Open Problems from CCCG 2002

Erik D. Demaine  
*Massachusetts Institute of Technology*

Joseph O'Rourke  
*Smith College, jorourke@smith.edu*

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Open Problems from CCCG 2002

Erik D. Demaine∗ Joseph O’Rourke†

The following is a list of the problems presented on August 12, 2002 at the open-problem session of the 14th Canadian Conference on Computational Geometry held in Lethbridge, Alberta, Canada.

Boxed problem numbers indicate appearance in The Open Problem Project (TOPP); see http://www.cs.smith.edu/~orourke/TOPP/.

Great Circle Graphs: 3-colorable?

Stan Wagon
Macalester College
wagon@macalester.edu

Is every zonohedron 3-colorable when viewed as a planar map? This question arose out of work described in [RSW01]. An equivalent question, under a different guise, is posed in [FHNS00]: Is the arrangement graph of great circles on the sphere 3-colorable? Assume no three circles meet at a point, so that this graph is 4-regular. Circle graphs in the plane can require four colors [Koe90], so the key property in this problem is that the circles must be great. All arrangement graphs of up to 11 great circles have been verified to be 3-colorable by Oswin Aichholzer (August, 2002). See [Wag02] for more details.

References


3-Manifolds Built of Boxes
Joseph O’Rourke
Smith College
orourke@cs.smith.edu

A result in [DO01, DO02] may be interpreted as follows: For any polyhedral 2-manifold homeomorphic to a sphere $S^2 \subset \mathbb{R}^3$, all of whose facets are rectangles, adjacent facets either meet orthogonally or are coplanar. This raises the analogous question one dimension higher: For any polyhedral 3-manifold homeomorphic to a sphere $S^2 \subset \mathbb{R}^4$, all of whose facets are rectangular boxes, is it true that adjacent facets lie either in orthogonal 3-flats or within the same 3-flat? Very roughly, must a 3-manifold built from boxes be itself orthogonal?

References


Visibility Product Characterization
Tom Shermer
Simon Fraser University
shermer@cs.sfu.ca

Let $P$ be a polygon, treated as a region in the plane. Define (for lack of a better term) the visibility product $\text{VP}(P)$ to be the following four-dimensional set:

$$\text{VP}(P) = \{(x_1, y_1, x_2, y_2) \mid (x_1, y_1) \in P, (x_2, y_2) \in P, (x_1, y_1) \text{ can see } (x_2, y_2)\}$$

Two points can see one another if the line segment between those points is a subset of $P$. Thus VP is something like a set product capturing visibility. Determine the structure of $\text{VP}(P)$, characterize the set, find an algorithm to construct it, and determine if it has utility.

3D Orthogonal Graph Drawings
David Wood
Carleton University
davidw@scs.carleton.ca

Does every simple graph with maximum vertex degree $\Delta \leq 6$ have a 3D orthogonal point-drawing with no more than two bends per edge? An orthogonal point-drawing of a graph maps each vertex to a unique point of the 3D cubic lattice, and maps each edge to a lattice path between the endpoints; these paths can only intersect at common endpoints. In this problem, each path must have at most two bends, that is, consist of at most three orthogonal line segments (links).

There are several related known results. Two bends would be best possible, because any drawing of $K_5$ uses at least two bends on at least one edge. If $\Delta \leq 5$, two bends per edge suffice [Woo03]. Two bends also suffice for the complete multipartite 6-regular graphs $K_7$, $K_{2,2,2,2}$, $K_{3,3,3}$, and $K_{6,6}$ [Woo03]. In general, there is a drawing with an average number of bends per edge of at most $2 + \frac{5}{6}$ [Woo03]. Additionally, three bends per edge always suffice, even for multigraphs [ESW00, PT99, Woo01]. This problem was first posed in [ESW00].

References


Sailor-in-the-Fog Generalization
Alejandro López-Ortiz
University of Waterloo
alopez-o@uwaterloo.ca

The venerable “Sailor in the Fog” problem asks for an optimal search strategy for a sailor to find the shoreline when lost in a fog offshore (a version was posed by Bellman in [Bel87]). There are many variations on this problem. For example, one version...
can be rephrased as follows: Find the shortest-length path from the center of a unit disk that intersects every halfplane whose bounding line (the shoreline) supports the disk. Note here the assumption is that the distance to the shore is known. This problem was solved by Isbell [Isb57].

A conference conversation suggested the following higher-dimensional generalization: Find the shortest-length path from the center of a unit ball that intersects every halfspace whose bounding plane supports the ball. This problem might represent a diver seeking the surface.

It came to light after the presentation that this problem was posed before, in a paper by V. A. Zalgaller [Zal92], for which there is apparently no published translation from the Russian. Nonetheless, the problem remains unsolved. See [Fin01] for more information.

References

Let $c_1$ and $c_2$ be two smooth, closed, convex, planar curves of the same length, each bounding a flat piece of paper. Choose a point $p_1$ on $c_1$ and a point $p_2$ on $c_2$, and glue the two curves to each other (according to arclength) starting with $p_1$ glued to $p_2$. The resulting single piece of paper forms a shape in space called a D-form by Helmut Pottmann, Johannes Wallner, and Tony Wills. The curves $c_1$ and $c_2$ join to form a closed space curve $c$ bounding two developable surfaces $S_1$ and $S_2$. These authors ask two questions in [PW01, p. 418]:

1. “It is not clear under what conditions a D-form is the convex hull of a space curve.”

2. “After some experiments we found that, surprisingly, both $S_1$ and $S_2$ were free of creases, but we do not know whether this will be so in all cases.”

References