AN INTRODUCTION TO LOGIC

FROM EVERYDAY LIFE TO FORMAL SYSTEMS

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Introduction: Language and Rationality

A. Different Ways to Use Language

When we use words to communicate, we are not always trying to say something that is either true or false. When a person, for instance, says, “Please pass me that paper,” he or she has not said anything that is true or false. Rather, that person has made a request that will either be granted or not granted. Likewise, when a mother tells her child, “Always hold an adults’ hand when you go across the street,” she has not said anything that is true or false. Rather, she has issued a command. If someone asks, “Sir, where is the White House located?,” that individual has not made an assertion about any state of affairs, but has asked a question. There are many uses that language can be put to: making requests, issuing commands, asking questions, and expressing emotion. Making assertions that are either true or false is only one of them. It is that particular usage that we are most concerned with in logic.

A proposition is a description of some state of affairs that is either true or false. One way of determining whether a proposition is true is to actually observe the state of affairs it describes and see if that state of affairs corresponds to the description given. For instance, if someone declares that it is raining outside, one way of determining if this is actually true is to look outside and see if rain is falling. If rain is falling, then the proposition, "It is raining outside" is true. If rain is not falling, then the proposition is false.

However, we do not always use direct observation in order to decide whether a proposition is true or false. Often, we infer the truth or falsity of a proposition. Although we might not be able to see or hear what is going on outside, if someone enters from outside wearing a wet raincoat and is carrying a wet umbrella, we would normally conclude that the proposition, "It is raining outside" is true. Logic is concerned with how we reason from certain propositions accepted as true (e.g., Jones has just entered from outside wearing a wet raincoat and carrying a wet umbrella) to different propositions not otherwise known to be true (e.g., It is raining outside.) In everyday life and in formal systems, logic is the study of the forms of correct inference.
Although we might infer the truth of the proposition, "It is raining outside", from the wet raincoat and wet umbrella, normally we could still establish its truth or falsity independently by simply walking outside. But this kind of direct observation is not always possible. Suppose, for instance, that the safe in Jones' house has been robbed and we suspect Brown. Since the robbery has already taken place, it is not possible to directly observe who committed the robbery. The actual commission of the robbery is a historical fact no longer available to direct observation. This, however, does not mean that it is impossible to determine the truth as to whether Brown committed the robbery. It merely means that establishing who committed the robbery by direct observation is impossible. But by using logic, it is possible to infer whether or not Brown committed the robbery.

Suppose the police are called and they find fresh fingerprints on the safe. If those fingerprints match fingerprints previously taken from Brown's hands, then, it is reasonable for them to conclude that Brown robbed the safe. In court, the prosecutor could argue from the fact that Brown's fingerprints were on the safe (and other facts, e.g., that Brown had no witnesses to establish where he was at the time of the robbery, and that Brown had a history of robberies) to the conclusion that Brown robbed the safe in Jones' house. Brown's defense attorney, on the other hand, could present arguments to show that Brown's fingerprints on the safe were not sufficient evidence to conclude that Brown had robbed the safe. For example, what if Brown may have accidentally touched the safe while cleaning around it? What if Brown, with Jones’ permission, had recently looked inside the safe at a rare coin?

It is only because human beings are able to make logical inferences that we are able to know so many things to be true without direct experience of what is described. History and science would be impossible without logic. No human being has traveled from the earth to the sun. Yet, we know that the distance from the earth to the sun is approximately 93,000,000 miles. No thermometer has ever been placed at the interior of the earth. Yet, we know that the temperature of the earth's core is approximately 4,000 degrees centigrade. Such facts are known, not from direct experience of the situations described, but by inference from other facts already accepted as true.
We depend on inferences not only in academic disciplines such as history and science, but in our everyday personal interactions with one another. We make inferences whenever we form an opinion about a person's intentions based on his or her actions. We do not directly observe other peoples intentions. Rather, a person’s intentions are inferred from what we do observe. A common source of misunderstanding is the failure to appreciate that, while we may observe what a person does, we do not observe the person's intentions.

**B. Rationality**

Every situation is open to multiple interpretations depending on the perspective from which it is viewed. In order to know what a person's intentions are, it is necessary to see that person's actions from that person's point of view. When we ask a person why he or she is doing something, we are asking that person to express in words the assumptions which give meaning to what that person is doing. We are, in short, asking for an explanation of his or her actions.

Being rational means accepting the responsibility of putting actions and situations into a framework that others can use to understand those actions and situations. Since different people may interpret the same action or situation in different ways, each must attempt to communicate to the other the different assumptions he or she brings to bear on the case at hand. One is rational to the extent that one is committed to maintaining and strengthening one’s relationship with others through the exchange of information and different points of view.

When a person communicates to others the framework within which he or she is acting, then others are able to understand what that person is doing. This also forms a basis for understanding some of the things the person has done in the past and will do in the future. A rational framework gives meaning to a particular thing, situation, or activity by clearly indicating its relationship to other things, situations, and activities. By establishing such relationships, a rational framework makes possible inferences from present situations to past and future situations, and from a particular case to other similar cases.
Consider the following:

If you see Tamara walk to a window, open it, and climb out onto the ledge, in order to know what Tamara intends requires that you engage in inquiry. You might ask her why she is climbing onto the window ledge. Your question might elicit one of the following responses: (1) the dirty window panes irritate her, and she is climbing onto the ledge in order to clean them; (2) the building is on fire and all exits are blocked except the windows; or (3) she has grown tired of the loneliness and futility of life and has decided to commit suicide.

Given the first explanation of why Tamara is climbing onto the window ledge, others may expect her to clean the window and then climb back inside. Based on the second and third explanations, however, she would be expected to leap from the ledge, instead of climbing back inside. But even though we might expect her to leap in the second and third cases, the reasons why she is expected to leap are different. Though she may leap in either case, the meaning of her leaping, or what she intends to accomplish by leaping, is different. In the second case, by leaping she means to save her life, while in the third case, by leaping she means to end her life.

An essential part of being rational is recognizing that when we see someone do something, it is not always possible to see what that person intends by what he or she is doing. One common source of misunderstanding often occurs when two people assume that they understand what one another means, when, in fact, they don't understand. As a result, people often argue in disagreement when, in fact, they agree. In other cases, people believe they are in agreement when, in fact, they are not. By requiring that the assumptions underlying our judgments and actions be spelled out as clearly and precisely as possible, logic facilitates the sharing and exchange of different points of view. We learn to avoid assuming that we know what another person means or intends without having to explain how we know. When we characterize what someone does or says, we should be prepared to explain why we believe that particular interpretation is true.
What has been said about understanding what another person is doing applies equally to understanding our own actions. We understand our own actions to the degree that we have articulated to ourselves a framework that gives meaning to those actions. But even this is not sufficient for one who is committed to being rational. A person may believe that she understands what she is doing, but the ultimate test of this is whether she is able to communicate this understanding to others. For, if no one can understand what she means, it is not obvious that she is making sense at all. Clarifying our actions and thoughts to others is an essential part of clarifying our actions and thoughts to ourselves.
CHAPTER 1:
THE STRUCTURE OF ARGUMENTS

1.A. Distinguishing Arguments from Non-arguments

Not any collection of propositions is an explanation or argument. Consider the following passage:

Wilson walked down the long, dimly-lit corridor. There were doors on each side of the hallway, each about five feet apart. As he passed he could hear the muffled sound of sleeping children on the other side of the closed doors.

The above passage is certainly a collection of propositions, each of which may be true. But there is no suggestion that the truth of any one of them is meant to follow from the truth of the other two propositions. In order for a collection of propositions to be considered an explanation or argument, the person who asserts those propositions to be true must intend that the truth of a particular one of the propositions (the conclusion) should follow from the truth of the other propositions (the premises).

In an explanation or argument, the collection of propositions is organized with the intention that our acceptance of certain of those propositions as true (the premises) will lead us to accept the truth of another of those propositions whose truth is not otherwise established (the conclusion). Explanations and arguments are similar. In an explanation we attempt to show why a proposition that is not expected to be true is nonetheless true. Thus, we explain why the earth goes around the sun, although it seems that the sun goes around the earth. In an argument we attempt to show why a proposition that may not be considered true should nonetheless be accepted as true. An explanation is often after the fact. An argument is often before the fact. However, unless the speaker intends to establish the truth of some proposition (the conclusion) on the basis of our accepting the truth of certain other propositions (the premises), that speaker is offering neither an explanation nor an argument.
The **standard logical form** for any explanation or argument is to list the premises first and the conclusion last.


premise 1
premise 2
. .
premise n

conclusion

This is written, laterally, as:

Premise 1 / premise 2 / … / premise n // conclusion

The premises are supposed to be connected in a way that shows why their truth makes the conclusion true. Often people use words to signal whether a claim they have made is intended to be a premise or a conclusion. These words are called premise-indicators and conclusion-indicators. A word, phrase, or symbol is a premise-indicator if what follows it is the premise of an explanation or argument.

**Examples of Premise-Indicators:**

Since
Let us assume that
Whereas
Because
Given that
. .
A word, phrase, or symbol is a conclusion-indicator if what follows it is the conclusion of an argument or explanation.

**Examples of Conclusion-Indicators**

Therefore
It follows that
So
Hence
Thus
Accordingly

To illustrate the relationship between premises and conclusions, consider the following episode:

1. Mrs. Jones had had it. She was sick and tired of her son
2. Billy, who failed to pick up after himself, help around the house, or contribute toward paying the bills. She had asked him time and time again to help and he always said he would, but he never really made any attempt to do so.
3. She made up her mind that this was not going to continue.
4. “Billy,” she said, as he was about to leave the next morning.
5. “I have something to tell you. When you come back this evening, the locks are going to be changed on the doors.
6. I am not going to let you stay here any longer.”
7. Billy was shocked. He knew he neglected his duties, but it had never occurred to him that his mother would ever say such a thing to him.
8. “Why are you going to do that?” he asked.
9. “Because you don't help out and nobody who stays under my roof is going to be a freeloader. So, I'm going to lock
17. you out.”
18. Mrs. Jones looked straight into Billy's eyes as she said this, because she wanted him to know that she meant it.
19. Billy didn't know what to say. He couldn't believe that his mother would actually do what she was threatening.
20. Often, he had found that the best way to deal with her when she was angry was simply to say nothing at all. So, he merely nodded, picked up his coat and walked out the door.
21. When Billy arrived home that night he found his clothes on the sidewalk. He couldn't believe it, and rushed to confront his mother in the house. But when he tried to open the door, his key wouldn't fit the lock.
22. “Mother,” he cried. “How could you do this to me? I thought you loved me, but you don't care anymore about me than you would a stray cat. How could you lock me out like this? How?” he screamed.
23. Mrs. Jones knew he would appeal to her motherly feelings toward him, but she controlled her responses and answered him evenly.
24. “I love you dearly, Billy,” she said, her voice slightly quivering, “but you use my love as a way of avoiding your responsibilities. It is no good for you to get in the habit of using people simply because they care about you, and that is what will happen if I let you keep using me.
25. So, I've decided that it's in your own best interest to learn your lesson now rather than later. You will only hurt yourself by taking advantage of the people who love you. That is why I've put you out.”
26. “You don't really love me,” Billy cried back. “You don't really care, because if you did, you wouldn't put me out
48. with no place to go. What kind of love is that? No,
49. you don't really care about me. You just want to have
50. people around who do only what you say do, that's all.”

In lines 7 through 10, Mrs. Jones announces to Billy that she does not intend to allow him to continue residing in her house. In line 14, Billy asks her to explain why she intends to put him out. In lines 15 through 17, Mrs. Jones explains why she intends to put her son out. Her explanation has the following standard form:

No person that is a freeloader is a person who can reside in my house.
You are a freeloader.
You are not a person who can reside in my house.

No person that is a freeloader is a person who can reside in my house. / You are a freeloader. // You are not a person who can reside in my house.

The explanation has the following logical form:

No A is B  A = person that is a freeloader
  x is A       B = person who can reside in my home
  x is not B
  No A is B. / x is A. // x is not B.

In line 37, Mrs. Jones says that she loves Billy, but in line 46-49, Billy attacks that claim, and argues that Mrs. Jones does not really love him. Billy's argument has the following standard form:

If you are a mother that loves her child, then you will not turn
that child away with no place to go.
You have turned your child away with no place to go.
You are not a mother that loves her child.
If you are a mother that loves her child, then you will not turn that child away with no place to go. / You have turned your child away with no place to go. // You are not a mother that lover her child.

Billy's argument has the following logical form, called Modus Tollens:

If P then not Q
Q
Not P
Or: If P then not Q. / Q. // not P

Both Mrs. Jones' explanation and Billy's argument are collections of statements presented with the intention that one proposition, the conclusion, be accepted as true because of its relationship with certain other propositions, the premises, which are assumed to be true. The truth of the conclusion is supposed to follow from the truth of the premises. Thus, we may assume that Mrs. Jones had long held that no freeloader could reside in her house, but it was only when she realized that her son was indeed a freeloader that she was forced to the conclusion that she had to put him out. In a similar fashion, if Billy believes it is true that if a mother loves her child, then she will not turn him away, then his being kicked out will force him to the conclusion that his mother does not love him. Since both explanations and arguments are collections of statements where the truth of the premises are supposed to establish the truth (or falsity) of the conclusion, they have the same basic structure and so, in the remainder of this text, explanations and arguments will be referred to generically as “arguments.”
1.A.1. **Exercises on Indicators:**

Give other examples of premise-indicators and conclusions-indicators.

1.A.2. **Exercises on Distinguishing Arguments from Non-Arguments:**

For each of the following, indicate whether it a) is an argument in the generic form by writing “A” or b) is not an argument by writing “NA” on the line provided.

1. I jog every morning because that's the only way I can avoid gaining too much weight.

2. If you have never sampled Chinese cooking, then you cannot say that you prefer American to Chinese cooking. Likewise, if you have never been to Athens, you cannot say that you prefer Paris to Athens. In general, if a person has not sampled all the options available to him, he cannot prefer that which he is familiar with over that which he has no knowledge of.

3. American influence is falling in foreign affairs. Inflation is driving prices sky-high. Pollution is poisoning us with cancer-causing agents, and, global warming is increasing.

4. If I were rich, I'd buy you the highest mountain, but I am not rich. That is why I cannot buy you the highest mountain.

5. I don't know why I should go to college. I don't know why the sun rises every day.
6. Mice are rodents. Rats are rodents. And, hamsters are rodents.

7. The Arabs are the primary oil exporting people in the world today. Most of the Arabian people are in a state of war with Israel. But the United States supports Israel.

8. Since religious freedom requires that the state show no favoritism between religions, it appears that there can be no religious freedom in America, for the American constitution was created in accordance with Christian principles and these principles are often in conflict with the principles of Islam, Hinduism, and many other religions.

9. The state can order its citizens to face death on the battlefield upon a declaration of war. That is why if a citizen refuses, that citizen is subject to arrest and prosecution.

10. Capitalists are used to exploiting other people. Women are the most exploited of all classes of people. Women make up the majority of churchgoers, too.
1.A.3. Exercises on Distinguishing Premises from Conclusion:

For each of the following arguments, pick out the conclusion and write it on the line below.

1. Since I'd marry you if I loved you, it follows that I cannot marry you, for unfortunately I do not really love you.

2. Either Jones loves music or Jones is trying to trick us. Let us assume that Jones is an honest man and is not trying to trick us. Then, it would appear that Jones really does love music.

3. The coal miners voted to strike because the coal contracts did not meet their demands.

4. Every time I read a book it gives me a headache. So, as far as I'm concerned, I can do without reading because my health is more important to me.

5. All millionaires make interest-free loans to their friends. Accordingly, if I were a millionaire, I'd make interest-free loans to my friends, too.

6. Since this is a red wine, it cannot be a Chablis.

7. Washington, D. C. has become more multi-racial over the last twenty years. Chicago, Detroit, Atlanta, and Philadelphia have too. Thus, the trend is that all large cities are becoming more multi-racial.
8. The Israelis believe they were invaded from Lebanon, and therefore, that they were justified in invading Lebanon.

9. John studies hard and is very conscientious. Hence, John will do well in college because that is what it takes.

10. John doesn't appear to have made it to heaven because he said that if he went to heaven when he died, he'd come back to tell me about it. But, he hasn't been back to say anything to me.

11. Almost every time I eat spinach, I get sick. Thus, if I eat this spinach, then I'll probably get sick.

12. Since no \( K \) is \( R \) and some \( R \) is not \( T \), it follows that some \( K \) is not \( T \).

13. Some \( S \) are not \( P \) because some \( S \) are \( M \) and some \( M \) are not \( P \).

14. Some \( S \) are not \( M \) and some \( M \) are not \( P \). Therefore, some \( S \) are not \( P \).

15. Since all \( A \) are \( B \), it seems that some \( A \) are \( C \) because some \( B \) are \( C \).

16. No \( Y \) is \( Q \) and all \( Q \) is \( J \), so no \( Y \) is \( J \).
17. If I see someone being robbed I would try to help them because I would want someone to try to help me if I were being robbed.

18. Since either you love me or you would not treat me so badly, I surmise that you don't love me, for you do treat me badly.

19. If it is raining outside then there are clouds in the sky, so there must be clouds in the sky because it is certainly raining outside.

20. Some people are taller than others; some are heavier than others; some can run faster than others; and some can swim better than others. Such facts show that it is useless to insist that every person is equal. We are all unequal.

21. Each person is different from each other person and each person has something to offer that another person does not have to offer. Thus, people are equal because people are different.

22. I'm not going to college, for college simply teaches you to survive within the system and the system is decaying.

23. Over the centuries human beings have developed a great deal of knowledge about many different kinds of things. Most of this knowledge has been codified and is taught within our universities. That is why I am going to college.
24. People lose confidence in their ability to adapt to new situations when they stop running, jumping, bending, stooping, crawling, and stretching their bodies into different positions. As a result, people become set in their ways.

25. People stop stretching their bodies into different positions because they become set in their ways and lose confidence in their ability to adapt to new situations.

26. “. . .the intellectual skills normal people display are limited, in practice, not so much by innate restrictions. . .as by the amount of motivating energy they are able to bring to bear on the tasks in question. It is the need to think that fuels the conceptual elaboration; while conversely, the level of skill reached evidences the strength of the need that underlies it. This train of thought leads us to expect that limits in channel capacity are rarely reached; and that many individuals' levels of accomplishment will alter dramatically, for better or worse, as their emotional states alter.”

27. Marx was a materialist only by expedience. His purpose was to show the limits of capitalism. Since no actual socialist societies existed, he could not present an actual alternative to capitalism. But by assuming that material accumulation for self-interest and other assumptions basic to capitalism held, Marx was able to show that capitalism was internally inconsistent and would necessarily destroy itself. Only by hypothetically accepting materialism could Marx prove the transient nature of capitalism. But none of this means that Marx personally was a materialist.
B. Logic in Everyday Life

Logic makes clear the criteria we use for deciding whether a particular conclusion follows from a given set of premises. It is important to recognize that one does not take a course in logic in order to learn how to be logical. People are usually logical. Whether a person is educated or not is irrelevant. Logic is basic to how human beings communicate and interact with one another. To illustrate this, consider the following exchange which takes place between Ms. Flotmos and her three-year-old daughter, Jamie:

1. “If you clean up your room, then I will take you for a treat,” said Ms. Flotmos to her daughter, Jamie.
2. Jamie was excited by this and hurriedly cleaned up
3. her room. When she finished, she went to her mother and
4. said, ”Well, I'm finished! Can we go for the treat now?”
5. Her mother looked at the room. “You did a wonderful
6. job, Jamie, but why do you speak as if we ought to
7. be going for a treat? You know I am trying hard to lose weight”.
8. Jamie was startled. “But, mother,” she cried, “you
9. said that if I cleaned up my room, then you would take me
10. for a treat .”
11. “True,” Ms. Flotmos replied, “but what does that have
12. to do with me going for a treat?”
13. “But mother,” the little girl cried, “you said
14. you would take me for a treat if I cleaned up my room, and
15. I cleaned up my room, so you're supposed to take me for a treat, like you said.”
16. But, I didn't merely say that I would take you for an ice cream treat ,
17. Jamie. I said that if you cleaned up your room, then, I
18. would take you for an ice cream treat ," replied
19. Ms. Flotmos. Ms. Flotmos was very concerned about her daughter's
20. intellectual development and always took time to discuss
21. issues with her.
22. “Understand carefully, now Jamie. I really didn't promise
23. that I would take you for a treat,” she said, as she kissed
24. her daughter on the forehead.
25. “But, you did say that if I cleaned up my room, then
26. you would take me for a treat, and that's why I cleaned up
27. my room so fast and so well—so you would take me for a treat. And, now you say that
you didn't say you would take
28. me,” Jamie screamed in disbelief at her mother. “Why did
29. you say you would take me if I cleaned up my room and now
30. you won't do what you promised?” Jamie cried.
31. Ms. Flotmos was a bit alarmed by Jamie's tone of voice
32. and wanted to quickly get the situation in hand.
33. “Now, Jamie,” she said firmly, “I've told you before about
34. twisting the truth. I didn't say that I would take you for a treat. What I said was “If
you clean your room, then I
35. will take you for a treat.”
37. and I did clean my room.”
38. “Well, I'm certainly happy to see that you've cleaned
39. your room, but I really don't like to be misquoted. I
40. didn't just say, ‘I will take you for a treat.’ I said, ‘If you
41. clean your room, then I will take you for a treat.’ Please try to understand: when I say
“when”, I say something with an ‘e’ in it, but I am not saying ‘e’. However,
42. since you seem to have your heart set on going for a treat, get
43. your coat and let us go. I love you so that I can't stand to
44. see you so distressed and upset.”
45. Jamie ran and got her coat and away she and her mother
46. went for a treat.

Her mother had said if Jamie cleaned her room, then she (Ms. Flotmos) would take Jamie for a
treat, and because Jamie cleaned her room, Jamie felt that her mother was supposed to take her
for a treat. While acknowledging what she had said and acknowledging that Jamie had cleaned
up her room, Ms. Flotmos nonetheless felt that she was not obligated to take Jamie for a treat.
We intuitively recognize that, though Jamie is only three years old, she is reasoning correctly.
Any person who reasons as Ms. Flotmos did would be illogical or a liar or both. Technically, Ms.
Flotmos is correct in holding that she did not unconditionally say that she would take Jamie for
an ice cream treat. Rather, she made a conditional statement, namely, “If you clean up your
room then I will take you for an ice cream treat.” The statement, “I will take you for an ice cream
treat,” (where ‘I’ refers to Ms. Flotmos) is a logical inference from the truth of the conditional
statement Ms. Flotmos made and the truth of the statement describing what Jamie did.

The statement “I will take you for a treat” is inferred by means of the following argument:

If you (Jamie) clean up your room, then I (Ms. Flotmos) will take
you for a treat.
You (Jamie) do clean up your room.  
I (Ms. Flotmos) will take you for a treat.

Jamie is justified in drawing the conclusion she did. If her mother spoke the truth when she said,
“If you clean up your room then I will take you for a treat,” and it is true that Jamie did clean up
her room, then her mother is obligated to act in such a way that the conclusion that she would
take her daughter for a treat is made true. If she does not do that, then, either she was not telling
the truth when she made the conditional statement or she is illogical.

The conclusion that Ms. Flotmos will take Jamie for a treat follows from the truth of what Ms.
Flotmos said and the truth of what Jamie did, just as the conclusion in the following argument
follows from the premises given:

If it is raining outside, then there are clouds in the sky.
It is raining outside.  
There are clouds in the sky.
Both of these arguments have the same argument form, called Modus Ponens:

\[
\begin{array}{c}
\text{If } A \text{ then } B \\
A \\
B
\end{array}
\]

In the study of logic, it is important that questions of form be distinguished from questions of content. The truth or falsity of a proposition is independent of the form of that proposition. When Ms. Flotmos said to Jamie, “If you clean up your room, then, I will take you for a treat,” the statement she made had the propositional form, “If A then B,” where:

\[
\begin{align*}
A &= \text{you clean up your room} \\
B &= \text{I will take you for a treat.}
\end{align*}
\]

Whenever one is given a statement of the form “If p then q,” and another statement “p,” and one assumes that both “If p then q” and “p” are true, then one is justified in inferring the truth of the statement referred to by “q.” Any person who does not acknowledge that the truth of “q” follows from the truth of “If p then q” and “p” is illogical. Only if one of the premises is false is one justified in not ascribing truth to such a conclusion.

Logic ignores content and focuses purely on questions of form. We have seen that in representing an argument in standard form, we ignore such particular features as whether the conclusion occurred at the beginning, in the middle, or at the end of the speaker's presentation. In representing the logical form of an argument, we ignore the subject matter of the propositions that make up the argument, and focus on their propositional forms and the manner in which these propositional forms are combined into argument forms.
C. Aristotelian Logic

A system of logic provides us with a way of representing propositions and their combinations into arguments so that it is possible to decide whether an argument so represented is acceptable or unacceptable. In this text, we introduce two generic systems of logic: Deductive logic and Inductive logic. Deductive logic is the primary model of reasoning used in mathematics, science, and legal proceedings. It has long been considered the gold standard of reasoning, in that a deductively sound argument is able to force agreement, even if the proposition agreed to is distasteful. Though many might intuitively reject the claim that all women are animals, if they accept the premises that all women are humans and all humans are animals, they are nonetheless forced to acknowledge that all women are animals.

For good deductive arguments, if the premises are true then the conclusion must be true. But with good inductive arguments, if the premises are true, the conclusion is not necessarily true, but only more probably true. The conclusion of an inductive argument is never guaranteed, even when all available evidence supports the conclusion. Inductive arguments are most prominent in probability, statistics, and the experimental sciences. We will explore the basic elements of inductive logic in Chapter 6. Chapters 2, 3, 4, and 5 will explore developments in deductive logic from its beginnings in Aristotle to the development of modern computers.

The first system of deductive logic to be introduced in this text is called categorical (or Aristotelian) logic. It is a notation system in which propositions are represented as combinations of two categories of things – the subject class and the predicate class. An example would be the (false) proposition “All dogs are cats”, where ‘dogs’ is the subject class and ‘cats’ is the predicate class. In terms of this system, every proposition has one of the following categorical propositional forms (no matter what the statement is about):

- A: All S are P's
- E: No S are P
- I: Some S are P
- O: Some S are not P
Following are some propositions which have **A** forms:

All jobs are wonderful.
All human beings are animals.
All flowers are plants.
All businessmen are honest.

Following are some propositions with **E** forms:

No freeloaders are people who will reside in my house.
No dogs are cats.
No lawyers are judges.
No mothers are women.

Following are some propositions which have **I** forms:

Some horses are big.
Some dogs are vicious.
Some cats are fish.
Some quarks are charmed.

Following are some propositions with **O** forms:

Some engines are not efficient.
Some dogs are not animals.
Some advertisements are not truthful.
Some men are not human.
It is important to remember that the form that a proposition has is independent of what that proposition is about and, therefore, independent of whether it is true or false. Form does not determine content, and content is independent of form. Following are some valid argument forms expressed in the categorical or Aristotelian system of logic:

All A are B
All B are C
All A are C

All J are K
No K are L.
No J are L

Some S are M
All M are P.
Some S are P

All A are B
Some A are not C.
Some B are not C
1.C.1. Exercises on Recognizing Categorical Propositions

For each of the following categorical propositions, (1) indicate its truth-value by writing T if the proposition is true and F if the proposition is false, and (2) write out the form of the proposition.

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Truth-Value</th>
<th>Categorical Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All dogs are animals.</td>
<td>T</td>
<td>All D are A</td>
</tr>
<tr>
<td>2. No animals are insects.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. All fish are living things</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Some living things are fish.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. No living things are insects.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Some rocks are not living.</td>
<td>T</td>
<td>Some R are not L</td>
</tr>
<tr>
<td>7. Some rocks are living.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Some dogs are living things.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. All living things are animals.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Some fish are not living.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1.C.2. **Exercises on Constructing Categorical Propositions:**

Using the following class concepts, construct two true and two false propositions for each of the propositional forms indicated.

**Class Concepts:** round things, brown things, dogs, cats, lizards, canines, felines, reptiles, mammals, animals.

<table>
<thead>
<tr>
<th>Truth-Value</th>
<th>Categorical Form</th>
<th>Proposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T)</td>
<td>All S are P.</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>All S are P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>All S are P.</td>
<td>All round things are brown things.</td>
</tr>
<tr>
<td>(F)</td>
<td>All S are P.</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>No S are P.</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>No S are P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>No S are P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>No S are P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>Some S are P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>Some S are P.</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>Some S are P.</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>Some S are P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>Some S are not P.</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>Some S are not P.</td>
<td>Some dogs are not canines.</td>
</tr>
<tr>
<td>(T)</td>
<td>Some S are not P.</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>Some S are not P.</td>
<td></td>
</tr>
</tbody>
</table>
### 1.C.3. Exercises on Recognizing Argument Forms:

For each of the following arguments, write out its argument form.

<table>
<thead>
<tr>
<th>Argument</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All dogs are animals. No animals are insects. No dogs are insects.</td>
<td>All D are A No A are I. No D are I</td>
</tr>
<tr>
<td>2. All fish are living things. Some living things are eaten. Some fish are eaten.</td>
<td></td>
</tr>
<tr>
<td>3. No rocks are living things. All living things are capable of feeling. No rocks are capable of feeling.</td>
<td></td>
</tr>
<tr>
<td>4. Some dogs are living things. Some living things are cats. Some dogs are cats.</td>
<td></td>
</tr>
<tr>
<td>5. Some fish are not living things. Some living things are not dogs. No fish are dogs.</td>
<td></td>
</tr>
<tr>
<td>6. No elephants are human beings. No human beings are mice. No elephants are mice.</td>
<td></td>
</tr>
<tr>
<td>7. All deer are mammals. All human beings are mammals. All deer are human beings.</td>
<td></td>
</tr>
</tbody>
</table>
D. Validity, Invalidity, and Refutations

Valid deductive arguments present premises which, if accepted, require that a certain conclusion also be accepted. But though a person may intend for the truth of a certain proposition to be conclusively established by the truth of the premises offered, this intention is not always realized. Consider the following argument:

Jones must be a Nazi because all Nazis hate Jews and Jones hates Jews.

Although the intention here obviously is to establish the truth of the proposition, “Jones is a Nazi,” the truth of that proposition does not follow from the truth of the premises given. This can be more clearly seen if the argument is put in standard form:

(a1) All Nazis are people who hate Jews.
     Jones is a person who hates Jews.
     Jones is a Nazi.

The argument has the same argument form as the following argument:

(a2) All tigers are animals that eat meat.
     Barack Obama is an animal that eats meat.
     Barack Obama is a tiger.

In each argument, the argument form is:

(a3) All A are B
    x is B
    x is A

In both cases, the premises might be true, but the conclusion does not necessarily follow. It may be true that all KKK members hate African Americans and it may be true that Jones hates
African Americans; however it does not necessarily follow that Jones must be a KKK member. Likewise, though it may be true that all tigers eat meat and it may be true that Barack Obama eats meat, it does not necessarily follow that Barack Obama is a tiger.

A deductive argument is one in which the maker of the argument intends for the truth of the conclusion to follow necessarily from the truth of the premises. Thus, if you accept the premises to be true, you must accept the conclusion to be true. But this intention cannot be realized if the maker uses an argument form from which it is possible to construct an argument where the premises are true but the conclusion is false.

DEFINITIONS

Valid Argument-- An argument is deductively valid if it has an argument form such that all arguments with that form transfer truth from premises to conclusion.

Invalid Argument-- An argument is deductively invalid if it has an argument form such that it is possible for an argument to exist with that form, yet have true premises and a false conclusion.

The development of procedures for deciding when an argument is valid or invalid is a central function of any system of deductive logic. A deductive argument is invalid when the truth of the premises of the argument does not establish the truth of the indicated conclusion. Unfortunately, when a conclusion is already known to be true independently of the truth of the premises offered, it is not always easy to see the faultiness of the argument. This is illustrated by the following categorical examples:

\[ (c_1) \quad \text{All dogs are canines.} \]
\[ \text{Some canines are vicious.} \]
\[ \text{Some dogs are vicious.} \]
(c₂) All guns are weapons,

Some weapons are used illegally.
Some guns are used illegally.

In each of these arguments, both the premises and the conclusion are true. Yet, neither (c₁) nor (c₂) is a valid argument because each has the following argument form, which is invalid:

(c) All A are B.
Some B are C.
Some A are C.

A technique commonly used to prove that an argument is invalid is to construct a different argument with exactly the same form as the argument in question, but where the premises are already known to be true and the conclusion is already known to be false. Such an example demonstrates that, for all arguments of a similar form, the truth of the premises does not guarantee the truth of the conclusion. Thus, the following argument proves that the arguments (c₁) and (c₂) are invalid:

(c₃) All dogs are animals.
Some animals are cats.
Some dogs are cats.

This argument has exactly the same form as (c₁) and (c₂). Yet, it is obvious here that the truth of the conclusion does not follow from the truth of the premises because, though in fact the premises are true, in fact, the conclusion is false. Arguing that the conclusion of (c₁) followed from the truth of its premises would be like arguing that the truth of the conclusion of (c₃) had to follow from the truth of the premises of (c₃).
The technique above involves producing a **refutation by analogy**. Thus, given a particular argument:

\( (e_1) \) No Muslims are Christians.
   No Christians are Hindu.
   No Muslims are Hindu.

Refutation by analogy involves extracting the form of that argument:

\( (e) \) No A are B.
    No B are C.
    No A are C.

and producing another argument having exactly the same form but where the premises are obviously true and the conclusion obviously false:

\( (e_2) \) No babies are adults.
    No adults are children.
    No babies are children.

or

\( (e_3) \) No men are elephants.
    No elephants are human.
    No men are human.

Both \( (e_2) \) and \( (e_3) \) have exactly the same form as \( (e_1) \), so that if the premises of \( (e_1) \) are taken as establishing the truth of its conclusion, then the premises of \( (e_2) \) and \( (e_3) \) should be taken as establishing the truth of their respective conclusions. But this could not be because we already know that the conclusions of \( (e_2) \) and \( (e_3) \) are false. As we will see in chpt.4, refutations by analogy are similarly constructed in the system of truth functional logic.
1.D.1. **Exercises on Categorical Forms:**

For each of the following invalid arguments, (a) specify the form of the argument and (b) construct a refutation by analogy using the following class concepts (p. 17):

Example: 

No dogs are cats. 
No cats are rats. 
No dogs are rats.

<table>
<thead>
<tr>
<th>argument form</th>
<th>refutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No A are B.</td>
<td>No dogs are felines. (T)</td>
</tr>
<tr>
<td>No B are C.</td>
<td>No felines are canines. (T)</td>
</tr>
<tr>
<td>No A are C.</td>
<td>No dogs are canines. (F)</td>
</tr>
</tbody>
</table>

1. Some gangsters are not educated. 
   Some educated people are not honest. 
   Some gangsters are not honest.

   a. argument form
   b. refutation

2. No horses are fish. 
   All fish are scaly. 
   No horses are scaly.

   a. argument form
   b. refutation
3. All cats are felines.
   All cats are animals.
   All felines are animals.
   
a. argument form
   b. refutation

4. Some students are serious people.
   Some serious people are very nice.
   Some students are very nice.
   
a. argument form
   b. refutation
E. Validity and Soundness

When we accept an argument, we do so believing that the argument is sound. A **sound deductive argument** is one where we accept its premises as true and the relationship between the premises and conclusion is such that it is impossible for the premises to be true and the conclusion false. A primary objective of this book is to clarify how to determine when the relationship between premises and conclusion is such as to imply the truth of the conclusion, given premises that are true. Thus, if we assume it true that all dogs are canines and we assume it true that all canines are mammals, then we must accept it as true that all dogs are mammals. This argument has the form:

\[
(g_1) \quad \text{All dogs are canines.}
\]
\[
\text{All canines are mammals.}
\]
\[
\text{All dogs are mammals.}
\]

This kind of a relationship between the premises of an argument and the conclusion of an argument is what is referred to when we say that an argument is valid. It means that acceptance of the premises requires acceptance of the conclusion. Thus, the following argument is also a valid argument:

\[
(g_2) \quad \text{All dogs are cats.}
\]
\[
\text{All cats are rats.}
\]
\[
\text{All dogs are rats.}
\]

Arguments \((g_1)\) and \((g_2)\) have the common form \((g)\):

\[
(g) \quad \text{All A are B}
\]
\[
\text{All B are C.}
\]
\[
\text{All A are C}
\]
Argument \((g_2)\) is a valid argument because if we accept the premises to be true, then we would have to accept the conclusion as being true. The difference between \((g_1)\) and \((g_2)\) is that, in \((g_1)\), we do in fact accept the premises of that argument to be true, whereas with argument \((g_2)\) in fact, we do not accept the premises of that argument to be true. But it is only because we do not accept the premises of \((g_2)\) to be true that we can avoid having to accept its conclusion as true. While \((g_2)\) is a valid argument, it is not a sound argument.

Definitions:

A deductive argument is **SOUND** if:

1. it has a valid argument form and
2. all of its premises are true.

A deductive argument is **FALLACIOUS** if:

1. it has an invalid argument form or
2. it has at least one false premise.

Irrespective of what a deductive argument may be about, if it has a valid argument form and if the premises are true, then it is impossible for the conclusion to be false. Thus, if it were true that all dogs are cats, and all cats are rats, then it would be impossible for there to be a dog which was not a rat. Thus, any argument with the same form as \((g)\) would be a valid argument.
1.E.1. Exercises on Constructing Sound and Unsound Arguments

Each of the following Argument Forms is Valid. For each, construct one sound and one unsound argument having the same form:

i. Categorical Forms

1. All X are Y  
   Sound:  
   Unsound:  
   All Y are Z  
   All X are Z

2. All X are Y  
   Sound:  
   Unsound:  
   No Y are Z  
   No X are Z

3. All X are Y  
   Sound:  
   Unsound:  
   Some X are not Z  
   Some Y are not Z

4. No B are C  
   Sound:  
   Unsound:  
   Some A are C  
   Some A are not B

5. All P are Q  
   Sound:  
   Unsound:  
   Some P are R  
   Some Q are R
F. Truth-Functional Logic

The second system of deductive logic to be discussed in this text is called Truth-Functional (or modern) logic. This system begins with simple propositions that do not contain other propositions, and represents them by alphabets such as “p,” “q,” and “r.” These simple propositions are then combined by logical connectives such as “and,” “or,” “if-then,” and “not” into compound propositions with the following kinds of propositional forms:

conditional -  if p then q  
conjunction -  p and q  
disjunction -  p or q  
negation -  not-q

Following are some compound propositions that have conditional forms:

If John goes to town, then Tom will go to town.  
If it is raining outside, then there are clouds in the sky.  
If the water is boiling, then the water is hot.  
If the economy expands, then unemployment will increase.

Following are some compound propositions that have conjunctive forms:

John is going to town and Tom will go to town.  
It is raining outside and there are clouds in the sky.  
The water is boiling and the water is hot.  
The economy will expand and unemployment will increase.
Following are some propositions that have **disjunctive** forms:

- John is going to town or Tom is going to town.
- It is raining outside or it is snowing outside.
- The water is boiling or the water is hot.
- The economy is expanding or unemployment will increase.

Following are some propositions that have **negation** forms:

- It is false that today is Saturday.
- I am not asleep.
- The water is not hot.
- It is not the case that unemployment will increase.

Propositions with these kinds of forms are then used to construct all the argument forms in the system. Some basic argument forms are:

\[
\begin{align*}
\text{If } p \text{ then } q & \quad \text{If it is raining outside then there are clouds in the sky.} \\
p & \quad \text{It is raining outside.} \\
q & \quad \text{There are clouds in the sky.}
\end{align*}
\]

\[
\begin{align*}
\text{If } p \text{ then } q & \quad \text{If the water is boiling then the water is hot.} \\
\text{not } q & \quad \text{The water is not hot.} \\
\text{not } p & \quad \text{The water is not boiling.}
\end{align*}
\]

\[
\begin{align*}
p \text{ or } q & \quad \text{The economy is expanding or unemployment will increase.} \\
\text{not } p & \quad \text{The economy is not expanding.} \\
q & \quad \text{Unemployment will increase.}
\end{align*}
\]
It is important to remember that the variables in categorical and truth-functional logic take different kinds of values: a variable used in the system of categorical logic represents a certain class of things, while a variable used in the truth functional system represents a complete proposition.

F.1. Exercises on Recognizing Truth Functional Propositions:

For each of the following propositions, use the following legend to state its propositional form and indicate whether the proposition is true or false:

Ax = this month is x. Bx = last month is x. Cx = next month is x.

m1=January m2=February m3=March m4=April m5=May m6=June m7=July
m8=August m9=September m10=October m11=November m12=December

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Propositional Form</th>
<th>Truth-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. If this month is January then next month is November.</td>
<td>If Am1 then Cm11</td>
<td>F</td>
</tr>
<tr>
<td>2. If this month is January then next month is not November.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. This month is March or this month is June.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. If next month is November and last month was September then this month is October.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. This month is July and this month is not July.

7. This month is May or this month is not May.

8. It is not the case that if next month is November then this month is January. not - (if Cm_{11} then Am_{1}) T

9. This month is not January and if next month is November then this month is October.

1.F.2. Exercises on Constructing Truth-Functional Propositions

Represent the class of days as follows: (Sunday = d_1, Monday = d_2, Tuesday = d_3, Wednesday = d_4, Thursday = d_5, Friday = d_6, Saturday = d_7)

Let Sx, Tx, and Ux be abbreviations for the following statement forms:

Sx = Yesterday was x.
Tx = Today is x.
Ux = Tomorrow is x.

Then the following examples have the abbreviated statement forms and truth-values indicated:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Propositional Form</th>
<th>Statement Form</th>
<th>Truth-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Today is Sunday or Yesterday was not Saturday</td>
<td>p or not-q</td>
<td>Td_1 or not-Sd_7</td>
<td>T</td>
</tr>
</tbody>
</table>
2. If tomorrow is Monday then today is Thursday

<table>
<thead>
<tr>
<th>Truth Value</th>
<th>Propositional Form</th>
<th>Statement Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>p and q</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>p and q</td>
<td>Sd₃ and Td₅</td>
</tr>
</tbody>
</table>

| T           | p or q             |
| F           | p or q             |

| T           | not-p              |
| F           | not-q              |

| T           | if p then q        |
| F           | if p then q        | If T d₅ then Ud₆ |
1. If today is Monday then yesterday was not Saturday.

   Today is Monday.
   Yesterday was not Saturday.

2. Tomorrow is Wednesday or tomorrow is Thursday.
   Tomorrow is not Wednesday.
   Tomorrow is Thursday.

3. If today is Friday then tomorrow is Saturday.
   It is not the case that tomorrow is Saturday.
   It is not the case that today is Friday.

4. Today is Monday or today is Tuesday.
   Today is not Tuesday.
   Today is not Monday

5. If (today is Wednesday or today is Thursday) then (yesterday was not Sunday and yesterday was not Monday).
   Today is Wednesday.
   Yesterday was not Sunday and was not Monday.
G. Validity, Invalidity, and Refutations

Valid deductive arguments present premises which, if accepted, require that a certain conclusion also be accepted. But though a person may intend for the truth of a certain proposition to be conclusively established by the truth of the premises offered, this intention is not always realized. In truth-functional form, we might be given the following argument:

Jones must be a Nazi because if somebody is a Nazi then they hate Jewish people and Jones certainly hates Jewish people.

As before, the intention is to establish the truth of the proposition “Jones is a Nazi.” The argument has the following standard form:

(b₁) If Jones is a Nazi then Jones hates Jewish people.
Jones hates Jewish people.
Jones is a Nazi.

Its argument form is:

(b) If p then q
q
p

This is also the form of the following argument:

(b₂) If Lady Gaga is a man then Lady Gaga is a human being.
Lady Gaga is a human being.
Lady Gaga is a man.
Argument (b₂) shows that in arguments of the form given by (b), the truth of the premises is not necessarily transferred to the conclusion, so that the premises of an argument of form (b) may be true, but the conclusion false. This means that the form of such arguments is itself not sufficient to guarantee that the truths of its premises are necessarily transferred to its conclusion. The form does not guarantee that true premises lead to true conclusions.

A deductive argument is one in which the maker of the argument intends for the truth of the conclusion to follow necessarily from the truth of the premises. Thus, if you accept the premises to be true, you must accept the conclusion to be true. But this intention cannot be realized if the maker uses an argument form from which an argument can be constructed where the premises are true but the conclusion is false.

**DEFINITIONS**

**Valid Argument**—An argument is deductively valid if it has an argument form such that all arguments with that form transfer truth from premises to conclusion.

**Invalid Argument**—An argument is deductively invalid if there is an argument with the same form where the premises are true and the conclusion is false.

A deductive argument is invalid when the truth of the premises of the argument does not establish the truth of the indicated conclusion. Unfortunately, when a conclusion is already known to be true independently of the truth of the premises offered, it is not always easy to see the faultiness of the argument. The development of procedures for deciding when an argument is invalid is a central feature of both categorical and truth-functional logic. This is illustrated in the following examples:

(d₁) If Paris, France is in the USA then Paris, France is in North America
     Paris, France is not in the USA.
     Paris, France is not in North America.
(d_2) If Jay Z is a lion then Jay Z is a feline.
   Jay Z is not a lion.
   Jay Z is not a feline.

In each of these arguments, the premises and the conclusion are true. Yet neither argument is valid because they share a common argument form, which is invalid:

(d) If p then q
   not p
   not q

The invalidity of (d) is shown by the following argument:

(d_3) If Barack Obama is a woman then Barack Obama is a human being.
   Barack Obama is not a woman.
   Barack Obama is not a human being.

It is certainly true that if Barack Obama is a woman then Barack Obama is a human being. And if Barack Obama is in fact a man, then it is also true that Barack Obama is not a woman. But if in fact Barack Obama is a man then it is certainly false that Barack Obama is not a human being. Thus, the premises of (d_3) would be true yet its conclusion false. This proves that any argument of form (d) fails to transfer truth from premises to conclusion. (d_3) is a *refutation by analogy* of (d).
Thus, given the following argument:

( f₁ ) If Washington, D.C. is not in Europe, then Washington, D.C. is not in France.  
Washington, D.C. is not in France.  
Washington, D.C. is not in Europe.

a refutation is produced by extracting its form

(f) If not-p then not-q  
    not-q ————  
    not-p

and producing another argument with the same form but where the premises are true and the conclusion is false:

( f₂ ) If Barack Obama is not human then Barack Obama is not a mother.  
Barack Obama is not a mother  
Barack Obama is not human

An invalid argument is one where the truth of its conclusion is not necessitated by the truth of its premises. The conclusion of an invalid argument may be true but it is not because the premises of that argument are true. When the conclusion of an invalid argument is true, its truth derives from information that is not included in the premises of that argument.
1.G.1. Exercises on Truth Functional Forms:

For each of the following invalid arguments: (a) specify the truth-functional form of the argument; and (b) construct a refutation by analogy.

1. If today is Monday then tomorrow is Tuesday.
   Today is not Monday.
   Tomorrow is not Tuesday.
   
   form: \( \text{If } p \text{ then } q \)  
   refutation: \( \text{If } \text{BO is a woman then } \text{BO is a human.} \)  
   \( \text{not-}p \) \( \text{BO is not a woman} \)  
   \( \text{not-}q \) \( \text{BO is not a human} \)

2. If Bill has had his morning coffee then Bill is not be sleepy.
   Bill is not sleepy.
   Bill has had his morning coffee
   
   form:  
   refutation:  

3. Either Susan is the murderer or Susan is being framed.
   Susan is not the murderer.
   Susan is not being framed.
   
   form:  
   refutation:
4. If Mary does not get a raise then Mary will quit her job.

   Mary does get a raise. __________________________

   Mary will not quit her job.

form: refutation:

more exercises needed
**H. Sound Deductive Arguments**

When we accept an argument, we do so believing that the argument is sound. A sound deductive argument is one where we accept its premises as true and the relationship between the premises and conclusion is such that it is impossible for the premises to be true and the conclusion false. A primary objective of logic is to clarify how to determine when the relationship between premises and conclusion is such as to imply the truth of the conclusion, given premises that are true.

Definitions:

A deductive argument is SOUND if:
(1) it has a valid argument form and
(2) all of its premises are true.

A deductive argument is FALLACIOUS if:
(1) it has an invalid argument form or
(2) it has at least one false premise.

1.H.i Exercises on Constructing Sound and Unsound Arguments:

Each of the following Argument Forms is valid. For each, construct one sound and one unsound argument having the same form:

ii. Truth Functional Forms

1. If p then q

    Sound: not-q

    Unsound

    not-p
2. If not-p then not-q  
   Sound: Unsound
   not-p
   not-q

3. Either not-p or not-q  
   Sound: Unsound
   q
   not-p

4. If p then not-q  
   Sound: Unsound
   q
   not-p

5. If not-p then q  
   Sound: Unsound
   not-q
   p

1.H.1. Exercises:

Indicate, for each of the following statements, whether it is true (T), or false (F).

1. If an argument is valid, then the truth of the premises
   necessarily implies the truth of the conclusion.  
   __________

2. An argument is invalid if any other argument of identical logical form can have true
   premises and a false conclusion.  
   __________
3. An unsound argument must be invalid.

4. A sound argument must have true premises.

5. An unsound argument must have two false premises.

6. An unsound argument must have a false conclusion.

7. An unsound argument must have one false premise.

8. A refutation of an invalid argument must have a false premise.

9. A refutation of an invalid argument must have a false conclusion.

10. An argument is valid if the truth of the conclusion of the argument (or any argument of identical logical form) is necessarily determined by the truth of the premises of the argument.