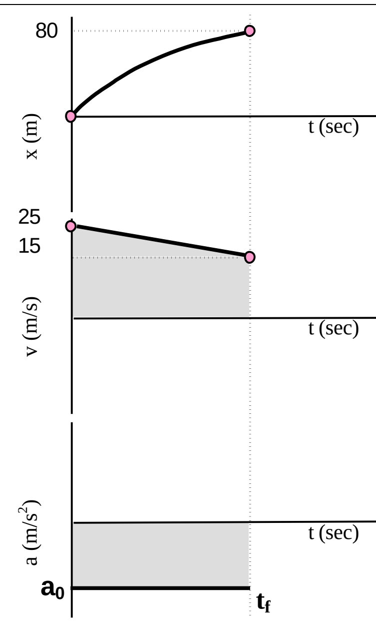
How do students know that position is parabolic when the velocity is linear? They have already learned that relationship when they graphically derived the equations of constant acceleration. We also emphasize correctly drawing the graph details. For example, in this problem a position graph that begins with a vertical slope or ends with a horizontal slope is a common error that is quickly addressed.</td>
<td>
</td>
</tr>
<tr>
<td><u>Step 4</u>
Solve for the unknown variables by applying the area relationships.</td>
<td>Area under v = change in x
$20t_f = 80 \rightarrow$ t_f = <u>4 seconds</u>
Area under a = change in v
$a_0 t_f = -10 \rightarrow a_0$ = <u>-2.5 m/s^2</u></td>
</tr>
</table>

Figure 5. Graphical analysis solution of a braking car.

For example, *Walking and Talking the Graphs* is an activity rich in calculus concepts. Students are exposed to position, velocity and acceleration graphs in that order. In the first few days of kinematics students need to walk only the position-time graphs. But soon they must also walk the same graph shape as a velocity-time graph. "But where do I start?" is invariably the response of the first student asked to walk a velocity-time graph. This leads to a "time out" from the activity as each laboratory team is given a chance to discuss the question. Students normally come to the conclusion on their own that it makes no difference. Further discussion leads to the conclusion that an infinite number of position-time graphs, differing only by a constant, exist for any given velocity-time graph. However, only a single velocity-time graph exists for a given position-time graph. Thus before learning calculus students readily understand that derivatives are unique and that integral functions vary by a constant that they can calculate when given the proper information.

Another important calculus concept is continuity. Students quickly realize that functions that are not smooth will have discontinuous slopes. The position-time graph shown in Figure 6 is a typical example. Even before students learn to calculate the slope of the graph, they realize that something strange is happening at the peak of the curve because of their difficulty in walking the graph and explaining their motion. The cause for this difficulty becomes more clear when they find that the velocity graph is discontinuous. Further discussion questions are raised when students attempt to take the derivative of the velocity graphs or later apply F=ma to understand the cause of this motion.

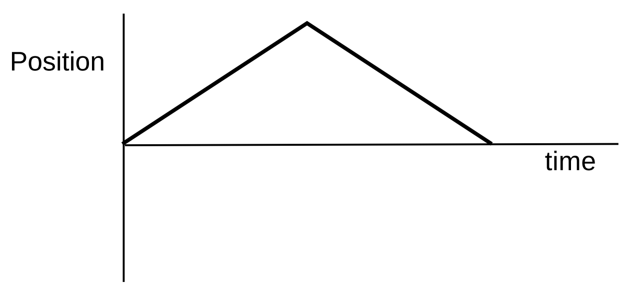


Figure 6. Position graph with a discontinuous slope at its peak.

Helping Students Take Control of their Learning

Our work on devising procedures for students to take control of their learning is ongoing; however, some basic elements are already in place. Through the use of concept maps (for example, Figure 4), students are made aware from the first day of class how kinematics fits into dynamics and how dynamics fits into the study of physics. At the beginning of the study of kinematics, we give students learning goals that will help them direct and assess their efforts. Practice problems, quizzes and tests (see Figure 7) are provided in addition to the regular homework assignments. Although these activities are optional, we have found that most students use them to measure their progress and identify problem areas.

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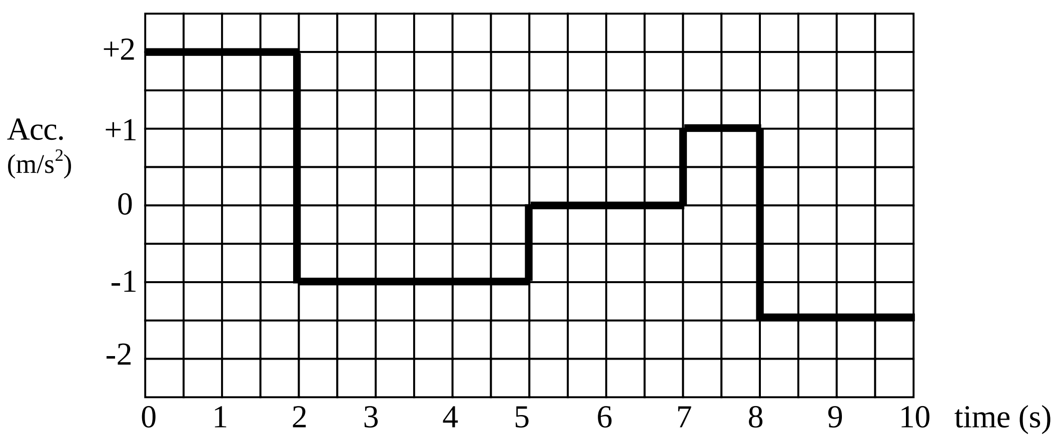
We encourage students to design their own practice problems. For example, the practice problem in Figure 7 can be altered by changing the graph shape, changing the motion variable, or modifying the questions asked about the graph. We also urge students to view problems from the perspective of a teacher and to devise their own word questions. For example, there are only five variables used to describe uniformly accelerated motion. These are displacement (Δd), initial velocity (v_o), final velocity (v_f), acceleration (a), and time (t). Because two independent equations relate the three motion graphs (area under a vs. t = Δv and area under v vs. t = Δx), they can expect to see values for three out of the five variables in a word problem, leaving them to solve for the two that are unknown. Students who have mastered the subject realize that there are only a finite number of possible problems that we can pose. Once they master the necessary manipulations, students can solve any problem of this type.

Carefully draw velocity and position versus time graphs for the motion shown below. Assume an initial velocity of –2 m/s and an initial position of –1 m. Show all calculations.



What is the average acceleration from t = 2 seconds to t = 7 seconds?
What is the instantaneous velocity at t = 3 seconds?
What is the average velocity from t = 2 seconds to t = 7 seconds?
What is the displacement from t = 1 second to t = 5 seconds?
What is the distance traveled from t = 0 seconds to t = 10 seconds?

Figure 7. Practice problem used to help students assess their own learning.

Student Response

We introduced the curriculum described in this paper in one trial introductory physics class during year 1 and, given its success, in all introductory physics classes the following year. Students greeted this change from a traditional curriculum enthusiastically. Course evaluations were positive. “I was surprised by how much I like physics” was an often-repeated student comment. Student enrollment for second-year physics courses (AP Physics C and Modern Physics) increased and, for the first time in school history, girls outnumbered boys in AP Physics C. In addition to observing the positive change in student attitudes, we also noticed a marked improvement in student understanding. We are developing a detailed formal assessment process that will (1) measure how this curriculum affects student attitudes and learning, and (2) compare these measurements to a control group taught in a more traditional way.

The effectiveness of graphical analysis for helping students learn calculus has been indicated through a number of channels. Numerous students who had participated in the first-year trial class reported that they had a substantial advantage over their classmates in 12^{th}-grade calculus. Their instructors also supported these claims. That same year the AP Physics C class was composed of both students from the trial class and students exposed the previous year to a more traditional approach. Students from the trial section made the transition to a calculus-based approach to kinematics far more easily than their classmates.

The following year, all of the student in AP Physics C had previously been exposed to graphical analysis. Because students already conceptually understood the calculus approach to kinematics, a substantial amount of class time was saved and students still showed a solid understanding of the subject. Interestingly, although students were given much practice using constant acceleration equations to increase their problem-solving speed for the AP exam, about half of the class continued to use graphical analysis because it helped them visualize the motion. Our message that deep understanding is more important than speed apparently made an impression.

Conclusion

We have developed a system of laboratories, activities, discussions and homework assignments that use a learner-centered approach to teach kinematics through graphical analysis. Observation of these classes and anecdotal student accounts indicate that this approach was successful for increasing conceptual understanding of kinematics as well as increasing student interest in the study of physics. A formal assessment process is needed to document the success of the approach and will begin in summer 2002.

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